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contents

Sraffa and Applied Economics: Joint Production

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> "In these circumstances there will be room for a second, parallel process which will produce the two commodities by a different method..."[1].

1. THE CLASSICAL METHOD

Is there a field of application for Sraffa's theory? There can be no doubt that Sraffa's work has so far almost exclusively been interpreted as a critique of the theory of value. As a consequence of discussions which were decisively influenced by Sraffa, the economic profession has in the last 15 years begun to accept the fact that the aggregate production function is an illegitimate tool of analysis. Wider implications for the neoclassical school are still being debated. Within the classical approach, the traditional form of the labour theory of value was superseded. But the empirical applications were few; they really consist only of some attempts to calculate wage curves. These attempts were necessarily inconclusive as far as the debate about capital theory was concerned while they did shed some light on the analysis of technical progress[2].

* I should like to thank P. Garegnani for valuable and extensive comments on an earlier version of this article.

[1] P. SRAFFA, *Production of Commodities by Means of Commodities*, Cambridge, CUP, 1960, p. 43.

[2] G. MARZI and P. VARRI have calculated wage curves from input-output data of the Italian economy. W. KRELLE has done the same for the Federal Republic of Germany (G. MARZI-P. VARRI, *Variazioni di produttività nell'economia italiana: 1959-1967*, Bologna, Il Mulino, 1977; W. KRELLE, "Basic Facts in Capital Theory. Some Lessons from the Controversy in Capital Theory", *Revue d'Economie Politique*, 1977, vol. 87, pp. 282-329). Such wage curves, if calculated and compared for different time periods, do not prove anything about the switch points between wage curves belonging to different technologies which compete in a given moment of time, but they do give indications as to the prevalent form of technical progress which went on between periods. According to Marzi and Varri the maximum rate of profit seems to have fallen in Italy as if mechanization (see B. SCHEFOLD, "Different Forms of Technical Progress", *The Economic Journal*, vol. 86, 1976, pp. 806-819) had increased but there is no presumption to expect this result to hold in general.

I believe that Sraffa's theory has a wider scope and can provide a framework for much of modern applied economics. This does not imply a universal transformation of the field, since the methods employed by good applied economists can sometimes be justified better on the basis of classical rather than neoclassical theories while both predict similar results.

Input-output analysis provides a good example of an important tool of applied economics based on a classical methodology. Although Leontief has taken pains to emphasize the compatibility of his conception with the neoclassical tradition[3], it is quite clear that it fits in much better with the classical. For it is the point of input-output analysis to regard the methods of production in use as given independently of relative prices, to derive conclusions from the technological interdependence and to consider the influence of changes in relative prices and in technical progress on the coefficients of the input-output structure only subsequently.

It is a fundamental principle of classical economics that it *separates* a) the determination of outputs, b) the determination of distribution and c) the analysis of the relations between the distributive variables and between them and relative prices. It is this which makes the classical approach a better basis for applied economics. It provides direct links between the essential magnitudes in the system (the "short chains of reasoning", which Marshall was looking for)[4], while the countless relations of interdependence in a general equilibrium system are a poor guide to any application. Input-output systems also serve to analyse "interdependence", but by treating the methods of production in use as given and the determination of prices and distribution as separate issues, the analysis of the dependence of activity levels on final output becomes manageable even if feedbacks between different industries have to be taken into account.

The strength of the classical approach is most visible in dynamic analysis. While it is hard to represent the evolution of the economic system even without technological change as a sequence of Walrasian equilibria empirically, there are now several quite successful large econometric models which capture the process of macroeconomic dynamics and of structural evolution by means of a combination of an input-output system for the representation of technology with a macroeconomic model for the representation of the evolution of effective demand in its interaction with distribution and government policy, and a demand model based on *aggregate* demand functions which may be differentiated according to socio-economic criteria. My education in classical

[3] W. W. Leontief, *The Structure of American Economy 1919-1935* (1941), sec. ed. 1951, White Plains, International Arts and Sciences Press, repr. 1976, p. 37.

[4] P. Garegnani, "The Classical Theory of Wages and the Role of Demand Schedules in the Determination of Relative Prices", *American Economic Review*, May 1983, pp. 309-313.

economics proved very useful for the understanding of the true functioning of these models which one encounters in research on the economics of energy. However, one must admit that the eclectic character of most econometric models does not allow an unambiguous interpretation of their theoretical background.

Input-output analysis is not the only area which could benefit from an interpretation along classical lines, but research into the empirical usefulness of modern classical theory has, among other things, been impeded because its proponents have tended to focus on the critique of the neoclassical theory of capital and distribution instead of on positive contributions. The present paper is concerned with some preliminary considerations which might lead towards an application of Sraffa's theory of joint production by discussing joint production input and output tables and by analysing the meaning of a possible underdeterminacy or overdeterminacy in the system if the number of processes used is not equal to the number of commodities (goods with positive prices) produced.

Other tools of the classical theory which had been used by the classical economists themselves and could also be applied by modern economists are not being discussed here but a parallel paper will deal with the classical analogue of the Marshallian supply curve.

We assume that distribution determines a uniform rate of profit (alternatively: a hierarchy of rates of profit). Demand for consumer goods is treated very simply as emanating from given social needs. The relationships between those needs are thought to reflect complementarity rather than substitutability. The needs evolve with the growth of wealth in different segments of the population, but corresponding Engel curves do not have to be considered, since we are dealing with a given long period position. We assume, however, that there may be different domestic processes of production to fulfill the same needs, and that their choice depends (if we abstract from habits, taste, ignorance, etc.) on the cost of providing the corresponding services, hence on relative prices. Goods which are close substitutes (where the rise in the price of one leads to an increase in the consumption of the other because they fulfill the same need) are then to be treated as the same commodity if they can only be distinguished according to taste and not as alternative means used in different methods of production for the fulfillment of the same need. This manner of treating demand has been successful e. g. in the explanation of changes in energy consumption. It relates to the classical view of the matter and will turn out to be helpful for the understanding of Sraffa's theory of joint production.

2. JOINT PRODUCTION AND ACCOUNTING

There seems to be an unsurmountable gap between the treatment of joint production in economic theory and in the literature on business administration. Models of general equilibrium and the von Neumann model determine prices of joint products but this determination does not seem to provide definite rules for those working in the field of business administration, for the latter regard the theory of prices of joint products as largely indeterminate and discuss "practical" rules for the setting of prices in diverse circumstances. Of course, no theory can be expected to provide a ready-made set of rules for pricing, given the complexity of every-day life. But one can show why classical theory may serve as a background to explain some accounting procedures and why these procedures fail in specific cases; the "gap" may thus be bridged.

The first mistake of the prevailing economic theory, most clearly visible in the von Neumann model, consists in the assumption that a definite complete list of the goods to be used and produced by any method of production can be established for each process independently of the others. If this were the case, few environmental problems would arise. We do not have *complete* knowledge of what the smoke of factories consists of; far less do we know about the synergetic external effects of different processes of production. In reality, the identification of those goods which are to be the objects of economic planning is the first step in the practical transformation of the material world which we call "production". The goods so selected are potential commodities; everything else is ignored until external effects are being felt. Sraffa's analysis of joint production therefore starts from the system of commodities and processes which are actually used and considers the use of alternative processes and the introduction of new goods only subsequently and one by one.

All production is joint production as far as "goods" are concerned. The traditional emphasis on single production of commodities in economic theorizing is not simply due to the fact that the theory of single product industries is much simpler than that of joint production; rather, it reflects economic practice according to which production originally is a purposeful activity, in general directed at obtaining *one* specific good which may be sold as a commodity. Goods which are produced jointly are usually turned into commodities only later in order to increase the profitability of the process. Although there are exceptions to this rule, e. g. in transportation, where any vehicle is introduced to carry many products, it is surprising how often one finds a single purpose at the origin of what later becomes a multiproduct industry. Even the ships which produced trips from Spain to the West Indies and back jointly were first used only to import treasure, not to export cloth.

The economic growth of industries seems to be characterized similarly by an initial disregard for joint costs; e. g. the infrastructure is often taken care of properly only after the industries have been set up. This is also the case for the joint costs of a national economy and even for those of the world which increase faster than our recognition of the global interdependence of many environmental problems. The reason is that the planning process starts with a simplified view of the world — a simplification which often implies violence.

The representation of joint production in economic models ignores this dynamic element. In consequence, the treatment of joint production in general economics is closed, but it does not lead to a view of the sequence of events in the evolution of joint production processes. By the same token, it lacks specificity; one does not find a morphology of joint production in economic theory.

The opposite picture emerges at the level of the theory of the enterprise. There does not seem to be a generally accepted theory for the determination of prices in multiple product industries in the field of business administration, but there are essentially two approaches; diverse variants are discussed for different applications of each to different industries in the literature. *Either*, one tries to *set prices*. To this end, it is thought necessary to ascribe costs to individual products by means of splitting up overheads, depreciation etc. according to various rules, and to add a normal profit in accordance with a target rate of return or some similar notion. According to this theory of full cost pricing in its several variants, firms are able to administer prices in imperfect markets freely, but within limits, so as to guarantee a satisfactory return at normal levels of capacity utilization. The principal drive of competition ought to be expressed not in higgling about prices but design, marketing, product innovation etc.[5].

Alternatively, it is being thought that market prices are given within a narrow range even in imperfect markets and under modern conditions, because there is competition between products which are close substitutes and because the entrepreneurs almost always have some notion as to what the traffic will bear. One then asks *where profits are made*, i. e. how the sales proceeds contribute towards the covering of the expenses and the profits of the various decision taking units within the firm so as to obtain a measure of their efficiency and a guide for future investment policies. Methods for ascribing profit contributions are again diverse. It is sometimes thought best to ascribe costs to that unit of an enterprise where these costs appear as direct costs. For e. g. the overheads of the division responsible for sales may be direct costs to the central management of the firm. One can thus go some way towards the reduction of overheads to

[5] See J. M. Blair, *Economic Concentration; Structure, Behavior and Public Policy*, New York, Harcourt Brace Jovanovich Inc., 1972.

direct costs and restore simple rules for profitability, but the limits of the approach are clearly visible, e. g. in the case of the overheads due to the intertemporal use of fixed capital. It is sometimes being said that business accounting is an art rather than a science[6].

But it is clear that, on Sraffa's assumptions, prices *are* determined. The discrepancy between the uncertainty as to the proper rules of accounting and the uniqueness of prices in theoretical systems requires an explanation.

First of all, it may be shown that the main basic rules of accounting all come to the same thing and are consistent with the theoretical solution based on long run prices, provided there is an equilibrium. Consistency then means that if the accounting rules are applied to individual or groups of processes in a Sraffa system, they imply accounting prices and ascriptions of costs which do not lead to other prices than the Sraffa prices themselves. Some examples may suffice to show this.

If it is being asked how the costs of two joint products are to be ascribed to the individual products at given market prices, the answer simply is that, looking at the process where the products are being produced jointly, any splitting up of the costs will be consistent with Sraffa prices, provided the market prices are equal to Sraffa prices, and the total costs (including normal profits) are equal to the sales proceeds. The arbitrary rule for splitting up the costs (e. g. according to weight or calorific value) is simply irrelevant to the "macroeconomic" determination of prices. If the same two products happen again to be produced jointly in another industry in different proportions as Sraffa suggests, a different ascription of costs according to any such rule will lead to a different result in the other industry, but the discrepancy does not matter since the purpose of the accounting procedure is only to provide a basis for calculation for the price to be set or the profit contribution out of equilibrium while we suppose that equilibrium prices have been determined "behind the back of the producer". (If the two processes were used within the same firm, a different accounting procedure would be used).

Alternatively, there is sometimes a distinction being made between main products and subsidiary products. It is being asked what the correct pricing of the subsidiary product is, given the prices of the main product. The price of the total output of the subsidiary product is calculated by deducting the proceeds from selling the output of the main product from the total joint cost of production. The result will be the Sraffa price of the subsidiary product in equilibrium conditions. In disequilibrium conditions, the calculated price of the subsidiary product may be anything and even negative, for if e. g. the price of the main product is very high, its cost

[6] E. S. Schmalenbach.

of production will be more than covered even if the subsidiary product is given away free.

Thirdly, the "marginal method" may be mentioned. Here it is being asked what the prices of two products should be if they can be produced in varying proportions. The rate of transformation is said to define the relative prices of the outputs. But this is equivalent to a calculation of prices, given a shadow rate of interest, by considering the equation of the process in actual use and one resulting from a (small and continuous) variation of the inputs and outputs at the margin. It is clear that one thus obtains two Sraffa processes — a result which is consistent with Sraffa prices on the assumption that the shadow rate of interest equals the general rate of profit.

In this way, some of the accounting rules may be rationalized on the basis of Sraffa's theory of prices in joint production systems: the pricing rules lead to "equilibrium" prices under "equilibrium" conditions. This is in itself not surprising. The explanation is in fact of interest only to the extent that the underlying theory is convincing. That this is the case will be argued by illustrating its explanatory power first in cases where it applies directly, second in cases, where there is a "disequilibrium" and where it has to be shown how an equilibrium is established in a real process of adaptation.

3. WHY "SQUARE" JOINT PRODUCTION SYSTEMS?

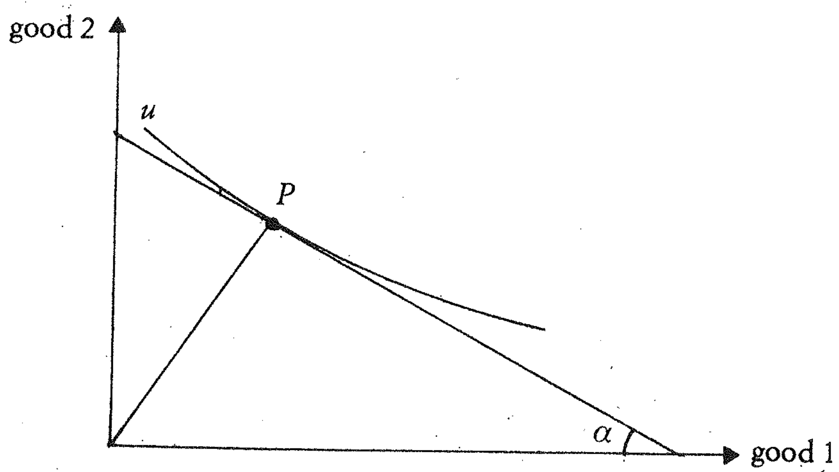
The strange and crucial assumption which allows one to determine the equilibrium seems to be that the number of commodities should be equal to that of processes. Economic theorists are sometimes puzzled by this because they do not know that this assumption will be fulfilled for the processes actually used and the commodities sold not only in Sraffa systems but also in von Neumann type systems with probability one, as I have shown elsewhere[7]. The proof is based on the assumption that a vector of final demand, i. e. the composition of output, is given. It is then shown that among all von Neumann systems capable of producing that vector at a given rate of growth (equal to the rate of interest) a system will be chosen for which the equality of the number of commodities with positive prices is equal to that of processes used except for a set of systems which is of measure zero in the set of all possible von Neumann systems of a given order.

The accountants and applied economists do not encounter an equality

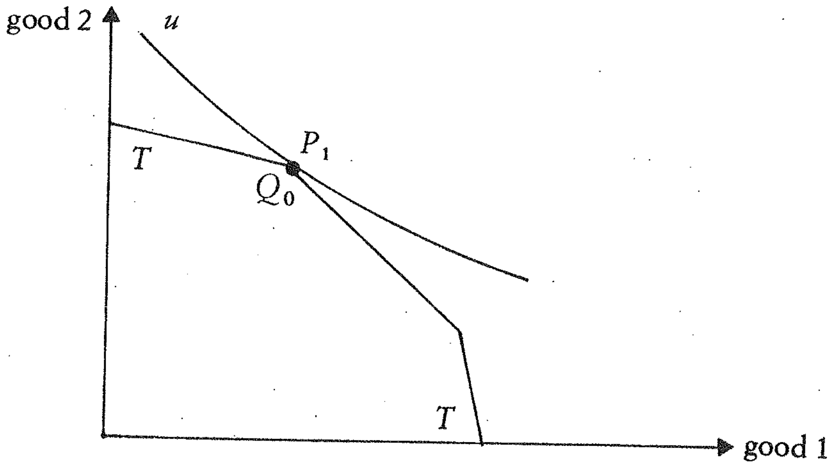
[7] B. SCHEFOLD, "On Counting Equations", *Zeitschrift für Nationalökonomie,* vol. 38, Heft 3-4, 1978, pp. 253-285.

of the number of processes and commodities in practice. The ordinary theorist, even if he should accept the assumption as an initial hypothesis, has never been told how these conditions reproduce themselves in the process of economic development. And none of them recognizes the usefulness of considering a system of prices of production, since it seems to be generally agreed that whatever happens to prices under changing conditions must be explained in terms of "supply and demand".

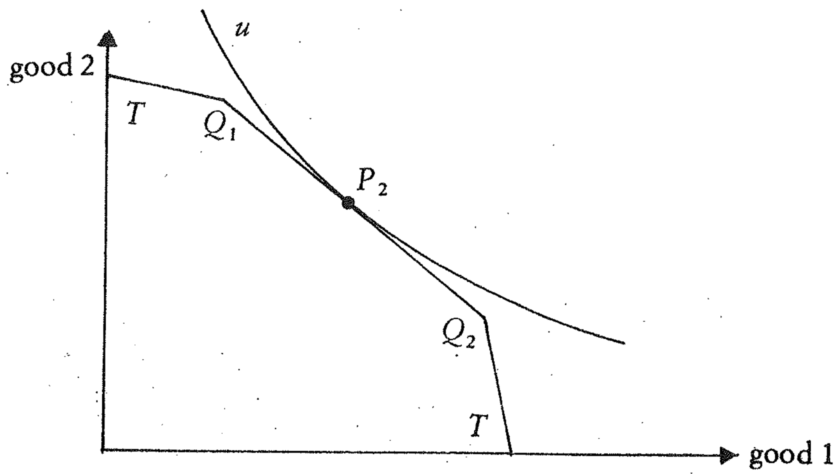
As a matter of fact, "square" systems do not necessarily result from general neoclassical assumptions. For if two commodities are being produced by one and only one process in rigid proportions, the marginal rate of substitution (as given by the slope of the indifference-curve u in the point of equilibrium P in the following diagram) will determine the relative price with $p_1/p_2 = tg\,\alpha$ unambiguously:



If there are, on the other hand, many potential processes of production which span the production possibility surface, there will be two cases: Either, a corner of the transformation "curve" (represented by a convex polygon T in the plane) will be the equilibrium point P_1 using *one* process Q_0:



or the equilibrium may be a point P_2 on a segment spaned by *two* processes Q_1 and Q_2:



Neither type of solution P_1, P_2, can be regarded as a fluke case since a small perturbation of the technology with given indifference curves (or of the indifference curves, given the technology) will leave the essential properties of the equilibria unchanged: there will be more commodities (two) than processes used (one) in P_1 while the number of processes used equals that of commodities (two) in P_2. The coexistence of several processes in actual use to produce the same set of commodities at positive prices is therefore not excluded and not exceptional in neoclassical theory but an excess of the number of goods with positive prices over the number of processes used is also not unlikely.

Casual observation indicates that we often encounter joint production processes yielding several commodities while additional processes producing the same commodities are not visible without close scrutiny. Common sense, the doctrines of accounting and neoclassical theory therefore all seem to contradict Sraffa's hypothesis of a necessary equality of the number of commodities and of processes used. We therefore have to ask from which assumptions the hypothesis follows and why it might be useful.

4. EXAMPLES FOR THE USE OF "SQUARE" SYSTEMS

I shall first illustrate the usefulness of the approach, taking the number of processes equal to that of commodities. Phenomena which might be classed under the general heading "supply and demand" can here be explained more specifically.

Consider, for instance, a Sraffa system in which the last process produces natural gas by means of labour and various commodities, and in which the second but last process produces coke and gas from coal by means of basics and labour. Assume that the two kinds of gas are substitutes. In such a system it may happen that — contrary to what happens in basic single product systems — a rise in the cost of gas production (due e. g. to an increase of labour requirements) leads to a *fall* of the price of coke and of all other prices if gas is a non-basic and coke a basic. The reason is, simply, that the rise in the price of gas contributes towards the expenses of the coke industry so that the price of coke may be lower and this, being the price of a basic, leads to a fall of all other prices, provided the remainder of the system works insofar like a single product system[8].

Similarly, if we take a step towards considering a situation in which the number of commodities exceeds that of processes and assume that a new consumption good results from known processes in a Sraffa system, the selling of the new commodity (which had previously been a useless by-product) will generate a revenue such that all prices fall, if the new commodity is produced in an indispensable process, but some prices may rise and some fall if the process is not indispensable. If, in the previous example, the second but last process does initially not produce any gas, the introduction of gas in that process (which is indispensable) will allow to reduce the price of coke and hence of all basics. But if a by-product emerges as a new commodity in the last process which now produces natural gas and some other non-basic, e. g. petroleum, the price of natural gas falls in the last process, the same happens in the coke-producing process, and this forces the price of coke to rise so that all other prices will rise.

Generally, the criterion for the choice of technique in the presence of joint production cannot be that of reducing individual prices in terms of the wage rate as in the case of single product systems. The reduction of the price of natural gas is directly beneficial only to gas consumers while the rise of the price of coke and all other basics affects all consumers.

These arguments rest on the assumption that a rate of profit (or, in practical applications, a hierarchy of rates of profits) may be regarded as given for the analysis of questions related to the choice of technique, e. g. in the area of energy economics. The applied economist is used to making this assumption in the appropriate context and he will not be surprised that joint production will lead to curious effects such as the ones which I

[8] The example is spelt out in numerical terms in B. SCHEFOLD, "Multiple Product Techniques with Properties of Single Product Systems", Zeitschrift für Nationalökonomie, vol. 38, Heft 1-2, 1978, pp. 29-53.

have described, provided the system of interdependent processes is small enough to be understood in intuitive terms. I should say that he then argues along the lines characteristic of classical economics. But research into the effects of changes in methods of production, long run demand conditions and distribution on interdependent joint production systems with a large number of equations is little known and it must be admitted that it has so far only rarely been undertaken with a view towards practical applications. Empirical input-output analysis has not yet been extended to take the scientific progress made by Sraffa into account.

The examples given indicate, however, that the approach might be fruitful because it allows to analyse effects of joint production on the economic system as a whole within a unified framework. Such effects cannot be understood satisfactorily by means of conventional approaches. For if the applied economist uses partial equilibrium analysis, the effects on individual industries and prices can be described but the global effect is lost whereas the use of input-output matrices in their present form is based on a preliminary elimination of joint production by means of aggregation procedures which are to some extent arbitrary and conceal specific effects such as that of a fall in one price causing all other prices to rise.

We now come to the counting of equations. It will be seen that the consideration of the "disequilibrium" conditions in which the number of equations is not equal to that of commodities provides the foundation for a better understanding of classical theory as well as of the diversity of accounting rules in disequilibrium situations.

5. COUNTING OF EQUATIONS I: THE CASE OF OVERDETERMINATION

It is easy to see what happens if the number of processes seems to exceed that of commodities. Some processes will then be more (and some less) profitable than others. Surplus profits may be consolidated as rents and accrue to those who control the causes for the permanence of the multiplicity of methods. The incomes of owners of land or of a patent are based on property rights; they can — but they need not be — identical with the entrepreneur who receives the ordinary profits. The surplus profits are temporary in the case of technical progress. Sraffa emphasized yet another case: the rents of obsolescent machines which are not being produced any more so that their capital cost need not be accounted for and the cost of production of their products reduces to that of raw materials (with normal profits) and labour.

But I want to use a *crude* example to show that one can go still further and include "domestic processes of production":

$$
\begin{aligned}
&CH \,\&\, C \,\&\, L \rightarrow WW\\
&CH \,\&\, O \,\&\, L \rightarrow WW\\
&I \,\&\, AH \,\&\, L \rightarrow RH \,\&\, WW\\
&P \;\;\&\;\; L \;\rightarrow RH
\end{aligned}
$$

In the first process central heating and coal and labour are used to produce warm water in houses, in the second central heating and oil and labour are used to produce warm water in houses, in the third insulation and additional heating and labour are used to produce a renovated house and warm water and in the fourth paint and labour are used to produce a renovated house. If we assume the prices of inputs *CH* (central heating), *C* (coal), *L* (labour), *O* (oil), *I* (insulation), *AH* (additional heating), *P* (paint) to be known on the assumption that the outputs are non-basics, we have four equations and two unknowns: prices of warm water (*WW*) and room heating (*RH*).

Everybody knows that such situations of overdetermination are frequent and that they may persist for some time. Among the causes, first habit and ignorance are certainly important. It has been estimated that if the main devices for saving energy which were known, in partial use in 1975 and which would have been profitable at 1978 prices had been used generally in 1975, energy consumption would have dropped by a third[9].

Second. But it is also possible that prices of inputs rise to make prices of output match. Thus, the price of coal may rise to match the price of oil, and the rise in the price of coal may be engendered by a rise in the wages of miners. Part of their wages then has the character of a rent. A spurt of demand may drive up all the prices of materials for insulation temporarily. Such differentials are assumed to get eliminated through competition in the Ricardian long run unless the cause for the differential is permanent and the corresponding rent can be appropriated.

Third. The prices of production are centres of gravitation. All costs, including the cost of insulation, tend to get reduced to the cost of production which is assumed to be given. The point is that it then becomes possible to analyse the process of disequilibrium with reference to an equilibrium defined by prices of production and hence an equality of the number of processes and "commodities" where the latter include various objects which receive a permanent rent such as lands, patents, workers with particular talents etc. Since the formation of habits and property

[9] Deutsche Shell Aktien Gesellschaft, "Perspektive der Energieversorgung", Oktober 1980.

rights are among the causes for lasting differentials, the analytical task involves questions of political economy. One has to decide which techniques will turn out to be "socially necessary". The accounting rules will reflect differences of the institutional set-up and thus conceal the fundamental similarity of different phenomena of overdetermination. There is again an equality between the number of positive prices and the number of processes in the pure case of Ricardian rent, if different kinds of land with positive rents are counted as so many "commodities".

6. COUNTING OF EQUATIONS II: THE CASE OF UNDERDETERMINATION

The converse case where there seem to be not enough processes is perhaps the more interesting. There can be no doubt that there are many joint production processes in industry with little or no possibility for a variation in the proportion of the outputs produced, and without additional processes being visible which might help to determine the prices of production simultaneously with the first according to the rule of counting of equations. Here, neoclassical tradition as well as the textbooks on business administration suggest that we rely on "demand" but Sraffa argues that, in such cases, conditions of "demand" will generally ensure that further processes will be used which are distinguished from the first by different proportions in which the commodities in question are used as outputs *or as inputs*, for otherwise the commodities could not be produced and used in the combination socially required.

In the simplest case the underdeterminacy of the price system is made to disappear by letting superfluous commodities disappear. If a main product cannot be produced as a commodity without also producing some by-product in excess of the demand from other producers or from consumers, the by-product cannot be a commodity with a positive price; hence it is not part of the system and does not cause an underdeterminacy. (In applied theory, it is not asserted that goods are free if and only if they are overproduced. In particular, if the by-product is a waste which must be removed at some cost, the cost of its removal has to be regarded as an *input* to the production of the main product. In either case there is no difficulty to the theory of price formation).

There can be no question, however, that the indeterminacy of "too few" processes being present does arise in a less trivial manner in phases of transition. The difficulty seems to be the greatest if the proportion in which joint outputs are being produced cannot be varied, i. e. if there is "rigid" joint production. Let us consider an example of such a disequilibrium: Nuclear power stations cannot vary their output of electricity quickly for technical reasons. The addition of a nuclear power station creating an adequate supply of electricity during daytime therefore leads to an excessive supply of electricity at night. As a result, the market price for electricity produced at night will fall. The classical approach rules out as irrelevant attempts to determine the extent to which the price of excess electricity will fall by means of considering the subjective element of demand in isolation. The clue to the problem is found in the observation that the fall of the price creates an opportunity for introducing new processes which use that electricity (otherwise the price might fall to an indefinitely low level). An example is storage heating which uses electricity produced at night to generate heat during the day which may also be produced directly by other means, e. g. central heating.

If these new processes are not sufficient to lead to a match of supply and demand, a process will be required which ensures that the correct proportion according to social needs is reached. In the circumstances, it is the direct transformation of electricity produced at night into electricity produced during the day by means of stations which use electricity produced at night to pump water, and this in turn is used to produce peak load electricity during day time.

We thus have two essentially different solutions to solve the indeterminacy in the case of rigid joint production. Symbolically, the first solution looks as follows:

$$\begin{aligned} NPS &\rightarrow ED \,\&\, EN \\ EN &\rightarrow H \\ CH &\rightarrow H \end{aligned}$$

The nuclear power station (*NPS*) produces electricity during the day and electricity during the night (*ED* & *EN*); electricity during the night is used to produce heat (*H*). On the assumption that the cost of *NPS* is known and that there is an alternative process which produces *H* and determines its price at given costs (central heating *CH*), the prices of *EN* and *ED* are determined by the first two equations. It turns out that one of the outputs has its price determined as an input.

The more direct and more elegant second solution is provided if there is a separate process transforming *EN* into *ED* by means of pumping stations:

$$\begin{aligned} NPS &\rightarrow ED \,\&\, EN \\ EN &\rightarrow ED \end{aligned}$$

For simplicity, additional inputs of known costs such as labour are not shown. Here, one output can be transformed into the other.

The indeterminacy may therefore be solved on the input side because the outputs have alternative uses as inputs in other processes. Or it may be solved because a process links inputs and outputs directly as in $EN \rightarrow ED$. Or (this is the third and most conventional solution), there may be a second process which produces one (or both) of the commodities as an output. E. g.:

$$NPS \rightarrow ED \ \& \ EN$$
$$C \rightarrow ED$$

Here, electricity during day time is produced by means of old coal-fired power stations which are still used to supplant nuclear electricity generation for peak-load production. In conventional terminology, *EN* would then be regarded as a by-product in the first process and its price would be explained accordingly.

Finally, there may be a case specially emphasized by Sraffa: a second process produces positive amounts of both commodities in different proportions. Examples of this with *rigid* joint production are perhaps not common, but there may be a variability of outputs which leads to the same result, as it turns out, since small variations of output which match demand may be represented by a linear superposition of two neighbouring processes.

The last possibility is illustrated by power stations which produce hot water and electricity jointly and where there is some substitutability between outputs. For instance, coal may be used in fluidized bed combustion (*FBC*) to produce hot water (*HW*) and electricity (*E*):

$$FBC \rightarrow \mathrm{HW} \ \& \ E$$

Since the proportions are variable, we may imagine that two processes are used which differ slightly so as to satisfy demand:

$$FBC_1 \rightarrow HW_1 \ \& \ E_1$$
$$FBC_2 \rightarrow HW_2 \ \& \ E_2$$

In the limit, the two processes may fuse into one and one obtains the usual marginal condition.

The last solution seems to lead back to Marshall and the neoclassical method where a substitutability of the outputs which are being produced jointly is assumed. Demand as derived from utility determines relative prices and outputs. Whilst some influence of preferences cannot be denied, the emphasis of the classical approach is different, however, because preferences are not the only social force which are admitted as influencing the composition of output. The concrete examples which I have chosen illustrate the political element in the determination of methods of production and hence of the proportions in which the outputs are being produced to fulfill *social needs* through adaptations of technology.

7. COUNTING OF EQUATIONS III: A MORE SYSTEMATIC PRESENTATION FOR A SPECIAL CASE

The abstract nature and the very generality of Sraffa's approach seems to have prevented theorists from attempting a systematic survey of those links between multiple product industries which lead to an equality of the number of processes and commodities in equilibrium. The preceding examples have shown that joint production of the same commodities on the outputside by means of different methods are not the only constellation which allows the equilibrium condition to be fulfilled. It follows that the transition from single product systems to joint production through the emergence of by-products and the discovery of new, parallel processes which also produce that by-product is only one among several adjustments leading to the dynamic correction of an overdetermination or an underdetermination of prices in consequence of an excess or a deficiency of the number of processes used with respect to the number of commodities produced.

However, because of the traditional emphasis on it, because of its analytical simplicity and because it leads to a straightforward comparison with the neoclassical approach, the case will now be considered in greater detail. The emergence of further products from an "original" single product process leads to shifts of relative (market) prices which allow new links with other existing processes or the introduction of recent inventions to be established such that a new system of prices of production is formed. A by-product will, it is assumed, be needed in definite quantity, either for consumption or as an input to other processes. Then, two main cases may be distinguished:

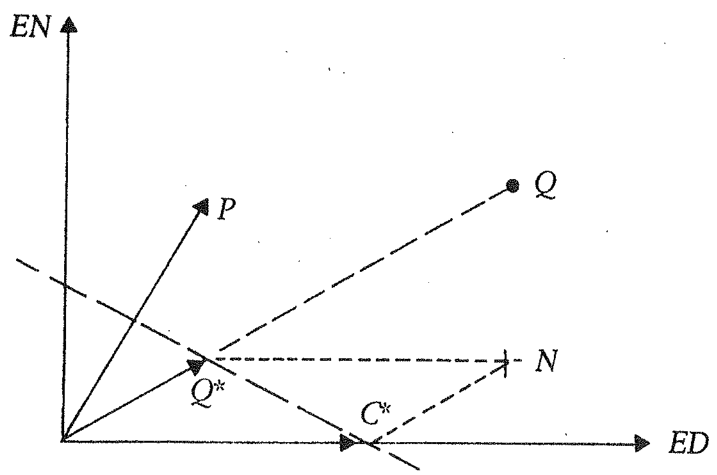
1) Either the original process, run at its original level, provides more than enough of the by-product so that this will not acquire a price and become a commodity.

2) Or the original process, run at its original level, does not provide enough of the by-product. It is then either expanded to the level required by the need for the by-product. In consequence, one should expect the original product to be overproduced and to receive a zero price (case 2a). However, the cheapening of the original product, with its established market, may also lead to the discovery of new uses for it in industrial (case 2b) or in domestic processes of production (case 2c). Or, finally, the original process is not expanded; the excess demand for the by-product will then have to be satisfied by an additional process (case 2d).

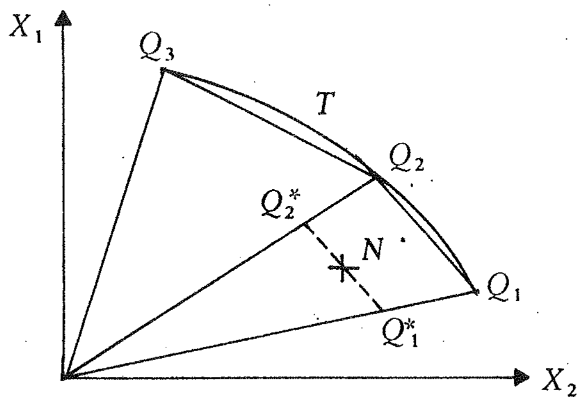
It is clear that analogous distinctions 1a, 1b, 1c, 1d for the first main case are also conceivable, although the existence of overproduction accompanied by a zero price seems most plausible then, because the by-product has never been marketed before. An example of 1a is provided by CO_2 which is a waste product, produced, among other things, jointly with cement, but also absorbed, therefore used as an overproduced input, in the drying of cement.

Insofar as the introduction of a differentiated tariff for electricity produced during the day (*ED*) and the night (*EN*) is older than nuclear power stations, the examples of the preceding sections do not fit into this framework. But insofar as *EN* may be regarded as a by-product of the production of *ED*, of which there is excess production, case 1 can be further illustrated: the case 1b may be identified with $EN \longrightarrow ED$ (pumping), 1c is exemplified by $EN \longrightarrow H$ (domestic storage heating) and the production of peak-load electricity $C \longrightarrow ED$ corresponds to 1d.

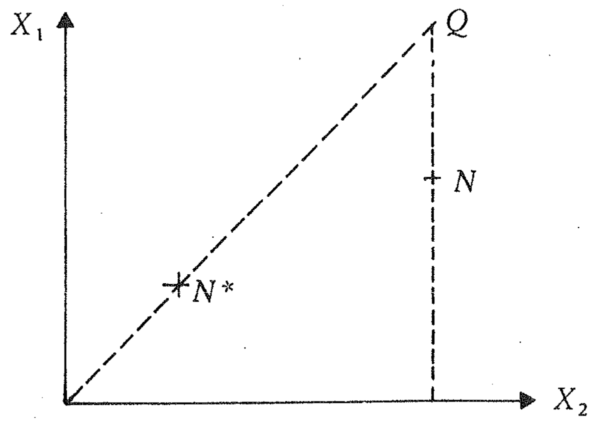
The last transition may be represented graphically in the following diagram showing the outputs of *EN* and *ED* on the axes. *Q* is the production of electricity by means of *NPS* alone such that the needs (point *N*) for *ED* are satisfied; *EN* is then overproduced. An equilibrium corresponding to case 1d is reached, if *NPS* produce only Q^* and coalfired power stations produce the amount C^* of *ED*. If both technologies happen to have the same input costs (including profits and wages), *P* will be the vector of relative prices for *ED* and *EN*.



Similar diagrams are obtained for the other cases. The next diagram shows how the possibility of a continuous substitution on the output side for outputs X_1, X_2 ("transformation curve" *T*) is to be replaced by a finite number of linear segments so that two (Q_2^*, Q_3^*) of the corresponding activities Q_1, Q_2, Q_3 will be used to satisfy given needs N. The transformation curve will by itself consist of linear segments, if discrete linear single product activities have constraints which lead to joint costs (e. g. space in a warehouse where various commodities are stored). The case of continuous substitution was exemplified above by electricity generation with fluidized bed combustion. The relative price is determined by the rate of transformation, but it is not necessary to introduce the rate of substitution derived from utility functions.

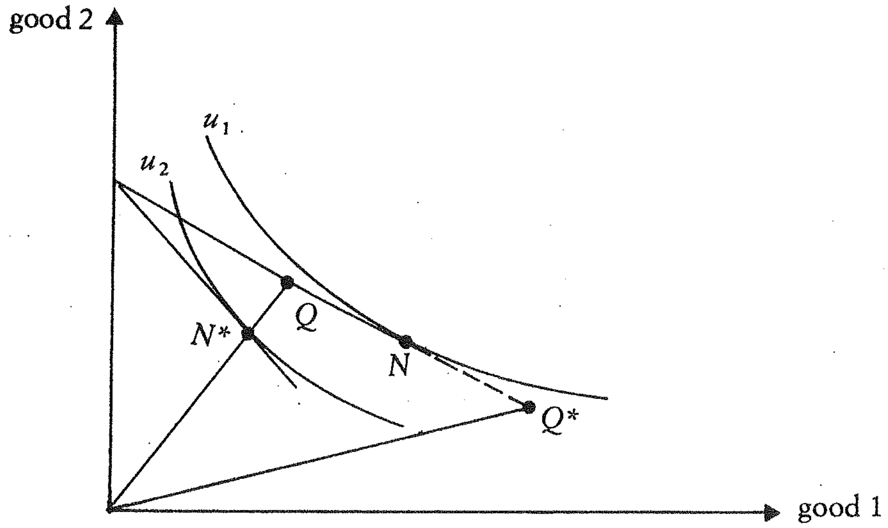


If joint production is rigid (the transformation curve T reduces to a point Q), if no alternative methods for the production of X_1 and/or X_2 are available and if X_1 and X_2 are pure consumption goods (so that there can be no determination of relative prices on the input side), one commodity is necessarily overproduced with respect to needs N and receives a zero price while the production price of the other is determined by cost. The one case resulting in an indeterminacy is the most unlikely and occurs only if, in addition to the preceding conditions, needs N^* happen to be in the same proportion in which the rigid production of the two goods takes place:



The theory thus asserts that given social needs can and will in the long run be satisfied through modifications of the system of production. By the same token, the theory denies that discrepancies between the production of original products and by-products on the one hand, and the needs for them on the other, will be matched through adaptations of the needs themselves, i. e. that the needs are so price elastic that changes in *relative prices alone* are sufficient to lead to a permanent adaptation of demand to supply. But such an elasticity of demand, apart from not being necessary according to the approach presented here, is not even plausible on neo-classical assumptions since the original product and the by-product cannot be expected to be substitutes for the same need — if they were, they could for most purposes conveniently be treated in terms of a single commodity. In fact, people are more likely to find new uses for *EN*, to convert *EN* (directly or indirectly) into *ED* or to produce part of *ED* without joint production of *EN* rather than to change their habits and to cook at night only because *EN* is cheaper than *ED*.

To repeat the argument in geometric terms: if needs *N* are not in line with a given process of production *Q*, it is not expected that a change in relative prices will cause needs to shift to some point N^* (resulting e. g. from a rise in p_1 with income and substitution effects and depending on indifference curves u_1, u_2) but that other processes (including disposal) will allow to adapt production to demand (as in the diagram, for example, a second process Q^*):



The consumer's sovereignty may thus be posited in the form of rigidly given needs and yet be realised through changes in technology, and this will result in an equilibrium with probability one in a Sraffa system, as was proved in the article "On counting" referred to above and has been shown here in a dynamic context by means of more intuitive methods. (In this paper, the absolute levels of different needs have to be regarded as given because the levels of employment, of wages and profits are given while the theory of steady growth starts from an increase of employment at a given rate, constant returns and from given rates of profits and of wages, hence also from *relative* levels of needs).

The few general hypotheses which have been advanced by economists to explain long term transformations of needs (in particular Engel curves) easily fit into this framework. More importantly, consumption by households may be viewed as the provision of services to fulfill a need by means of domestic processes of production. Insulation as an alternative to heating for the provision of the service "warm house" is a case in point (section 5). It has been observed that the growth of industrial production tends to increase the scope for commodity production at the expense of the domestic provision of services. Preferences for commodities do not seem to be sufficient for the explanation of this change which affects the behaviour of consumers fundamentally.

8. REVIVAL AND DEVELOPMENT OF THE CLASSICAL METHOD

In order to analyse the causes which modify the evolution of an economic system over time, the classical system stresses that technology should initially be regarded as given and proposes to consider the various influences leading to its modification only subsequently and one by one. The gradual rise in the consumption of electricity, for instance, leads to decisions concerning the choice of the power stations, this creates after some time an excessive electricity supply at night, and to this various answers can be and are found. The apparent underdetermination of relative prices of outputs is then turned into an overdetermination in that many methods compete for applications and, to the extent that they are actually used, give rise to surplus profits and losses. It is, perhaps, the most remarkable result of this enquiry that the apparent underdetermination of prices which seems to contradict experience, which so much worries the accountants and which leads to the call for a determination of relative prices by means of the neoclassical theory of demand (rate of substitution), may in fact only be a reflection of a lack of perspicacity: in reality, there are often many more processes than commodities in actual use; they — or, rather, the entrepreneurs using them — are rivals trying to establish monopolies and to defend what ought to be temporary rents. The energy producing subsystem is a case in point, for the examples presented in the previous sections all coexist in reality and are expressions of competitive forces in that sector of the economy. If attention is focused directly on a "final equilibrium" in which the technology mix exactly corresponds to demand as derived from preferences, the other forces influencing the chain of events, including what eventually counts as "preferences", will be lost from sight.

As in the examples above, substitution possibilities on the output side play a role at the same time as those between inputs. There is the well known variability of mutton and wool (sheep may be slaughtered earlier or later) but there are also the substitution possibilities on the input side: sweaters can be made from wool and from cotton. Fashion mainly determines the extent to which the substitutability is realized. The possibilities of substitution are very broad where overheads arise because different methods and products are linked only through costs of management and distribution as in a department store. Supply can then be varied to match needs exactly. Rates of transformation are obtained by finding the relevant constraints on production under conditions of normal capacity utilization and afford a rule for splitting up the joint costs.

The likely outcome seems to be a tendency towards overdetermination as I first realized in a discussion of the energy system[10] which quite obviously had to be considered as a group of competing processes with some prices regarded as given and some others as overdetermined. The classical foundation of this analytical approach is the choice of the "socially necessary technique". As we shall see, even the presence of "too many" processes in all sectors of the economy does not represent a chaotic state of overdetermination, since it may be assumed that the socially necessary technique has already been determined in all sectors of the economy except the one under consideration so that, in the case of single product systems, input prices in any given industry may be regarded as already determined. It can then be discussed which method (or which combination of methods) is socially necessary and determines output prices in the sector under consideration, while other methods yield rents.

Not sufficient attention has been paid so far to the question of how this procedure is to be made more rigorous in basic Sraffa systems (where the output price reacts back on input prices), to joint production (where the classification of "industries" is not straightforward) and to conditions where there remain "pockets of underdetermination".

According to one approach, the decisions about the methods of production to be regarded as "socially necessary" (e. g. solar versus nuclear energy) have to be taken in the light of estimates about the potential productivity growth of each, about the supply of raw materials, about political developments and the evolution of social values ("preferences"). Ricardo thus treated as socially necessary that technique which allowed to satisfy *total* demand in the long run in the cheapest way, i. e. the technique employed on the marginal land in agriculture and the most productive technique in manufacturing industry.

[10] B. SCHEFOLD, "Energy and Economic Theory", *Zeitschrift für Wirtschafts- und Sozialwissenschaften,* 1977, pp. 227-249.

Alternatively, and for more short run considerations, an average of existing methods in each sector of the economy has been defined as the socially necessary technique. This was the approach suggested by Marx in vol. III of "*Das Kapital*" in the determination of what he called "market value", while his concept of "socially necessary labour" in vol. I is, apart from the political and historical element, closer to Ricardo. The same principle of "averages" is also used in the construction of modern input-output tables where the coefficients reflect the average productivity in any industry.

In the future, it may become possible to extend Sraffa's theory of joint production further by developing new practical rules for aggregation and by rearranging the statistical data in order to construct *square* joint production input *and output* tables.

The socially necessary technique of classical economics is thus found by applied economists either by means of "technology assessments" or by means of "aggregation". The purpose is in both cases to abstract from short run disturbances (cf. the "market prices" of the classics). Both methods may have to be combined in order to deal with all possible situations. The "pockets of underdeterminacy" are eliminated if technology assessment yields estimates about limits to the range in which an underdetermined price may move (e. g. the future price of gasoline sets a limit to the possible variation of the price of liquefied coal as a future by-product of high temperature nuclear reactors). If this is not feasible, the product is likely to be new and therefore non-basic so that it is eliminated in the formation of the basic system.

The familiar construction of the basic system is therefore rendered more complicated only insofar as there will be overdetermination. A theoretically rigorous solution is then available: if competition prevails, that combination of processes will be chosen which maximizes the rate of profit, given the real wage[11]. However, in concrete circumstances the choice between "technology assessment" (selection of a method which is likely to dominate and which does not necessarily minimize costs) and "aggregation" (formation of averages) is to some extent a matter of judgement and of purpose: technology assessment is more appropriate for prediction and policy-making while aggregation serves the analysis of inter-industry relationships in the present. In practice, the availability of data and the access to information about industrial strategies will be decisive factors. Finally, where surplus profits have been consolidated into rents, the traditional method should be followed and the "marginal" process determines prices.

But, whatever method is chosen in the formation of the basic system from "socially necessary techniques", the classical method of regarding distribution, employment and the composition of output as given is clearly an essen-

[11] B. SCHEFOLD, "Von Neumann and Sraffa: Mathematical Equivalence and Conceptual Difference", *The Economic Journal*, March 1980, vol. 90, pp. 140-156.

tial element in the derivation of the equations which determine production prices; it is difficult to see how these procedures could be used in a neoclassical general equilibrium approach although they are, apart from the element of joint production, standard in input-output analysis (with rates of profit differing between sectors).

Once production prices have, empirically or conceptually, been derived at some level of aggregation, a disaggregation for a sector under particular consideration is again feasible in order to discuss the underdetermination or overdetermination of certain prices (other prices being given), with competing processes and the potential introduction of by-products, as we have done with examples taken mainly from the energy sector in sections 5, 6 and 7 above.

The classical economists themselves do not seem to have been aware of the possibility that production prices might have to be determined simultaneously in a group of joint production processes. They treated joint production as a special extension of single product industries by implicitly distinguishing between the main product (whose price was equal to cost of production) and a subsidiary product which would fetch as much as a close substitute. This is why Adam Smith argued that fur was cheap in a country where fur was used for clothing and where the main diet was meat[12], and why Marx called joint products the "excretions of production" and seemed to think that the problem of joint production was mainly that of capitalists trying to sell waste products to make some additional profit[13].

Natural processes — if we may use that expression — always feature joint production in that any transformation in an ecological system can be regarded to fulfill several functions, but the purposive act of human production is typically directed at the creation of *one* product. This, and the difficulty of accounting for costs and prices in the case of joint production, lead to the prevalence of single product systems and the emphasis on single production by the political economists in the period of classical analysis. I interpret the paper by Kurz[14] — which had been sent to me when this paper had already gone to the Journal — as yielding further evidence on this point. The classics did discuss disposal activities but were unable to provide a complete solution to the determination of relative values of joint products — hence Jevons's famous scorn for J. S. Mill who had invoked "supply and demand" in this context. But it seems also plausible that the increasing importance of chemical industry in the last third of the 19th century led to a greater awareness of the problem. At any

[12] H. D. KURZ, "Joint Production and the Influence of Demand on Relative Prices: A. Smith, J. St. Mill and Marshall", *mimeo*, 1980.

[13] K. MARX, *Das Kapital,* Band III (1894), Berlin, Dietz Verlag 1969, pp. 110-113.

[14] H. D. KURZ, "Joint Production in the History of Economic Thought", *mimeo* (60 pp.), 1984.

rate, I should like to advance the hypothesis for further historical investigation that the processes introduced in the manufacturing sector during the first industrial revolution lead less often to immediate joint production of commodities than the more traditional methods preceding and the more advanced methods following it.

Today, the classical theory of value can be extended quite naturally to multiple product industries, if Sraffa's suggestion "to count the equations" is taken up, as we have seen in this paper. Sraffa had discovered that the underdeterminacy of one process with two joint products leaves "room for a second, parallel process". Now we have seen that there are *incentives* actually to bring the number of processes used and commodities produced into equality. The point, however, is to transcend the formal analysis of the long period equilibrium and to analyse the movement of the economy by taking the long period position only as the frame of reference without identifying it with the actual state of the economy. Since the properties of joint production systems in equilibrium have largely been clarified, the task is to analyse movements of capital between industries which equalize the rates of profit. The obstacles to the tendency of an equalization of the rates of profit are diverse, but in the course of the evolution of the economic system institutions are being developed which support that tendency, such as the capitalization of rents and the *ex post* reevaluation of the means of production in capital markets. In a sense, the phenomenon which we have here classed under the heading of "counting of equations" represents a specific form which this tendency assumes in the case of joint production.

However, it is not possible to analyse the changes of prices, distribution and technology all simultaneously. One can, given distribution and the methods of production, analyse the formation of a general rate of profit with the transfer of capital between industries but such was not our purpose here. We have analysed the competition between methods of production at a given general rate of profit and at given levels of demand in terms of surplus profits which may be turned into rents in the case of an overdetermination and which are due to the profit contribution of excess commodities in the case of an underdetermination of prices. The basis of our analytical procedure therefore was provided by the classical methodological separation between the theories of distribution, value and output. It is this which had allowed us to identify the classical element in modern econometric models and which allowed us to reconcile the apparent contradiction between the conflicting interpretations of joint production in mathematical economics on the one hand and business administration on the other.

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