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contents

Joint Production and Technical Progress

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It is widely believed and quite possibly true that technical progress is of great importance and it is certainly true, if less widely recognized by economic theorists, that joint production is very far from being exceptional[1]. The purpose of this paper is therefore to consider how the presence of joint production affects certain familiar results of the neo-classical theory of the consequences of technical change, in particular the clearcut results obtained in the context of a two commodity, two primary input model. It might perhaps seem obvious that technical *progress* must necessarily move the "factor price frontier" outwards from the origin, when the only "factor returns" involved are those to primary inputs, but it will be shown that this is not so in the presence of joint production, even when there are no produced means of production. This will be shown by means of a simple numerical example but the results will then be presented in more general form in a subsequent section.

1. THE CONVENTIONAL RESULTS

It may be helpful first to sketch the background to what follows, by reviewing the familiar neo-classical analysis of the effects of Hicks-neutral technical change in one sector, in the context of a two-sector, two-factor model[2]. The two commodities may be labelled 1 and 2 and the two factors

[1] Cf. I. STEEDMAN, "The Empirical Importance of Joint Production", *Manchester Discussion Paper in Economics*, Number 31, 1982; "L'importance empirique de la production jointe", in C. BIDARD (ed.), *La production jointe*, Paris, Ed. Economica, 1984.

[2] These results are set out in many standard works; for one good textbook example, see M. CHACHOLIADES, *International Trade Theory and Policy,* Tokyo, McGraw-Hill Kogakusha Ltd., 1978, pp. 349-358. The classic reference is, of course, R. FINDLAY and H. GRUBERT, "Factor Intensities, Technological Progress and the Terms of Trade", *Oxford Economic Papers*, XI, 1959, pp. 111-121.

called land and labour. It is assumed that, say, commodity 1 is unambiguously the land-intensive commodity, *i. e.*, that at every ratio of rents to wages, the land-labour ratio will be higher in sector 1 than in sector 2. It readily follows that, if p_i is the price of commodity i, W is the rent rate and w the wage rate, then (p_1/p_2) is a monotonically increasing function of (W/w).

Suppose now that sector 1 experiences Hicks-neutral technical progress, *i. e.* that the land-labour unit isoquant in sector 1 contracts towards the origin in a radial fashion. (Or, equivalently, that the "factor price frontier" relating $[W/p_1]$ to $[w/p_1]$ expands outwards from the origin in a radial fashion). It will be clear that (p_1/p_2) falls for every given (W/w) or, in other words, that (W/w) rises for every given (p_1/p_2). Since the whole (p_1/p_2) versus (W/w) curve is different before and after the progress, we have two alternative bases for comparing pre- and post-progress real rent rates and real wage rates; we may *either* hold (p_1/p_2) constant in making our comparisons *or* hold (W/w) constant. The usual neo-classical procedure is, in fact, not to choose between these two alternative bases but simply to make both sets of comparisons.

Consider first the constant (W/w) comparisons. Both (W/p_1) and (w/p_1) will have risen — and risen precisely by the rate of technical progress. (W/p_2) and (w/p_2), on the other hand, will be unchanged, since the "factor price frontier" relating them will be unchanged. Now consider the constant (p_1/p_2) comparisons. As has already been noted, if (p_1/p_2) is to be constant, (W/w) must be higher in the post-progress case; it follows at once that (W/p_2) will be higher and (w/p_2) will be lower after progress, since their frontier is unchanged. And it then follows in turn that (W/p_1) will be higher and (w/p_1) will be lower, since $(W/p_1) \equiv (W/p_2)(p_2/p_1)$, etc.

To summarise, the conventional results are as follows: Hicks- neutral progress in the production of the land-intensive commodity will, at constant (W/w), *raise* real rents and real wages (unless they are measured exclusively in terms of the other commodity) and will, at constant (p_1/p_2), *raise* real rents and *lower* real wages[3].

2. A TWO FACTOR, TWO COMMODITY EXAMPLE

We may now consider a very simple example of an economy using constant returns to scale processes, in which homogeneous land and homogeneous labour are used to produce two commodities. There are no produced means of production and rents and wages are paid ex-post, so

[3] Cf., e. g., R. N. BATRA, *Studies in the Pure Theory of International Trade*, London, Macmillan, 1973, p. 147.

that there are no interest payments. The common neo-classical assumptions of free disposal of commodities and of zero factor prices for factors less than fully employed will both be made. This model of production (without technical progress) has been set out before[4], but it will perhaps be helpful to the reader to present it in full once again.

TABLE 1

Process	*Labour*		*Land*		*Commodity 1*		*Commodity 2*
P_1	4	+	5	→	0	+	5
P_2	1	+	1	→	1	+	1
P_3	1	+	3	→	2	+	0
P_4	5	+	9	→	8	+	0

Table 1 shows the four available processes of production; P_1 produces only commodity 2, P_2 is a joint products process and P_3 and P_4 produce onnly commodity 1. If the fixed supplies of labour and land are 8 units and 12 units respectively, then the production possibility frontier is as shown in Figure 1. At A only P_1 is used and there is unused land. Along

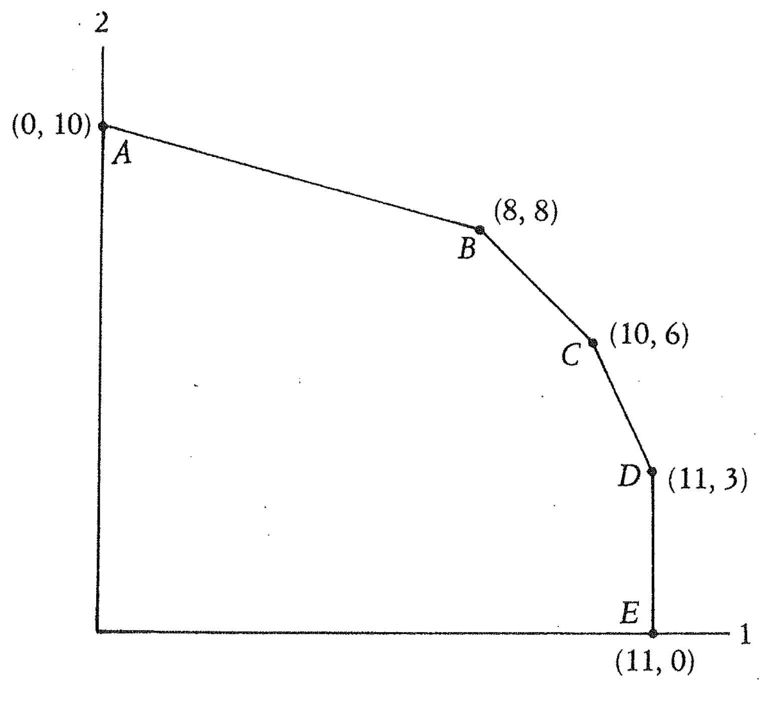


Fig. 1

[4] See I. STEEDMAN, "Joint Production and the Wage-Rent Frontier", *Economic Journal*, XCII, 1982, pp. 377-385.

AB both P_1 and P_2 are used but land is still not fully employed, since the ratio in which it is available (12/8) exceeds the ratio in which it is required in either process. At B itself, P_2 is used alone and land is again underemployed, for the same reason as before. Along BC and at C itself, however, P_2 and P_3 are used and *at* C (though not on BC) *both* land and labour are now fully utilised. This full utilisation of both factors obtains, in fact, all along CDE. On CD processes P_2, P_3 and P_4 are all in use, while at D itself, along DE and at E, processes P_2 and P_4 are used. (In every case, one draws the *separate* land and labour constraints for each combination of processes and then takes the "most binding" constraint, at any given ratio of the two outputs, as being part of the production possibility frontier $ABCDE$). The section DE is vertical because production is at D, while varying amounts of commodity 2 are freely disposed of. Since it is only on CDE that *both* land and labour are fully employed, and since the commodity price ratio is infinite along DE, we shall focus our attention on the section CD and on those commodity prices — $(p_1/p_2) > 1$ but finite — which lead competitive producers to be on CD.

In Figure 2 we show three alternative real rent-real wage frontiers; in each case W is the rent rate and w the wage rate, the sections "c" and "d" correspond to the corners C and D in Figure 1 and the arrows show the direction of movement as (p_1/p_2) rises. Figure 2 (a) is the rent-wage frontier when commodity 1 is the standard of value; Figure 2 (b) is the frontier when $p_1 + 6p_2 = 1$ defines the standard; and Figure 2 (c) is the frontier when commodity 2 is the standard. To see how these frontiers are obtained consider, for example, section "c" in Figure 2 (c). At corner C processes P_2 and P_3 are used, so we see from Table 1 that:

$$w + W = p_1 + p_2$$

and

$$w + 3W = 2p_1 \qquad [1]$$

It follows at once that:

$$W = -2p_2 + w$$

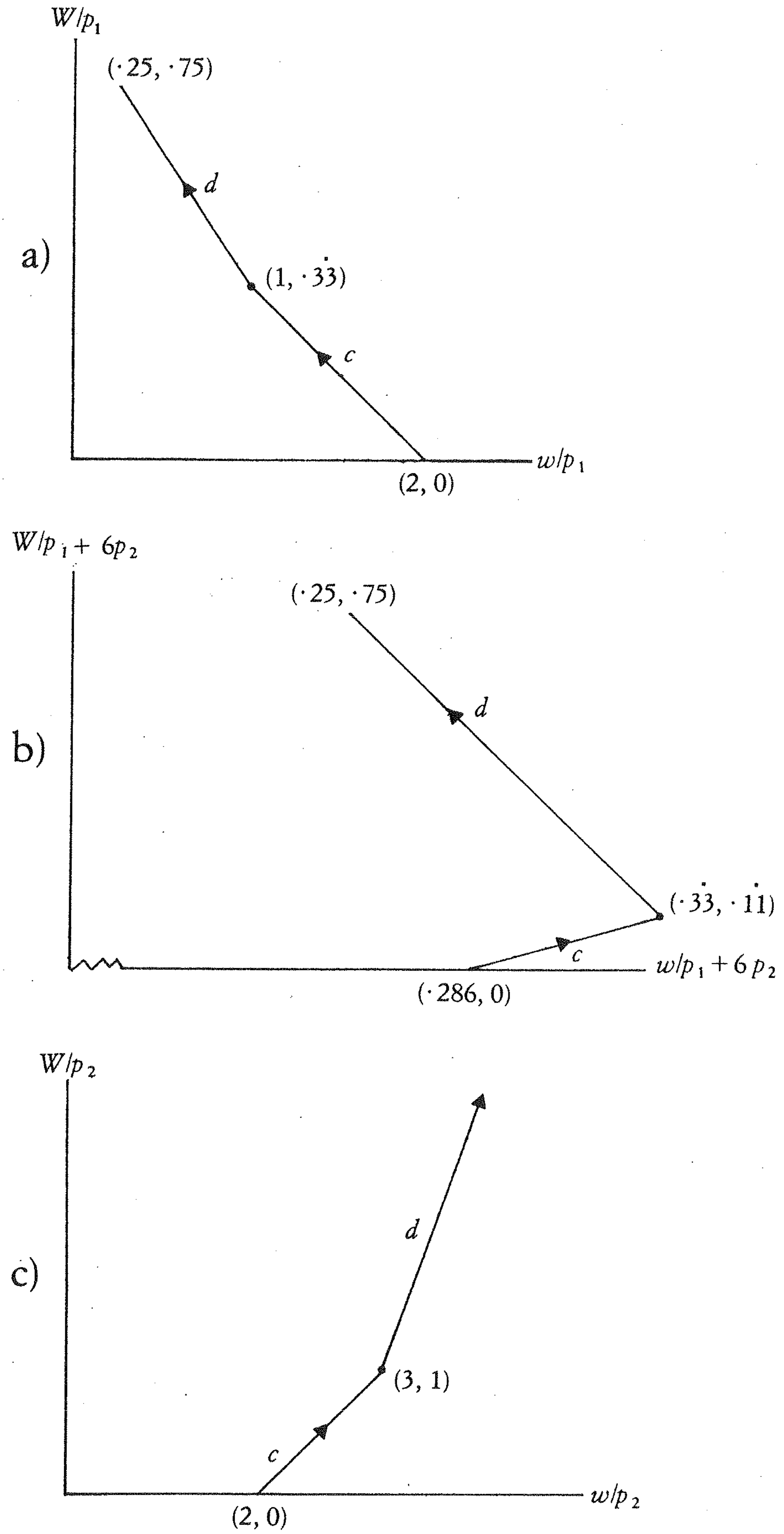
or

$$(W/p_2) = -2 + (w/p_2), \qquad [2]$$

which is the equation of "c" in Figure 2 (c). Moreover, from [1] and [2], $(p_1/p_2) = [(w + 3W) / (w - W)]$, so that (p_1/p_2) rises as (W/w) rises. All the other branches of the frontiers shown in Figures 2 are obtained in a similar way. (Many of the figures have been rounded off).

It is, of course, a striking feature of Figures 2 that not all sections of these real rent-real wage frontiers are downward sloping. Such sections

Fig. 2


a)
W/p1
(·25, ·75)
d
(1, ·33)
c
w/p1
(2, 0)
b)
W/p1 + 6p2
(·25, ·75)
d
(·33, ·11)
c
w/p1 + 6p2
(·286, 0)
c)
W/p2
d
(3, 1)
c
w/p2
(2, 0)

contrast sharply with the necessarily downward sloping frontiers obtained from single-products systems and it has been shown how upward sloping frontiers can upset familiar comparative statics results concerning changes in demand, labour supply and real wages, and the Rybczynski and Stolper-Samuelson theorems[5]. It will be noted, however, that everywhere in Figures 2 (p_1/p_2) is positively related to (W/w). This constant feature suggests that commodity 1 may be thought of as the land-intensive commodity (and 2 as the labour-intensive one), in line with the standard result concerning relative factor intensities, and the corresponding movements of relative commodity prices and relative factor prices. (See above, first section).

Technical Progress. We now consider the effects of technical progress and shall concentrate on the case of "neutral" tecnical progress in processes P_3 and P_4, which constitute the unambiguous "commodity 1 sector". (Since P_2 also produces commodity 1, in addition to commodity 2, it could, of course, be suggested that we are not allowing for progress in the "full" sector 1. But P_2 cannot be classified unambiguously and, indeed, it is not self-evident how the concepts of neutrality, bias, etc. should be generalized to the case of joint production. See further below, however). Suppose that, with unchanged inputs, the outputs from P_3 and P_4 increase to $(2.2+0)$ and $(8.8+0)$, respectively, representing 10% neutral progress in the unambiguous sector 1. The new production possibility frontier is

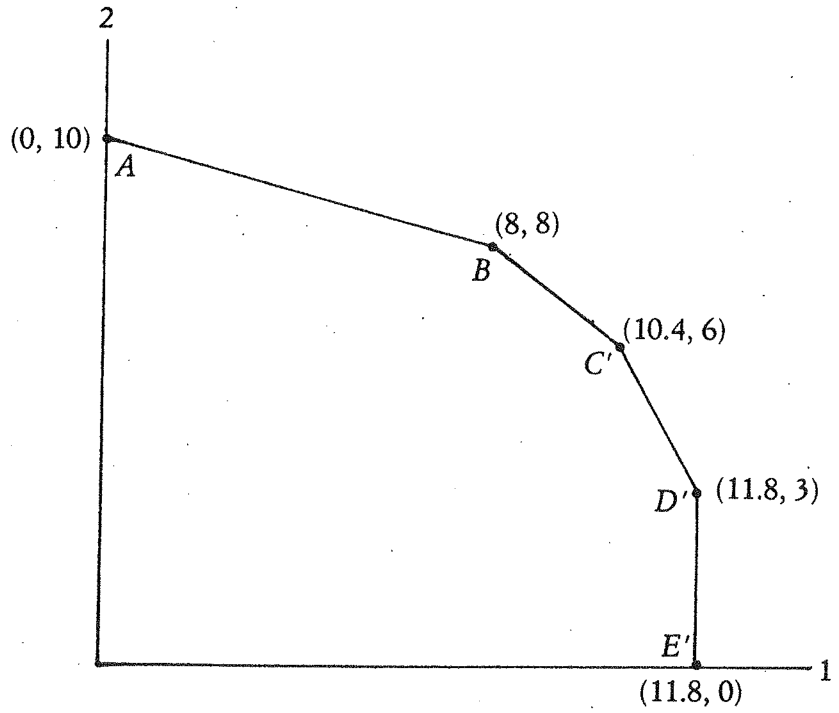


Fig. 3

[5] Cf. my "Joint Production", *op. cit.*

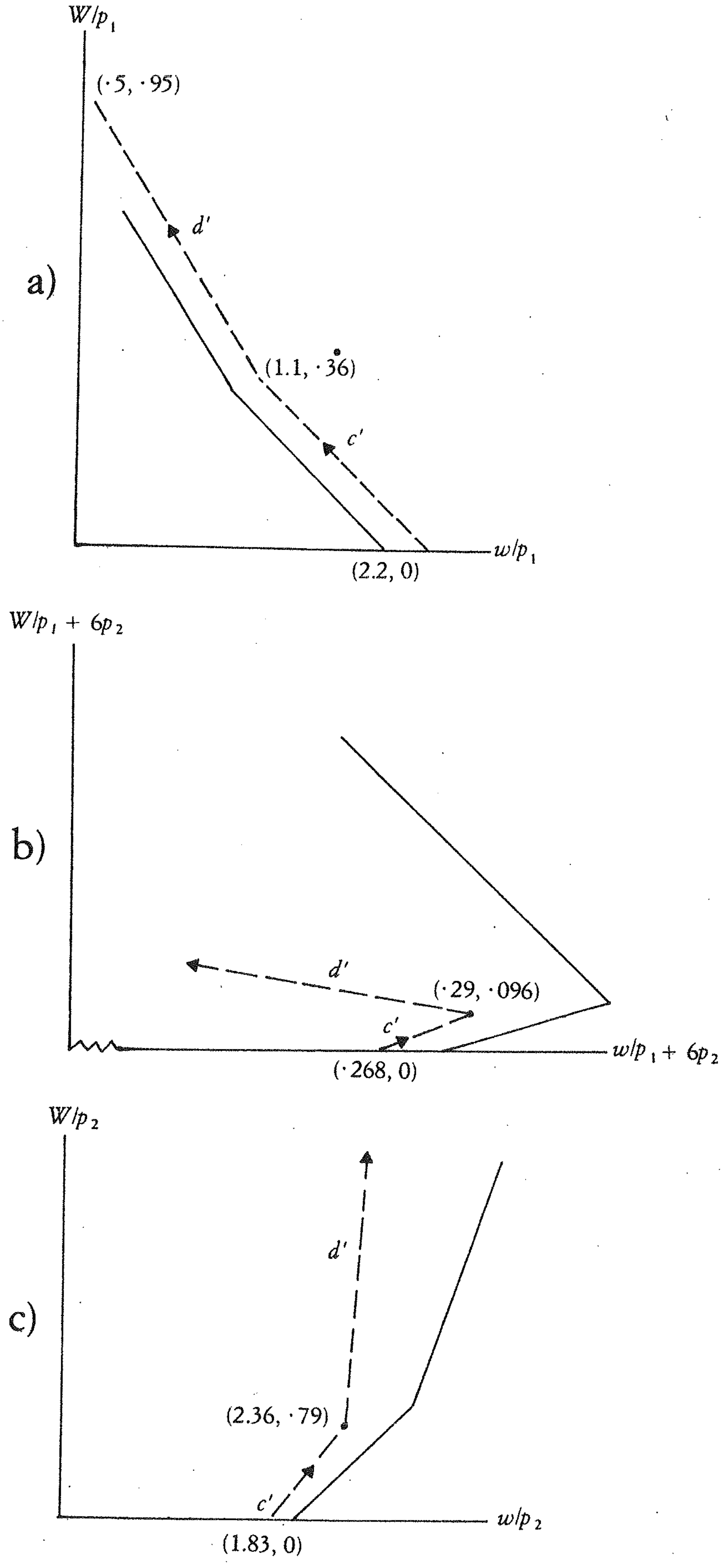
shown in Figure 3, where corners A and B are as in Figure 1 but corners C', D' and E' have all moved horizontally to the right, as compared with C, D and E in Figure 1. It does *not* follow, however, that every real rent-real wage frontier has moved out from the origin. Figure 4 reproduces Figure 2 in its solid lines and shows also, in dashed lines, the frontiers *after* the 10% progress in P_3 and P_4. In Figure 4 (a) the progress has indeed moved the frontier outwards but in Figures 4 (b) and 4 (c) technical *progress* has moved the frontiers towards the origin. (The new frontiers are, of course, derived in just the same way as the old ones).

In all three cases, the rent-wage *ratio* at the switch from "*c*" to "*d*" is unchanged by the progress. (This would indeed hold for any standard of value). But in Figures 4 (b) and 4 (c) the absolute values of *both* the rent and the wage have fallen at each rent-wage ratio. This result is quite contrary to the standard neo-classical theory of technical progress in the 2×2 model. (On the other hand, at any given (p_1/p_2) it can be shown that the real rent has risen and the real wage has fallen, which is entirely in line with that standard theory, if commodity 1 is taken to be the land-intensive commodity, as suggested above).

Further Cases. It has already been noted that, in the presence of joint product processes, it is not self-evident how one should generalise the usual neo-classical 2×2 analysis of technical change. In the above example only P_3 and P_4 — the unambiguously sector 1 processes — were subject to 10% technical progress. Suppose now, by contrast, that in addition to those changes, the output of commodity 1 from process P_2 also increases by 10%, the inputs and the output of commodity 2 being unchanged. In this new case, the real rent and real wage will both increase, as a result of the progress, at every rent-wage ratio (unless commodity 2 is the standard), as in standard theory. At constant (p_1/p_2), however, we do *not* obtain the conventional result, for it is readily seen that, on both "*c*" and "*d*", the real wage increases as a result of the technical progress (while the standard theory says that only the rent will increase, the wage falling). Thus whichever way we interpret "progress in sector 1", at least one of the conventional results fails to carry over to the joint products case.

(In order to consider "biased progress" it is, of course, appropriate to change the inputs and not the outputs of Table 1, reducing the land and labour inputs, in different proportions, in the relevant processes. It is clear from continuity considerations that "biased" input reductions will *not* necessarily restore all the conventional findings and it is left to the interested reader to construct suitable examples. Rather than present such examples, it is more interesting to note here that, in the presence of joint product processes, one has to consider "output neutrality or bias", as well as "input neutrality or bias", when defining types of technical change).

Fig. 4



a)
W/p_1
(·5, ·95)
d′
(1.1, ·36)
c′
(2.2, 0)
w/p_1
b)
$W/p_1 + 6p_2$
d′
(·29, ·096)
c′
(·268, 0)
$w/p_1 + 6p_2$
c)
W/p_2
d′
(2.36, ·79)
c′
(1.83, 0)
w/p_2

3. INTUITION

Having checked all the calculations and confirmed the results obtained from the above examples, the reader may still say, "Yes, these results are correct — but *why* can a real rent-real wage frontier slope upwards and *why* can technical progress move such a frontier inwards towards the origin?". The purpose of the present section is to help to reduce the implied sense of puzzlement.

Consider, for example, corner C in Figure 1 and branch "c" in Figure 2 (c), where processes P_2 and P_3 are in use, and try the experiment of "imputing" to commodities 1 and 2 the total amounts of labour and of land required to produce them. Let l_i (L_i) be the amount of labour (land) in question for commodity i. From Table 1 we see that:

$$l_1 + l_2 = 1$$

and

$$2l_1 = 1,$$

so that $l_1 = l_2 = (1/2)$. In the same way, we have:

$$L_1 + L_2 = 1$$

and

$$2L_1 = 3,$$

so that $L_1 = (3/2)$ and $L_2 = -(1/2)$. We have imputed to commodity 2 a *negative* amount of land required in its production. (So that $[L_1/l_1] > [L_2/l_2]$ — the condition that commodity 1 be the more land-intensive one — holds with a vengeance). But the reciprocal of L_2 is the value of (W/p_2) when $w = 0$ — and this is now seen to be negative! The upward sloping frontier, in terms of commodity 2, should now begin to seem less strange. Moreover, once it is realized that a joint production system may impute a *negative* amount of land use to some commodity, it will be seen that one ought to have no confident *a priori* expectations as to how technical progress will affect real wages and real rents. Our examples are really *not* surprising at all.

4. GENERALIZATION

In the above examples produced means of production were deliberately excluded, in order to emphasize that the possibility of non-neo-classical consequences of technical progress derives from joint production as such, and not from the interaction of joint production with capital theoretic complications. But we may now consider a system using n processes to produce n commodities by means of m types of primary input and inputs of the n types of commodity, there being a uniform rate of interest on the value of these latter. If the jth columns of B, A, E represent the outputs from, produced inputs to, and primary inputs to the jth process, at the unit level of operation, then:

$$wE = p\,[B - (1 + r)\,A];\; ps = 1 \qquad [3]$$

where w and p are row vectors of primary input "wage rates" and commodity prices, respectively, r is the interest rate, and s is a column vector representing the (composite commodity) standard of value. From [3]:

$$dwE = -\,wdE + dp\,[B - (1 + r)\,A] + p\,[dB - (1 + r)\,dA] \qquad [4]$$

and

$$dps = 0 \qquad [5]$$

if r is constant.

Case 1. If relative commodity prices are held constant, so that $dp = 0$, it follows from [4] that:

$$dwE = [-\,wdE - (1 + r)\,pdA + pdB] \qquad [6]$$

Any fall in E or A, and any rise in B, will increase the *RHS* of [6] but the effect on w depends, of course, on the structure of E. More specifically, let dE and dA both be zero, as in our numerical examples, so that:

$$dwE = pdB$$

If improvement is uniform *within any given process,* then $dB \equiv B\hat{t}$, for some diagonal matrix $\hat{t}$, and thus

$$dwE = (pB)\,\hat{t} \qquad [7]$$

It is clear from [7] that the presence of joint product processes — that is, the non-diagonal nature of B — can lead to no qualitative difference from the usual single product theory results. If improvement is uniform *for each given commodity,* however, $dB \equiv \hat{T}B$, for some diagonal matrix $\hat{T}$, and thus:

$$dwE = p\hat{T}B. \qquad [8]$$

It is clear from [8] that the non-diagonal nature of B *can* now lead to results which differ from the single product theory results. Thus both [7] and [8] confirm what was found above in our examples.

Case 2. If relative primary input prices are held constant, so that $dw = kw$ for some scalar k, it follows from [3], [4] and [5] that:

$$k = [-(wdEx_s) - (1+r)(pdAx_s) + (pdBx_s)] \qquad [9]$$

where $x_s \equiv [B - (1+r)A]^{-1} s$. In words, x_s is the (hypothetical) activity vector required to produce the standard bundle s for consumption and to maintain steady growth at a rate equal to r. If $x_s \geq 0$ then the *RHS* of [9] rises with every fall in E or A and with every rise in B. But if x_s contains one or more negative elements — *i. e.* the processes (B, A, E) cannot produce s and maintain growth at rate r — then k may respond "perversely" to some changes in (B, A, E)[6]. More specifically, suppose once more that dE and dA are zero, so that [9] becomes:

$$k = (pdBx_s)$$

In the *process* improvement case defined above:

$$k = (pB)\hat{t}x_s \qquad [10]$$

and joint production, since it can give rise to an x_s with negative elements, can involve k responding "perversely" to some elements of $\hat{t}$ in [10]. In the *commodity* improvement case, however, we have:

$$k = p\hat{T}Bx_s$$

or

$$k = p\hat{T}q_s \qquad [11]$$

where q_s is the *gross output* vector required to support s for consumption and growth at rate r. In our example A was zero and hence $q_s \equiv s$; in this case [11] shows that k must respond positively to the elements of $\hat{T}$, as stated above. More generally, however, the (hypothetical) vector q_s could have some negative elements, provided that the corresponding rows of Ax_s, the (hypothetical) capital stock vector, were also negative. Only if this is so can *commodity* improvement fail to raise all elements of w, when

[6] If x_s contains one or more negative elements it does *not* follow that s is an uninteresting standard in which to measure real primary input prices: see *ibid.*, pp. 380-381.

those elements are held in fixed proportions to one other. Again, then, [10] and [11] confirm our earlier findings.

It will be clear to the reader who has followed the arguments so far that we may develop [6] and [9] in various ways (for example, by writing $dB = \hat{T}_1 B\hat{t}_1$, $dA = \hat{T}_2 B\hat{t}_2$, $dE = -\hat{T}_3 E$) and that no very general results can then be expected. It can still be said, however, that the presence of joint production, by giving rise to a non-diagonal B and to the possibility of a non-semi-positive x_s, does mean that definite results are even harder to obtain than in the (very special) single products case.

5. CONCLUSION

It has been seen that, when joint production is allowed for, technical *progress* can actually move the primary-input-price frontier *inwards* towards the origin, when certain standards of value are used for measuring real wages, rents, etc. Moreover, familiar neo-classical results concerning the effects of technical progress, at either constant relative commodity prices or constant relative primary input prices, are no longer valid. Since joint production is so very widespread in real economies[7], these findings suggest that the standard neo-classical theory of the consequences of technical change is of little value. The challenge is, of course, to create an alternative and superior theory, able to take the fact of joint production in its stride.

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[7] Cf. note 1 above.