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contents

Technical Innovations, Life of Equipment and Effective Demand*

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1. INTRODUCTION

That technical innovations may play an active role in overcoming the problems of effective demand that curb long term growth has often been recognized, and insisted upon, in the economic literature, in particular as a counter argument against the idea of permanent stagnation advocated by underconsumption theories. These debates, still alive during the '40s and early '50s, were mostly felt to be obsolete by economists who witnessed the remarkable growth performance of the '50s and '60s. The interpretation of the lasting stagnation which set in with the seventies as the downswing of a "long wave" (or Kondratieff cycle) — a cycle of about 50 years' duration in economic life — has revived the interest in the contribution of technology to growth, and in Schümpeter's seminal work on the technological explanation of long waves. As is well known, Schumpeter was reluctant to attach great significance to the problem of effective demand. In his view the cyclical bunching of innovations plays an essential role in pushing the economy out of the stationary equilibrium towards which it would otherwise be gravitating. Some authors, such as Freeman and his school[1], while developing critically and in original directions Schumpeter's hints about the emergence of innovation clusters, have tried to marry up this idea to Keynes' notion of effective demand. This preoccupation has brought again to the fore the old notion that technical innovations sustain

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[1] Cf. C. Freeman, J. Clark, L. Soete, *Unemployment and Technical Innovation: A Study of Long Waves in Economic Development,* London, Frances Pinter, 1982.

demand through their influence on investment activity and the propensity to consume. In using this notion to obtain a cyclical model of long term growth, one is faced with the problem that different innovations (and thus heterogeneous clusters of innovations) may vary to a great extent in their effects on demand. The attempt to come to grips with the above problem has led to a distinction being drawn between the demand effects of two stylized types of innovations, namely product and process innovations. It is suggested that the former, which would be prevalent during the early phase of industry's life cycles driving the upswing of a long wave, sustain demand more than the latter which would be prevalent during the maturity phase of industry's life cycles. The reason is that product innovations sustain investment, increase the propensity to consume and have a lower impact on the growth of labour productivity, which may lead to a fall of total workers' expenditure through its effect on employment[2].

It is suggested in the present paper that far more stringent restrictions than those indicated above must be imposed on the composition of the innovation-flow and on the methods in use to establish a sufficiently systematic influence of innovations on effective demand. Indeed if the demand effects of technical innovations are looked at through sharper lenses, further differences tend to emerge that go well beyond the distinction between product and process innovation. In the pages that follow it will be argued in this sense through the analysis of the influence of embodied technical progress[3] on the life of equipment, and thus on gross investment[4].

Embodied innovations may affect the life of fixed capital in either or both of the following ways.

a) Through faster loss of competitiveness of the older vintages of machinery in the production of a given commodity.

b) Through the fall of the demand for the old products when new substitute products are introduced by new firms possibly operating in completely new industries, or, more generally, when the product innovation is itself embodied.

[2] Cf., e.g., G. Dosi, "On Engines, Thermostats, Bicycles and Tandems, or, moving some steps towards economic dynamics", Brighton, SPRU, University of Sussex, 1982, Mimeo, pp. 2-18.

[3] One can always use the term "technical progress" in the sense of a net addition to the set of technical possibilities. However, to use the word progress, in comparing two elements within this set and to provide a measure of the extent to which one is superior to the other, may be highly ambiguous. On the assumption that product innovations do not alter the use value of the given consumption commodity, the macroeconomic rate of technical progress can be measured for indecomposable systems by the rate of increase of the real wage at the given and constant rate of profit. The term embodied technical progress refers to the case where the introduction of a new method and/or product can only take place through the introduction of a new type of durable capital good.

[4] "Whatever may be its effect on net investment, technical progress will normally raise gross investment, in so far as it hastens obsolescence and shortens the life of existing capital". R. C. O. Matthews, *The Trade Cycle*, Disgwell Place, J. Nisbet and Co. Ltd., Cambridge, CUP, 1959, p. 68.

With respect to both *a*) and *b*) channels of action, the present paper argues that the relationship between embodied innovations and the life of equipment is likely to be unsystematic.

As far as *a*) is concerned the critical arguments of section 3. rely on the multisector dimension of technical progress, with its effect on the structure of commodity prices. These price effects may be such that: *i*) technical progress in one sector may determine — in other sectors — the return of an otherwise obsolete method on the frontier of the best practice methods; *ii*) technical progress in one sector may cause a later scrapping of the older (and technically obsolete) vintages of machinery in other sectors. At an abstract and highly general level of analysis, phenomena discussed in *i*) and *ii*) are by no means irrelevant flukes. The opposite seems rather to be true. This raises the question of why historical facts provide after all relatively abundant evidence of early replacement of machinery stimulated by innovations. Although answers to such a question can be easily found, from the viewpoint of the present paper it will be crucial to observe that the influence of innovations on the life of machinery through *a*) is conditional upon the occurrence of specific and historically determined empirical circumstances.

A similar conclusion will be reached after the analysis of channel *b*). Here the important point is that the investment activity stimulated by a given innovation depends on the pervasiveness of the obsolescence triggered by it. An old product can be made completely obsolete by the innovation or it may survive because it is better suited for particular uses. As it turns out (even abstracting from changes in the propensity to consume), the displacement effects of a product innovation (and thus its influence on equipment life and gross investment) depend, in a non stationary economy, not only on the size of its relative share in total output obtained after diffusion, but also on the relative share obtained in individual industry markets. Thus the cyclical behaviour of the flow of innovations can be causally associated — through *b*) — with the cyclical behaviour of investment only if the pervasiveness of the obsolescence effects of innovations does not change throught time. But the above property is itself dependent upon a host of empirical circumstances. Even when the prevailing empirical conditions are such that a higher rate of embodied technical progress has the effect of shortening the life of machinery, an increase in gross investment does not follow of necessity. To the extent that the shortening of equipment life is foreseen it may induce capitalists to adopt fixed capital-saving methods of production. In the case of process innovations this may be expressed in terms of the familiar technical frontier. If the frontier of the best practice methods shifts as the result of a persistent rate of embodied technical progress, productive systems will not be on the frontier because the adoption of the new methods is conditional upon the life of the existing machinery. The attempt by capitalists to get closer to the shifting frontier through an early replacement policy may find an obstacle in the increase in overhead costs that go with it. However, under appropriate states of distribution, they may find it possible to approach the frontier through a movement within the set of technical possibilities in the direction of fixed capital saving processes of relatively short durability. This alternative has been considered occasionally and rather vaguely in the literature, the suggestion being that machines of shorter durability would be constructed in periods of faster technical progress. The present paper considers a further specification of the same alternative, hinging upon an increase in the desired rate of capacity utilization. The desired rate of capacity utilization is thus shown to be related to expectations about future technical innovations, through their influence on the expected life of durable capital goods.

When the shortening of equipment life is combined with the adoption of fixed capital saving methods the macroeconomic behaviour over the trade cycle may undergo important modifications. With reference to the class of trade cycle models where the phase of expansion is ultimately halted by the ceiling of full employment, the present paper suggests that the changes in the technical coefficients mentioned above should be expected to result in shorter phases of depression and boom. This is clearly at variance with the idea that investment activity stimulated by innovations is conducive to longer phases of boom in periods of faster technical progress. In the conditions stated above technical innovations would at best sustain a long wave upswing in output from the side of supply rather than from the side of aggregate demand, and clearly this effect would take place only in so far as long term output growth is supply driven, as it is in the trade cycle model mentioned before. Should one attempt to marry a different trade cycle model with technology-driven long waves in output, the attempt may fail altogether. The point is not irrelevant in a situation, like the present one, where there are various forms of evidence of a shortening of equipment life and of an increasing diffusion of capital-saving techniques. This leads back to the central thesis of the present paper: the relationship between technical innovation and investment is not a systematic one, unless exceedingly restrictive conditions are imposed on the pattern of technical change. Markedly different modes of cyclical growth seem to be compatible with the idea, to some extent questionable in itself[5], that periodic clusters of innovations tend to occur over time. Thus, an excessive abstraction from the many dimensions (both quantita-

[5] A recent critical examination of the empirical evidence rejects the hypothesis of the bunching of innovations during long term depressions. Cf. S. Solomou, "Innovation Clusters and Kondratieff Long Waves in Economic Growth", *Cambridge Journal of Economics*, X, 1986, pp. 101-12, which also gives references to the relevant literature.

tive and qualitative) of technical progress deprives one of the possibility of understanding which form of interaction between technology and growth is relevant under the prevailing historical circumstances.

2. MACHINES AND PROCESSES

2.1. The attempt to consider the effect of embodied technical progress on the life of equipment calls forth a suitable representation of technology that makes it possible to deal both with qualitative changes in production (in the form of new vintages of machinery) and with changes in relative prices; at the same time the age composition of productive capacity must be explicitly recognizable in each moment of time. These are, to some extent, conflicting objectives. As is well known, qualitative changes in technology are easily handled if productive processes are integrated vertically through time, as in neo-Austrian models[6]; but this procedure — apart from the restrictions on technology imposed thereby[7] — amounts to considering all capital goods as intermediate products. Capital goods (circulating and fixed alike) are concealed from the view of the analyst, and their prices with them. All problems of sector interdependence that may crucially affect the life of equipment would be ruled out[8]. To obtain some of the advantages conferred by the vertical view of technology, without losing the possibility of dealing with problems of sector interdependence, only fixed capital goods are here considered as intermediate products. Following a well established procedure[9], this is done by integrating the process using a given machine with the process producing that machine, on the assumption that only circulating capital and labour inputs are used in the latter[10]. As a consequence one can identify in

[6] Cf. J. Hicks, *Capital and Time A Neo-Austrian Theory*, London, Oxford University Press, 1973.

[7] As is well known, the main restriction is that no base commodities are allowed; cf. H. Hagemann, H. D. Kurz, "The Return of the Same Truncation Period in Neo-Austrian and more General Models", *Kyklos*, XXIX, Fasc. 4 1976, pp. 678-708; E. Burmeister, "Synthesizing the Neo-Austrian and Alternative Approaches to Capital Theory", *The Journal of Economic Literature*, XII, June 1974, pp. 413-56.

[8] The neo-Austrian description of a productive technique does not give information about the interindustry relationships. Thus, whenever interindustry relationships are crucially relevant for the analysis, neo-Austrian vertical integration must be abandoned, even though it may be feasable for the economic system under consideration (see footnotes 7 and 10).

[9] Cf. B. Schefold, "Different Forms of Technical Progress", *The Economic Journal*, LXXXVI, December 1976, pp. 806-19.

[10] It is clear that the attempt to consider machines as intermediate products, through vertical integration, leads to an infinite regress if machines are directly produced by means of machines. However, since integration is not complete, the synchronization problems facing neo-Austrian models in the presence of fixed capital in producer goods industries do not arise in the present context. The synchronization problems in neo-Austrian models have been stressed in S. Baldone, "Integrazione verticale e transizione", *Economia Politica*, I, April 1984, pp. 81-8.

each process an age interval, corresponding to the construction and to the installation of the durable equipment, in which the process gives no output. This interval is assumed to be of length d for every process[11]. The sector which is the result of this partial vertical integration will be called the "integrated sector" or, where no confusion arises, simply "sector". Such a vertical integration makes it possible to deal, in a simple way, with *product* innovations in the durable capital goods industries, bringing *process* innovations into other industries.

As will be seen, the present representation of technology must also be in the position to deal with the influence of persistent changes in equipment life on the desired working time of machinery. In a model of production where only flow magnitudes exist, a specific working time pattern for the durable capital used in a given process, is implicitly fixed together with the description of the process[12]. The way in which a particular working time pattern will result in a particular description will depend — inter alia — on whether the process is specified in continuous or in discrete time. For the sake of simplicity a discrete time representation is here assumed to be available. More precisely, it is assumed that there exists a time interval of conveniently short length, so that the time distribution of the flow of imputs and outputs within this interval can be ignored. Such an interval is taken as the unit time interval and is labelled "day". Although production may take place continuously at least throughout definite sections of the day, the age of a process can be and is always expressed by an integer number of "days"; r is the daily rate of profit, which is taken as exogenously given and constant. The attempt to allow for changes in the desired daily working time of machinery (also referred to as the desired rate of capacity utilization) brings with it the need to introduce heterogeneous labour. Indeed, day-time labour is non-homogeneous to night-time labour. However, on the usual strong assumption that the real wages for the different kinds of labour have the same composition, labour can be aggregated and reduced to homogeneous labour. This way out is adopted also here, on the understanding that it can only provide a first approximation to a more rigorous solution[13].

[11] If a process is truncated before the age d, this means that the order for the corresponding machinery has been cancelled. Customers cancelling the order must pay the suppliers for the costs of production already sustained. If a process of age $t > d$ is truncated, there is then a corresponding machine of age $t - d$ which is scrapped. A process of age u is said to be in its constructive phase if $u \leqq d$; if $u > d$ the process is said to be in its productive phase.

[12] Cf. on this point section 4 of H. D. Kurz, "Normal Positions and Capital Utilization", *Political Economy*, II, number 1, 1986. Operating times of fixed capital are more explicitly dealt with, at the loss of other aspects that are fundamental in the present context, within the flow-fund models introduced by N. Georgescu-Roegen.

[13] For a more general treatment of heterogeneous labour introducing vectorally non-comparable real wages for the different kinds of labour, cf. I. Steedman, "Heterogeneous Labour and 'Classical' Theory", *Metroeconomica*, XXXII, February 1980, pp. 39-50.

To avoid problems of pure joint production only one machine of a given age (it may well be a composite commodity) is assumed to exist in each process making use of fixed capital; old machines are not transferable across sectors[14].

2.2. The pages that follow are often concerned with problems of technical choice. These are dealt with under the rational choice criterion that among different productive processes, the one which has the highest present value (that is, the highest present value of its net proceeds) is to be preferred. The relevant discount rate is provided here by the exogenously given and constant rate of profit r[15]. Since productive processes have a long life-time, the prices used to evaluate their profitability are presumably those expected to rule under normal market conditions. In what follows it is assumed that, when a given technique is in use, normal prices coincide with the production prices univocally determined by that technique and by the exogenously given rate of profit. It is further assumed that if a new technique is introduced at time $t = 0$ normal prices converge after a time interval of length s, $0 \leqq s < \infty$, to the new production prices[16].

3. EMBODIED PROCESS INNOVATIONS AND THE LIFE OF EQUIPMENT

3.1. The existence of an inverse relationship between the rate of embodied technical progress and the economic life of durable capital goods seems to be deeply rooted in the mind of most economists. Indeed the economic literature appears, on several occasions, to have confirmed that idea through studies that have considered the problem using different sets

[14] The properties of fixed capital, single product systems displaying such characteristics have been studied in B. SCHEFOLD, "Fixed Capital as a Joint Product and the Analysis of Accumulation with Different Forms of Technical Progress", in L. Pasinetti (ed.), *Essays on the Theory of Joint Production*, London, Macmillan, 1980, pp. 138-217; S. BALDONE, "Fixed Capital in Sraffa's Theoretical Scheme", in L. Pasinetti, *op. cit.*, pp. 88-137; P. VARRI, "Prices, Rate of Profit and the Life of Machines in Sraffa's Fixed Capital Model", in L. Pasinetti, *op. cit.*, pp. 55-87; P. TANI, "Troncabilità dei processi in un modello multisettoriale con capitale fisso intransferibile", *Rivista Internazionale di Scienze Sociali*, XLIX, October-December 1978, pp. 435-468.

Some properties of more general fixed capital systems where many machines are jointly employed and where old machines are transferable, are analysed in N. SALVADORI, "Fixed Capital within Linear Models of Production and Distribution", Catania, 1985, Mimeo.

[15] The choice as to the substitution of a process in use with a new one is carried out at the pre-innovation production prices corresponding to the given rate of profit r. At these prices the present value of each process in use is zero; thus the new process is introduced only if its present value is non-negative at r. Under the present assumptions about technology the post-innovation real wage at r is not lower than the pre-innovation real wage at r.

[16] As will be clear later on, the length of the interval of convergence may be relevant to the problem under discussion.

of assumptions and various analytical tools[17]. The common denominator of the studies in question is that they are carried out within a partial analytical framework, that is, one in which the effect of technical change on the structure of relative prices is not taken into account. Reasoning within such a framework, one is led to the conclusion that as long as capitalists do not expect a further acceleration of technical progress in the future[18], the above relationship must take a negative (or at least a non-positive) sign. However, as Belloc has recently observed[19], the same conclusion does not necessarily hold within a global framework of analysis. The point has obvious implications for the relationship between technical change and demand; so it deserves to be discussed with some care in the context of the present analysis. The discussion here is carried out by supposing that a discrete and unexpected change in the best practice method of production occurs at time $t = 0$ in each integrated sector, and by investigating how the economic life of the processes in use at time $t = 0$ in such sectors is affected by that change.

3.2. Before proceeding further one must dispose of a difficulty that arises in connection with the fact that changes in the method of production in not only one but many sectors are considered here. If in the former case one can show that the change in relative prices caused by the change of technique cannot possibly induce capitalists to go back to the old method, the same is not necessarily true in the latter. To understand the nature of the difficulty one can consider the situation where the new methods are introduced one by one. Each method switch would be now unambiguously profitable with respect to the change in relative prices caused by it. Nevertheless a return to an old method of production may occur after the change in prices caused by a subsequent method switch[20]. Likewise one cannot be sure that, after a multiple method switch, capitalists in some sectors would not go back to the old method. The implications for the analysis of technical obsolescence are obvious and somewhat disturbing.

[17] While a detailed list would be far too long, it is worth quoting here V. L. SMITH, *Investment and Production*, Cambridge Mass., Harvard University Press, 1966, ch. V, pp. 128-161, giving various references to the relevant literature. For a more recent example, also supplied with bibliographical notes, see S. J. NICKELL, *The Investment Decisions of Firms*, Disgwell Place, J. Nisbet and Co. Ltd., Cambridge, CUP, 1978, ch. 7, pp. 126-30 and p. 147.

[18] As W. Fellner pointed out, if entrapreneurs expect a temporary acceleration of technical progress, they may postpone the adoption of innovations; cf. W. FELLNER, "The Influence of Market Structure on Technological Progress", *Quarterly Journal of Economics*, LXV, November 1951, pp. 556-77.

[19] B. BELLOC, *Croissance Economique et Adaptation du Capital Productif*, Paris, Economica, 1980, pp. 204-8.

[20] To my knowledge this possibility was first clearly pointed out in H. SIMON, "Effects of Technical Change in a Linear Model", in T. C. Koopmans (ed.), *Activity Analysis of Production and Allocation*, New York, John Wiley, 1951.

In this as in other cases a gap seems to arise between the possibilities foreseen by abstract analysis and the facts of economic history. Some understanding of the reasons why the return to old methods of production appears to have been infrequent in history may, however, be obtained. One type of explanation refers to the emergence of some well-defined historical forms of technical change. A trivial case arises when the new method implies a pure saving of imputs. Along the same line of argument and with reference to single product systems, *T.* Fujimoto has recently shown that the above problem cannot arise for a new method which is of the capital-using, labour-saving type and if the rate of profit is constant[21]. This would imply that in the new method: *a*) the unit requirement of labour should be no higher than in the old method; *b*) the unit requirement of each non-labour input should be no lower than in the old method. The same conditions would be fulfilled in a fixed-capital single-product system under the form of mechanization defined by Schefold[22].

As it turns out, such historical forms of technical progress involve no chance of a return of old methods not only when the analysis is carried out in terms of prices of production, but also for a much wider range of relative prices. It has been convincingly argued[23] that this apparently fortuitous coincidence is not a coincidence at all. Indeed, the emergence of methods that share the properties referred to above can be understood if one resorts to more general assumptions about price behaviour (possibility of non-convergent market prices) and to more realistic assumptions about expectations (uncertain expectations) than those usually adopted in the literature dealing with the choice of techniques. The idea is that, because of the uncertainty about the future behaviour of market and normal prices, capitalists can rely only upon those forms of technical progress for which the order of profitability between alternative methods is invariant in the face of *relatively* large changes in (relative) commodity prices at the prevailing level — and at higher levels — of the real wage[24]. It should be added that what "relatively large" actually means is of course a matter of capitalists' guessing and speculation; it may turn out that relative price changes are, ex-post, larger than was allowed for when

[21] T. Fujimoto, "Inventions and Technical Change: a Curiosum", *The Manchester School,* LI, March 1983, pp. 16-20.

[22] Cf. B. Schefold, "Different Forms", *op. cit.*; this author has suggested that mechanization in his sense conforms to mechanization as described by Marx.

[23] Cf. A. Salanti, "Prices of Production, Market Prices and the Analysis of the Choice of Techniques", *Metroeconomica,* XXXVII, February 1985, pp. 97-117.

[24] On the one hand, uncertainty about the future sectoral patterns of technical change, and the possibly non-convergent behaviour of market prices make uncertainty about commodity prices rather strong; on the other hand the possibility of a persistent fall of the living standards of the working population must be perceived as rather remote.

investment decisions were taken. The conclusion is that returns to old methods may occur, but they are likely not to be very frequent; capitalists' expectations about profitability cannot be systematically disappointed without generating a different course of action, that is, the search for methods that would be profitable under a still larger range of relative prices. According to the above interpretation, the appearance of the so-called historical forms of technical progress sharing such convenient properties is not all that surprising. These properties, together with the long-run upward pressure of the real wage, ensure that technical progress does not move in circle.

Under this assumption one can now refer to a second order of considerations that help to bridge the gap between abstract analysis and historical facts. They have to do with the plausible dynamics of technical change for what may be called different technology systems[25], such as those represented by the steam-engine and the electric motor as sources of motive power in factories. Consider two alternative technology systems α and β. As long as the cost of production with the best practice β methods is close to that obtained by the best practice α methods the superiority and the potentialities of the β technology are not firmly established within the engineering community. At this stage innovations may still proceed prevalently along the path described by the pre-existing dominant technology α which may apparently offer a higher pay-off. Their effect would be further to prolong the life of the old technology or, in other words, they could make a return to it profitable. This occurence would be less probable, once the potentialities of the new technology start to be fully grasped, so that it acts as a "focusing device"[26] for technological research and its applications in many sectors.

If the above considerations help us to understand why technical change has not historically moved in a circle, they should not suggest that the possibility of returns to old methods and to old technologies can be considered as an irrelevant fluke. Indeed returns to old methods may occur, but they are likely to be only local phenomena on the sequence of the best practice methods, while returns to old technologies are likely to occur only under specific historical circumpstances (still-competing technological paradigms) of a temporary nature. The possibility of returns to old methods provides an implicit partial critique of the proposition that

[25] It has been suggested that the way in which alternative technology systems emerge, prevail and develop is akin to that of paradigms in scientific research; hence the term technological paradigm has been introduced. Cf. G. Dosi, "Technological Paradigms and Technological trajectories. A Suggested Interpretation of the Determinants and Directions of Technical Change", *Research Policy*, n. 2, 1982, pp. 147-62.

[26] Cf. N. Rosenberg, C. R. Frishtak, "Technological Innovation and Long Waves", *Cambridge Journal of Economics*, VIII, March 1984, pp. 11-13.

embodied technical progress fosters an early scrapping of machinery. What has still to be duscussed is whether the proposition may apply when such a possibility is ruled out.

3.3. The assumption of instantaneous price convergence is now temporarily added to the assumptions about price behaviour that have been introduced. This allows dealing with the scrapping decision in a very simple way[27]. At the new prices P the present value of a new type process must be zero; thus an old type process is kept in activity only if it yields a non-negative present value over all or part of its residual life at the new prices. The analysis can therefore be carried out in terms of a comparison of the new with the old prices P^*. In a fixed capital single product system of the type described in section 2., when cost reducing processes are introduced in each sector and the possibility of returns to old methods just discussed is ruled out, the new prices of each finished good in terms of the wage rate is lower than the old ones: $\hat{P} < \hat{P}^*$. Since prices converge instantaneously, this means that the value of current labour costs increases with respect to the value of current output in each old type process at time $t = 0$. It is convenient to consider first what this implies in the particular case in which the structure of the relative commodity prices does not change with the new technique. The fall of the price of each commodity in terms of the wage rate would imply here the fall of the current net proceeds of each old type process during its productive life. For a sufficiently large improvement in best practice technology and thus for a sufficient increase of the wage rate, the truncation of the process would therefore take place before it could reach the pre-innovation economic life. The same conclusion would still hold true for at least one sector also with a non-constant relative price structure. The sector in question would be the one producing the finished good whose price, in terms of any other finished good, falls at time $t = 0$. Quite the opposite conclusion may however be reached for some, or possibly all, other sectors. It could be the case that the change in relative prices brought about by technical progress is such that it is profitable to delay rather than to anticipate the truncation of some old type processes that are in use at time $t = 0$. This effect depends on a change in relative prices. Thus, if the sectoral patterns of technical progress are such that the structure of relative prices does not change through time, the effect cannot come about; this is however a case that may only occur as a fluke. Indeed the proposition that embodied technical change fosters earlier scrapping of machinery is far from being general from the point of view of the theory considered above.

[27] The result of the following discussion can easily be modified to take into account of non-instantaneous price convergence. See below, footnote 28.

3.4. If economic history provides relatively abundant evidence in that sense, this must be explained with the scant realism of some assumptions of the theory and/or with the occurrence of specific empirical conditions[28]. As it turns out, the unwarranted assumption of a constant structure of production prices is unnecessary to explain the historical evidence of early replacement accompanying the introduction of new productive methods. However, it is crucial to observe that the explanation of the above evidence does not rest upon a systematic influence of embodied process innovations on the life of equipment that can be captured by a simple functional relationship. Such a representation is not available in as far as the above influence is conditional upon a multiplicity of empirical conditions[29] that cannot be reasonably assumed as constant through time. For this reason no systematic relationship can be established between the pace of embodied process innovations and the flow of gross investment. This conclusion is reinforced by the analysis of the following section, where the author is prepared to concede — for the sake of the argument — that the prevailing pattern of embodied technical progress is such that the life of machines can be systematically related to the pace of innovations. Would technical progress invariably raise gross investment? The answer appears to be in the positive, as long as technical progress is expected to be of the "once and for all" type; this is the case when capitalists hold static expectations about technical knowledge so that no further changes of it are expected in the future. We have now to assess whether the same conclusion would hold true when technical

[28] Certainly, instantaneous price convergence is unrealistic and if prices adjust with a considerable time lag, within such an interval a sufficiently high unexpected acceleration of technical progress would lead to early replacement. Slow price adjustment means, however, that changes in current prices depend also on past technical change; thus, the influence of a slow price adjustment on the life of machines would be less easy to ascertain when technical progress is not of the once and for all type, but it takes place at each point in time, at a possibly variable speed. Still, one could attempt to interpret the historical evidence about replacement by considering how the form of the observed time flow of innovations combines with the lag structure of price adjustment. Further considerations have to do with the fact that technical progress does not fall like manna from heaven. Indeed, an early replacement policy may allow for faster learning; when the results of endogenous technical progress cannot be easily taken up by imitators an early replacement policy may give rise to substantial competitive advantages. Thus, replacement policies are influenced by the height of barriers to imitation which depend on the nature of the technology involved, on market structures, etc. (Cf. M. Kamien, N. Schwartz, *Market Structure and Innovation*, Cambridge, Cambridge University Press, 1982). Last, but not least one must take into account that as long as labour costs add up to a high share of current variable costs, and process innovations allow for persistent increases of the real wage, early replacement seems to be favoured. True enough, the rise of the real wage referred to above must be understood as a long run phenomenon, while in the present context also cyclical movements should be considered. When, in accordance with a Marxian perspective, process innovations are introduced to restore profitability after periods of fast rising wages, early truncation of the old processes is easily explained provided negative price Wicksell effects are limited to commodities whose weight in current variable cost is relatively low; indeed this must be the case if the above condition about the relative weight of labour costs is satisfied.

[29] See above, n. 28.

progress is not of the once and for all type. Before proceeding in this direction, it should be mentioned that historians are inclined to depict the process of technological change as occurring through a long sequence of frequent minor innovations usually following a major technological breakthrough not yet available for economic application[30]. This view seems to be very far from the idea of a process occurring through well-defined and drastic changes taking place at distant points over time. To the extent that capitalists adapt their expectations to the observed pace of technical progress, non-static technological expectations are therefore more realistic. The analysis of the following section applies to situations where capitalists face a persistent rate of change in the state of technical knowledge which is *correctly* perceived as part of the normal state of affairs[31]. The aim of the analysis is to compare the investment activity in alternative systems characterized by different such rates. Under the usual strong stability assumptions the same analysis may be interpreted as describing the persistent effects on investment behaviour of a persistent change in the rate of technical progress.

4. PERSISTENT RATES OF TECHNICAL PROGRESS

4.1. The preliminary question to be answered is whether a persistent shortening of equipment life would have no effect, in the long run, on aggregate demand, because as has been argued[32], the increase in gross investment would be matched by an equal increase in gross business saving. R. Eisner criticized this view on two accounts[33]: *i*) The increase in gross business savings would not take place because it is usual business practice in any case to amortize the equipment in its first years of life. *ii*) Even if the increase in gross business savings exactly matches the increase in gross investment, the effect on aggregate demand would not be neutral. The reason is that a share of the former disposable income that goes to business savings would have been saved any way, so that the

[30] Cf. N. ROSENBERG, "The Historiography of Technical Progress", in N. Rosenberg, *Inside the Black Box: Technology and Economics*, Cambridge, CUP, 1982, pp. 6-8.

[31] This assumption shares with the assumption of static expectations the fact that technological uncertainty is ruled out. In the opinion of the author phenomena discussed in section 4 would be relevant also when technological uncertainty is taken into account and in particular when capitalists' degree of uncertainty about future technical possibilities increases.

[32] "In the long period, if entrapreneurs foresee the rate of technical progress correctly and adjust their depreciation allowances accordingly, any increase in gross investment that may result from a speeding up of technical progress will be offset by higher gross business saving". R. C. O. MATTHEWS, *op. cit.*, p. 68.

[33] Cf. R. EISNER, "Technological Change, Obsolescence and Aggregate Demand", *The American Economic Review*, XLVI, March 1956, pp. 103-4.

increase in gross business savings would be partly compensated by a drop in family savings.

The above arguments, however correct, do not allow us to conclude that a persistent increase in the rate of embodied technical progress would necessarily increase aggregate demand. Indeed a persistent increase in the rate of technical progress inducing a non-temporary shortening of equipment life may trigger changes in productive techniques with compensatory effects on demand[34].

4.2. The first case to be considered is that the producers of durable capital goods react to the shortening of the *economic* life of equipment by constructing machinery with a shorter *physical* useful life and which are thus less sturdy and cheaper[35]. This is here assumed to occur through the saving of material and labour inputs in the construction of machinery. An extreme, if quite abstract, possibility is also contemplated. This would arise if capitalists choose completely to avoid the use of fixed capital.

4.3. Other options may be open to the capitalists facing a persistent increase in the rate of obsolescence. One such alternative is related to the desired daily working time of fixed capital[36]. A longer working time implies higher labour costs during the utilization of machinery, in that a night shift or simply overtime work has to be introduced. The higher labour costs are therefore due to the wage differential imposed by economic and institutional factors on the use of night and overtime work[37]. It is also essential to observe that a high rate of capacity utilization is likely

[34] Arguing along quite ortodox lines R. Eisner approached the problem as follows. A shorter life of machinery amounts to a higher rate of depreciation and hence, ceteris paribus, to a higher service cost of capital. The ensuing substitution of labour for capital would bring about a fall of the capital output ratio that would adversely affect the demand for gross investment. Cf. R. Eisner, *op. cit.*, p. 99. The above argument, as it stands, is not tenable because it is exposed to the difficulties of traditional capital theory. The fact that the author tries to limit the implications of his (wrong) theoretical argument with the weak, because empirical, argument of the low interest elasticity of the demand for investment is also open to objections. Cf. *ibidem*, pp. 102-3.

[35] "It would seem natural that if a machine is expected to be used for a shorter time, it should be made less durable than if it is expected to be used for a longer time". J. Stiglitz, H. Uzawa, Introduction to Part II of, J. Stiglitz, H. Uzawa (ed. s), *Readings in the Modern Theory of Económic Growth*, Cambridge Mass., M.I.T. Press, 1969, p. 124. The same point is also insisted on by: J. G. Williamson, "Optimal Replacement of Capital Goods: the Early New England and British Textile Firms", *Journal of Political Economy*, LXXIX, November-December 1971, pp. 1321-2; N. Rosenberg, "On Technological Expectations", in N. Rosenberg, *Inside the Black Box, op. cit.*, pp. 108-9.

[36] The choice of the desired rate of capacity utilization is nothing but a choice of technique. Unless specific restrictions about technical coefficients are imposed, phenomena discussed by the reswitching debate may therefore arise. Cf. section 4 of H. D. Kurz, "Normal Positions", *op. cit.*, pp. 44-51.

[37] The present analysis does not consider the possibility that other inputs, beside labour, may be regarded as different commodities bearing a different price during day-tyme and night-time. Electricity may be a case in point.

to lead, through faster wear and tear, to a shorter life of the machinery[38]. Just to give an example, if the useful life of machinery (as measured by total working hours) is not affected by daily working hours, then the life (in terms of days) of a machine run with one daily shift is twice as long as the life of the same machine if run with two daily shifts[39].

4.4. The forms of technical change described in paragraphs 4.2. and 4.3. share the following property: the new process introduced thereby, if compared with the old process (producing the same constant quantity of final output at each age of its productive phase), is characterized by a shorter physical life and by a lower requirement of material inputs at each date during the period of construction of machinery[40]. For this second property they are referred to in the present paper with the label: fixed capital saving (FCS) technical change. As far as the utilization period of machinery is concerned, the adoption of a FCS process may have less predictable effects on technical coefficients. A longer daily working time for plant and equipment, in particular a continuous operation of machinery, leads to drastic gains of efficiency in some branches of production, such as steel or glass making. In other branches continuous operation involves a loss of efficiency through more frequent maintenance and repair services. Likewise, it is difficult to decide a priori how the construction of relatively short lived machinery will affect its efficiency pattern. However, if we confine our attention to the (aggregated) labour requirement, the wage differential between day and night work suggests that one must be prepared to allow for a (aggregated) labour requirement which is higher or, at least, not lower for the FCS process at each date of the utilization period of machinery. The problem is now to assess whether there is some truth in the proposition that in face of a persistent and sufficiently high increase in the pace of embodied technical progress FCS processes would be preferred.

4.5. To do this the following thought experiment is carried out[41]. It is supposed that a persistent and uniform rate ϱ of labour saving technical

[38] See J. M. Keynes, *The General Theory of Employment, Interest and Money*, London, Macmillan, 1936, pp. 66-73.

[39] The relevance of this point for decisions about desired utilization rates and investment is insisted on by P. Taubman, M. Wilkinson, "User Cost, Capital Utilization and Investment", *International Economic Review*, XI, June 1970, pp. 209-15. Strangely enough G. C. Winston in an otherwise useful article, seems to suggest that the above point is not relevant because changing utilization may not decrease the equipment's useful life (as in the example in the text); cf. G. C. Winston, "The Theory of Capital Utilization and Idleness", *Journal of Economic Literature*, XII, December 1974, p. 1308.

[40] For a more accurate description see the appendix, section B.

[41] The reader who is interested is invited to refer to section C. of the appendix.

progress prevails throughout the set of technical possibilities. Technical progress is everywhere embodied, as long as fixed capital is required in production[42]. If technical progress takes the form described above, for a given and constant rate of profit r one can easily identify a constant equilibrium price vector for finished goods and a real wage $\bar{w}(t)$ such that $\bar{w}(t+1) = \bar{w}(t)(1+\varrho)$[43]. The pattern of technical progress that has been assumed is indeed very peculiar; it has been selected in order to obtain the property of a constant relative price structure which, as it has been argued in the previous section, is one of the sufficient conditions yielding a non-increasing relationship between the rate of embodied technical progress and the economic life of machinery. Thus, given r, the optimal life of a process is a non-increasing function of ϱ. Conversely, since machines have constant efficiency, the optimal life of a process is a non-increasing function of r, given ϱ. At each date t in each integrated sector capitalists can choose to start either the best practice α process or the best practice β process for that sector. The latter, if compared with the former, satisfies the conditions which define a FCS technical change. Clearly parametric changes in r and in ϱ generate different optimally truncated systems. The crucial question that the present thought experiment attempts to answer can now be posed as follows. Is there a (non-empty) set of attainable rates of profit such that only α processes would be in use if $\varrho = 0$, while for a sufficiently high ϱ, FCS processes would be in use in one or more sectors? As the appendix shows, it is possible to specify sufficient conditions under which the above question must take a positive answer. One such condition requires that FCS technical change does not lead to a loss of efficiency in terms of the material input requirement at each date during the productive phase of a FCS process. This is, in itself, a very restrictive requirement; it is worth insisting, however, that it is by no means necessary.

These results can be interpreted in the sense that, under appropriate states of distribution, a drastic shortening of equipment life (induced by embodied technical progress) tends to favour the adoption of FCS methods in as far as they would lower the burden of overhead costs. However, there is no implication that the above effect on costs is sufficient to overcome other effects related to the possibly higher current input requirements of the FCS processes, during their productive phase. Only detailed knowledge of the data can tell which effect will prevail and this is very much in accordance with the spirit of the present paper. How this conclusion bears on the relationship between embodied technical prog-

[42] See above, section 2., p. 118.

[43] Correspondingly, a price system for machines can be determined such that the replacement cost of the u days old machine (with $u \geqq 0$) used in sector k equals the present value at r of its future net proceeds. Of course a machine of vintage t is replaced with a machine of vintage $t + 1$.

ress and effective demand remains to be considered. Before proceeding in this direction it should be observed that the same conclusions that have been drawn by comparing alternative economic systems characterized by different rates of labour-saving embodied technical progress can be extended to the case where the comparison is carried out between systems sharing a given rate of labour-saving technical progress, but where this is respectively embodied and non-embodied.

4.6. The question to be answered in the present paragraph regards the impact of a persistent increase in rate of embodied technical progress on the gross investment ratio i_t of a given economic system. Obviously enough, i_t may crucially depend on the growth performance experienced by the given economy at time t, and on its composition of output. To avoid both difficulties one is temped to resort to the usual assumption that the economy is in a state of balanced growth at a given rate g, $0 \leqq g < r$[44]. The present analysis is carried out by looking at the ordering among the gross investment ratios obtained in the following three situations:

i) $\varrho = 0$, that is, no change in the book of blueprints; only untruncated methods in use.

ii) $\varrho > 0$, only α methods are available, truncated α methods used in each sector.

iii) $\varrho > 0$, untruncated β methods used in each sector.

As intuition suggests, i_t (g) is likely[45] to be larger in *ii*) than in *i*). This is the usual result: shorter equipment life leads to a higher gross investment ratio. However, i_t (g) is likely to be markedly lower in *iii*) than in *ii*), and indeed it may well be lower in *iii*) than in *i*). The above results (proved in paragraphs C. 3 and C. 6 of the appendix), together with the analysis of the previous paragraph, suggest that even in the conditions under which a negative relationship can be established between the rate of technical progress and the economic life of machines, higher rate of technical prog-

[44] This assumption may appear in blatant contradiction with the other assumption of a persistent rate of embodied technical progress; indeed, if an economic system is following a balanced growth path, technical change would normally push the system away from the path. A closer inspection reveals, however, that the above contradiction does not arise, given the particular form of technical progress that is considered in the present section, which is ultimately purely labour saving. As the appendix shows, given the rate of growth and the methods of production in use at a specific date, the balanced composition of the output of finished goods would be independent of the rate of technical progress which would only affect the rate of increase of the real wage.

[45] The qualification refers to the fact that the orderings to be examined below do not hold for all types of price behaviour compatible with the assumptions of the paper. Such orderings may however hold also when price effects work in the "wrong" direction, while they would certainly hold if relative prices move in the "right" direction, or if they do not change at all. These propositions, together with the results presented in the text, are proved in the mathematical appendix. It is worth adding that there is no a priori reason for expecting a particular direction of change in relative prices because no assumption is made about intersectoral differences in the methods of production.

ress should not be light-heartedly associated with higher gross investment ratios. The reason is that high rates of technical progress may themselves bring about adjustments in productive techniques that have compensatory effects on i_t (g). It should be recalled that the above balanced growth comparisons were carried out under the assumption that $0 \leqq g < r$. Indeed, a negative rate of growth for aggregate output is in general not compatible with balanced proportions because the durability of machines sets limits to the velocity with which the output of old machines (and hence the existing fixed productive capacity) is allowed to fall. Once undesired excess capacity comes into existence, the analysis cannot proceed in terms of a comparison between gross investment ratios, because such ratios may all be zero, no matter what the rate of change of the best practice methods on the book of blue prints is. However, in a situation of widespread undesired excess capacity the effect of shorter machine durability is unambiguous. As has been often recognized in the context of trade cycle analysis[46], the effect would be to increase the velocity with which excess capacity is eliminated.

4.7. The above arguments suggest that the comparison of the cyclical behaviour displayed (under the conditions specified below) by two alternative systems, one of them being characterized by a shorter durability of equipment and by the adoption of FCS methods, may reveal asymmetric results with respect to the duration of booms and depressions. If the economy with shorter equipment life should be expected to experience shorter depressions through faster elimination of excess capacity, it may not experience longer booms. Consider, for the sake of simplicity, a particular member in the class of aggregative trade cycle models where economic expansion is eventually halted by the ceiling of full employment[47]. In an extreme attempt at simplification and abstracting from time lags, the explanation of the upper turning point in this model is as follows. When the booming economy reaches the ceiling of the full employment of labour, the actual rate of output growth is constrained by the rate of increase of the labour force q_n and by the rate of increase of labour productivity ϱ. For this reason a gap must arise between the warranted rate of growth $G(t)$[48] and the actual rate of growth, so that excess

[46] See Harrod's famous remarks on this point in R. F. Harrod, "Notes on Trade Cycle Theory", *The Economic Journal,* LXI, June 1951, pp. 266-7.

[47] It is natural here to think of J. R. Hicks, *A Contribution to the Theory of the Trade Cycle,* Oxford, Clarendon Press, 1950; more explicitly concerned with the relationship between technical innovations and cyclical growth is, within the same class of models, Goodwin's celebrated "A Model of Cyclical Growth" as reprinted in R. M. Goodwin, *Essays in Economic Dynamics,* London, Macmillan 1982.

[48] For an interesting re-examination of Harrod's notion of a warranted rate of growth that stresses how this tool should be meant to apply in disequilibrium as well as in steady state cf. J. A.

capacity is bound to arise and the instability of the adjustment process brings the boom to an end. The warranted rate of growth $G(t)$ is determined by the long run gross savings ratio, given the methods of production in use at time t. It is the rate of growth at which the gross investment of the economy (assumed to be in balanced proportions[49]), given the methods of production in use, absorbs its full capacity savings corresponding to a desired gross savings ratio. The length of the time interval that the economy is allowed to spend on the ceiling of full employment must depend on the rate of increase of excess capacity which is a monotonically increasing function of the difference between the gross savings ratio and the gross investment ratio implied by balanced growth at the rate $(q_n + \varrho)$. Consider now two alternative systems sharing the same rate of increase in labour productivity ϱ, the same gross savings ratio and the same rate of increase of the labour force q_n. Technical progress is, respectively, embodied and non-embodied in such systems. The previous analysis has shown that there is no need, where technical progress is embodied, and the life of machines is shorter, for the system to have a higher gross investment ratio than the system where technical progress is non-embodied, while both experience the same growth rate. Thus there is no need for the periods of full employmenbt growth to be longer in the former system. The same analysis suggests that a persistent shortening of equipment life, if associated with the adoption of capital saving methods (whether or not related to a higher flow of innovations), should be expected to alter the period of the trade cycle.

5. EMBODIED PRODUCT INNOVATIONS

5.1. While the previous analysis has been mainly concerned with embodied process innovations, it is now time to consider the case of embodied product innovations. These may lead to the creation of completely new industries, to the entry of innovating firms in already existing industries or to the installation of new plant and equipment by old firms. An important distinction is usually drawn between the case in which the new product is a consumer good and that in which it is a producer good, although many goods can obviously be classified under both headings. The main reason for the distinction is that a new consumer good is likely to alter the consumer's perception of his needs and wants with the result of increasing

KREGEL, "Economic Dynamics and the Theory of Steady Growth: an Historical Essay on Harrod's Knife-edge", *History of Political Economy*, XII, Spring 1980, pp. 97-123.

[49] For a similar assumption and a justification thereof, cf. R. M. GOODWIN, *op. cit.*, pp. 130-1.

[50] Product innovations in the form of new machines have been assumed so far to be non-embodied.

his propensity to consume out of his income. A discussion of the influence of innovations on consumers' tastes goes beyond the scope of the present paper. It will suffice here to recall that before a systematic relationship can be postulated between the intensity of the innovation flow and the propensity to consume, very strong restrictions must be imposed on the qualitative characteristics of innovations.

To simplify our argument, let us now assume that the new commodity is either a producer good, or that it leaves the propensity to consume unchanged, if it is a consumer good. If the innovation proves successful it must replace, in part or completely, one or more of the existing goods in their physical uses, or simply as a share of consumers' demand. This replacement is nothing but the mirror image of the diffusion of the new product, a process requiring a certain lapse of time. It is argued below that the investment effects of a product innovation depend on the extent to which the displacement of the old products triggered by it is complete, or only partial. The distinction is best understood when the problem is considered in the context of the cyclical growth of output. During depression, investment is sustained by innovations, since the installation of capacity in the new industries is not curbed by the excess capacity faced by existing industries. The impact on investment depends, in this phase, on the displacement of existing goods by the new one. The same impact may possibly arise from the partial displacement of a product accounting (in value terms) for a relative large share in total output, or from a more complete displacement of a product whose share in output is lower. However, if observed over a longer time-span, the investment effects of the given innovation must be different in the former case as opposed to the latter. If the old product is made only partially obsolete by the innovation, the industry producing the former is not necessarily displaced by Schumpeterian "creative destruction". In as far as it loses part of its market to the new product, it will — in the first place — take more time before excess capacity is absorbed by the growth of aggregate demand during recovery and expansion; in the second place, after excess capacity is eliminated, the faster than average growth of the new industry will (at least partly) be compensated by the slower than average growth of the old industry in question. Thus, in all such cases of partial obsolescence the emergence of Schumpeterian "creative destruction" is dependent upon growth performance[51]. The flow of investment to be associated with a

[51] The circumstance has not gone unnoticed by economic historians: "In a rapidly growing economy, innovations may simply reduce the rate of growth in the (partially) obsolete sectors. A multi-purpose product or process may lose a part of its market to an innovation; but the growth of demand in the remaining markets, in which it is not obsolete or protected by market imperfections, may offset the loss. Indeed, such evidence as is available for the growth of competing production methods in the United States between 1850 and 1914 indicates that overall economic expansion

given innovation would be — ceteris paribus — markedly different when the obsolescence, triggered by it, it is not only partial. Thus, investment behaviour over the trade cycle must be sensitive to the pervasiveness of innovations within the relevant markets. This can be broadly defined for a given innovation as the extent of its market, after diffusion, relative to the former size of the market for the product made obsolete (whether partially or not) by it. It is worth insisting that the above concept introduces a further criterion to evaluate the demand effects of innovations, and this remark leads back to the main theme of the present paper.

Technical progress is necessarily a process with many dimensions (both for its sectoral location and for its qualitative features); the attempt to reduce such dimensions proves to be an obstacle to a proper understanding of its interaction with cyclical growth. Moreover, when the space where technical progress can be appropriately located is considered in its full dimensionality, the hypothesis (required by a strong version of the technological explanation of long waves) that its behaviour conforms to precise regularities, is easily falsified. In such conditions one could fruitfully start from the observation that the location of technical progress has changed over different historical periods, and set out to study how this circumstance has modified the relationship between technology and growth over such periods.

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usually, but not always, offsets the destructive potential of innovations". W. P. STRASSMAN, "Creative Destruction and Partial Obsolescence in American Economic Development", *The Journal of Economic History*, XIX, September 1959, pp. 335-49. The passage quoted above is on p. 338.

APPENDIX

A. MACHINES AND PROCESSES[52]

The productive method used in a given sector, say sector k, is here considered as unfolding through time, and it is called process. Inputs and outputs are classified according to the age (referring to the days of life already completed) of the process; thus $A_k(u)$, $B_k(u)$ stand for the vectors of material inputs and outputs which flow respectively into and out of a process k of age u[53]. The vector $L_k(u)$ is the corresponding input of heterogeneous labour. The price ratio w_f/w_g between a unit of labour force of type f and one of type g is assumed to be exogenously given. To obtain current labour costs the quantities of heterogeneous labour l_{fk} $f = 1, ..., g$ may be reduced to a quantity of homogeneous labour of type g through the scalar $l_k(u) = \sum_f l_{fk}(u)\, w_f/w_g$. Setting $w_g = w$ current labour costs for process k are then $l_k(u)\, w$. There are n integrated sectors, each producing a different finished good. The output of a k process of age $u > d + 1$ consists of a quantity of commodity k. Thus $b_{ik}(u) = 0$ if $i \neq k$. Since there is a production lag of one day (our unit time interval) and it takes d days to produce a machine, a k process of age $d + 1$ yields no final output $[b_{kk}(d + 1) = 0 \quad k = 1, ..., n]$ while $A_k(d + 1)$, $l_k(d + 1)$ are the inputs required to start the utilization of the machine employed in the process. Since no input is needed when the process is terminated, $A_k(T_k) = 0$, $l_k(T_k) = 0$ if process k ends its life at the age T_k. The assumption of constant efficiency of machines is formalized as follows:

$$b_{ii}(u + 1)\, k^a_{ji} = a_{ji}(u) \qquad b_{ii}(u + 1)\, k^l_i = l_i(u) \qquad i, j = 1, ..., n$$
$$u = d + 1, ..., T_k - 1 \qquad k^a_{ji} \geqq 0 \qquad k^l_i > 0.$$

A technique $A(u)$, $B(u)$, $L(u)$ $u = 0, 1, ..., T$ is a set of n processes, one for each integrated sector. For the sake of simplicity, it is assumed that all processes belonging to a given technique, have the same physical life T; thus $T_k = T$ $k = 1, ..., n$. Constant returns to scale are assumed.

[52] Appendix to section 2.

[53] Vector magnitudes are henceforth indicated with capital letters; conversely, capital letters refer to vector magnitudes with the following exceptions: T: physical life of a process; D: economic life of a process; R: maximum attainable rate of profit.

B. FCS TECHNICAL CHANGE[54]

With reference to a given technique α let $A_k^\alpha(u)$, $B_k^\alpha(u)$, $l_k^\alpha(u)$, $u = 0, 1, \ldots, T^\alpha$, be the process in use in *kth* integrated sector. If a FCS technical change occurs in the *kth* sector this must lead to the introduction of a process $A_k^\beta(u)$, $B_k^\beta(u)$, $l_k^\beta(u)$, $u = 0, 1, \ldots, T^\beta$, which, under appropriate normalization, displays the following properties:

0. $T^\beta < T^\alpha$

I. $B_k^\beta(u) = B_k^\alpha(u) \qquad 0 \leqq u \leqq T^\beta$

II. $A_k^\beta(u) \leq A_k^\alpha(u) \qquad 0 \leqq u \leqq d$

III. $l_k^\beta(u) < l_k^\alpha(u) \qquad 0 \leqq u \leqq d$

IV. $l_k^\beta(u) > l_k^\alpha(u) \qquad d < u \leqq T^\beta$

Conditions I, II and III above imply that for $D^\alpha = T^\beta$ the following inequalities hold true.

V. $$\frac{\sum_{u=0}^{d} A_k^\beta(u)\,(1+r)^{T^\beta - u}}{\sum_{u=0}^{T^\beta} b_{kk}^\beta(u)\,(1+r)^{T^\beta - u}} \leq \frac{\sum_{u=0}^{d} A_k^\alpha(u)\,(1+r)^{D^\alpha - u}}{\sum_{u=0}^{D^\alpha} b_{kk}^\alpha(u)\,(1+r)^{D^\alpha - u}}$$

VI. $$\frac{\sum_{u=0}^{d} l_k^\beta(u)\,(1+r)^{T^\beta - u}}{\sum_{u=0}^{T^\beta} b_{kk}^\beta(u)\,(1+r)^{T^\beta - u}} < \frac{\sum_{u=0}^{d} l_k^\alpha(u)\,(1+r)^{D^\alpha - u}}{\sum_{u=0}^{D^\alpha} b_{kk}^\alpha(u)\,(1+r)^{D^\alpha - u}}$$

Indeed, such inequalities may hold also for $T^\beta < D^\alpha$; if they hold at $D^\alpha = \bar{D}^\alpha > T^\beta$, they also hold for all values of D^α in the interval $[T^\beta, \bar{D}^\alpha]$. A restricted definition of FCS technical change is obtained by adding the following conditions to those listed above.

VII. $a_{ik}^\beta(u) \gtreqless a_{ik}^\alpha(u) \qquad i = 1, \ldots, n \qquad d < u \leqq T^\beta;$

VIII. VI. and VII. hold true for $D^\alpha = T^\alpha$.

We shall need to allow that a FCS technical change leads to the introduction of a process which avoids completely the use of machines. On the assumption that such a process would end its life at the age $T^\beta = \bar{T}^\beta$ and that (quite realisticly) $T^\beta > \bar{T}^\beta$ for a FCS process involving

[54] Appendix to paragraphs 4.2., 4.3. and 4.4.

the use of machinery, we can complete the definition of a FCS technical change as follows:

IX. $A_k^\beta(u) = 0, l_k^\beta(u) = 0 \qquad u = 0, ..., d$ if and only if $T^\beta = \bar{T}^\beta$.

C. PERSISTENT RATE OF EMBODIED TECHNICAL PROGRESS[55]

C.1. Consider a given economic system in which embodied technical progress proceeds regularly through time. For the sake of simplicity it is assumed that all machines have the same physical life, they are also supposed to have a constant efficiency in terms of the current physical requirement of each labour and material input, per unit of current output. The technique that is made first available at time t is here defined as:

$$A(u, t), B(u, t), L(u, t) \qquad u = 0, 1, ..., T.$$

The prevailing pattern of embodied technical progress is described by the following equalities:

$$\text{(I)} \qquad \begin{aligned} A(u, t) &= A(u, t+1) && u = 0, 1, ..., T; \\ B(u, t) &= B(u, t+1) && u = 0, 1, ..., T; \\ L(u, t) &= L(u, t+1)(1+\varrho) && u = 0, 1, ..., T, \varrho > 0. \end{aligned}$$

Clearly the set of techniques $A(u, t+h)$, $B(u, t+h)$, $L(u, t+h)$ $u = 0, 1, ..., T$, $h = 0, 1, 2, ...$ have a common maximum rate of profit R, and yield the same production prices for finished goods at the given rate of profit r. Since technical progress is embodied, the system of production in use at time t does not in general coincide with the best practice technique available at the same date. The system of production in use at time t in the economy under consideration is described by the following set of equalities:

$$\text{(II)} \qquad \begin{aligned} \bar{A}_t(u) &= A(u, t) && u = 0, 1, ..., T \\ \bar{B}_t(u) &= B(u, t) && u = 0, 1, ..., T \\ \bar{L}_t(u) &= L(u, t) && u = 0, 1, ..., d \\ \bar{L}_t(u) &= L(u, t-u) = L(u, t)(1+\varrho)^u && u = d+1, ..., T. \end{aligned}$$

It is worth stressing the difference between the notions of technique and of system of production that have been introduced above. A tech-

[55] Appendix to paragraphs 4.5. and 4.6.

nique is a set of processes, one for each integrated sector, written in the book of blueprints. A system of production is a set of processes, one for each integrated sector, as can be observed at some date in the economy. In the case here considered, technical progress being embodied, the system of production in use is the outcome of a persistent rate of change in the labour requirement of the best practice technique.

At each date t the prices ruling in the economy under consideration are assumed to coincide with the prices of production generated by the system of production in use at time t, at the exogenously given and constant rate of profit. This choice about price determination will be justified below. $\bar{w}$ is the real wage; it is a quantity of the composite commodity F, which is the standard of prices ($\bar{P}F = 1$). If no process is truncated before the end of its physical life, the prices at time t can be shown to be[56]:

$$\text{(III)} \qquad \bar{P}_t(r) = \hat{L}_t \bar{w}_t(r) [I - \hat{A}_t(r)(1 + r)]^{-1},$$

$\hat{A}_t$ and $\hat{L}_t$ being defined as follows. The generic element of the $n \times n$ matrix $\hat{A}_t(r)$ is:

$$\hat{a}_{ij,t} = \frac{\sum_{u=0}^{T} a_{ij}(u, t)(1 + r)^{T-u}}{\sum_{u=0}^{T} b_{jj}(u, t)(1 + r)^{T-u}}$$

The generic element of the $1 \times n$ vector $\hat{L}_t$ is:

$$\hat{l}_{j,t}(r) = \frac{\sum_{u=0}^{d} l_j(u, t)(1 + r)^{T-1} + \sum_{u=d+1}^{T} l_j(u, t)(1 + \varrho)(1 + r)^{T-u}}{\sum_{u=0}^{T} b_{jj}(u, t)(1 + r)^{T-u}}$$

The system of production in use at time $t + 1$ is

$$\bar{A}_{t+1}(u), \bar{B}_{t+1}(u), \bar{L}_{t+1}(u) \qquad u = 0, 1, ..., T,$$

where

$$\text{(IV)} \qquad \begin{aligned} \bar{A}_{t+1}(u) &= \bar{A}_t(u) \\ \bar{B}_{t+1}(u) &= \bar{B}_t(u) \\ \bar{L}_{t+1}(u) &= \bar{L}_t(u)(1/1 + \varrho) \end{aligned} \qquad u = 0, 1, ..., T.$$

The third equality above implies

$$\hat{l}_{j,t}(r) = \hat{l}_{j,t+1}(r)(1 + \varrho),$$

[56] For the relevant references see above, p. 9, n. 14.

which, taking into account (III) yields:

$$\text{(V)} \qquad \bar{P}_t(r) = \bar{P}_{t+1}(r) \qquad \bar{w}_t(r) = \bar{w}_{t+1}(r)\,(1/1+\varrho).$$

V. can be used to show that as long as capitalists are assumed to foresee correctly, with the pattern and speed of technical progress, also the resulting behaviour of the price system ($\bar{P}_t$, $\bar{w}_t$), the expected present value of the future net proceeds of all processes would be zero. To see this it is enough to notice that, indicating the expected magnitudes with the upperscript ^{e}, one obtains:

$$\bar{l}_{i,t}(u) = l^e_{i,t}(u)\,(1+\varrho)^u \qquad u = 0, \ldots, T \qquad i = 1, \ldots, n, \qquad \bar{w}^e_{t+u} = \bar{w}_t\,(1+\varrho)^u$$

yielding

$$\bar{l}_{i,t}(u)\,\bar{w}_t = l^e_{i,t}(u)\,\bar{w}^e_{t+u} \qquad u = 0, \ldots, T \qquad i = 1, \ldots, n.$$

Under the price system as determined in (III) and displaying the properties indicated in (V), no capitalist would expect to make extra profits by moving to a different sector. This shows that (III) is not at variance with the rules of competition.

It should be observed that the systems of production described in (II) and (IV) are not necessarily cost minimizing at r; at the ongoing prices extra profits may be obtained through appropriate truncations thereof. Indeed, under appropriate conditions of feasibility of truncations, it can be easily shown that for a sufficiently high level of ϱ, the optimal economic life of each process in the system of production in use at time t is shorter than its physical life.

Let the optimal truncation at r of the *ith* process in the systems of production defined in (II) and (IV) be identified by the economic lives $\bar{D}_{i,t}(r)$ and $\bar{D}_{i,t+1}(r)$, respectively. Since for a given truncation and for a given rate of profit relative commodity prices do not change through time, we must have:

$$\bar{D}_{i,t}(r) = \bar{D}_{i,t+1}(r) = \bar{D}_i(r)$$

(eliminating the time script). In what follows it is assumed that:

$$\text{(VI)} \qquad \bar{D}_j(r) < T \qquad j = 1, \ldots, n.$$

On the assumption that only optimally truncated systems of production would be in use, the appellation system of production and the notation introduced to represent such a system is understood to refer to an optimally truncated system; if not, explicit notice will be given.

C.2. Consider a second economic system, where technical knowledge does not change through time. The technique in use in this system coincides with the best practice technique available at a given date t in the other system. All magnitudes referring to the former are indicated with upperscript*, all magnitudes referring to the latter without it. Since all machines have a constant efficiency, no machine would be scrapped before the end of its physical life in the economy where $\varrho = 0$; this implies that all processes would last T days.

C.3. There are reasons to expect that in conditions of balanced growth a shorter life of machinery caused by a higher rate of embodied technical progress would imply a higher gross investment ratio. Let $i_t\,(g)$, $i_t^*\,(g)$ be the ratios between gross investment[57] and gross output in the two economies under consideration if they expand in conditions of balanced growth at the rate g, $0 \leqq g < r$. With reference to the economy where $\varrho > 0$, it can be easily shown that:

$$1/i_t\,(g) = \frac{\lambda_t\,(g)}{\bar{P}_t \bar{A}_t\,(g)\,(1+g)\,\bar{Q}_t} + 1;$$

where $\bar{Q}_t$ is the $n \times l$ vector of activity levels for the n integrated sectors, $\lambda_t\,(g)$ is the value of aggregate consumption of commodity F, which is the standard of prices ($\bar{P}F = 1$), and the generic element $\bar{a}_{ij,\,t}\,(g)$ of the matrix $\bar{A}_t\,(g)$ is obtained as follows:

$$\bar{a}_{ij,\,t}\,(g) = \sum_{u=0}^{\bar{D}_j} a_{ij}\,(u,\,t)\,(1+g)^{\bar{D}_j-u}.$$

No other consumption commodity, beside F, is assumed to exist. The diagonal matrix $\bar{B}_t\,(g)$ must also be defined; the jth element ($j = 1, \ldots, n$) on the main diagonal of this matrix is:

$$\bar{b}_{jj,\,t}\,(g) = \sum_{u=0}^{\bar{D}_j} b_{jj}\,(u,\,t)\,(1+g)^{\bar{D}_j-u}.$$

In the economy where $\varrho = 0$ the reciprocal of the gross investment ratio is, in obvious notation:

$$1/i_t^*\,(g) = \frac{\lambda_t^*\,(g)}{\bar{P}_t^* \bar{A}_t^*\,(g)\,(1+g)\,\bar{Q}_t^*} + 1;$$

[57] The definition of gross investment adopted here and consistent with the characteristics and purposes of the present model, differs from the usual definition on two accounts: *a)* The expenditure for machines is not included, since all machines are concealed from the view through vertical integration. *b)* The expenditure for the acquisition of all circulating capital inputs is included and indeed it exhausts the definition.

Again, the generic element of the matrix $\bar{A}_t^*(g)$ is:

$$\bar{a}^*_{ij,t}(g) = \sum_{u=0}^{T} a_{ij}(u, t)(1+g)^{T-u}.$$

The diagonal matrix $\bar{B}_t^*(g)$ is also introduced; the *ith* element ($j = 1, ..., n$) on the main diagonal of this matrix is:

$$\bar{b}^*_{jj,t}(g) = \sum_{u=0}^{T} b_{jj}(u, t)(1+g)^{T-u}.$$

Without loss of generality suppose:

(VII) $$\lambda_t(g) = \lambda_t^*(g);$$

then,

(VIII) $$i_t(g) > i_t^*(g)$$

if $$\bar{P}_t \bar{A}_t(g)(1+g)\bar{Q}_t > \bar{P}_t^* \bar{A}_t^*(g)(1+g)\bar{Q}_t^*.$$

The above inequality does not hold under all types of price behaviour that the model may yield. The important point is that it would hold even under moderate price effects acting in the opposite direction (indeed, under the present assumptions there is no reason to expect that price effects should act in a particular direction). This can be shown as follows:

(IX.I) $$[\bar{B}_t(g) - \bar{A}_t(g)(1+g)]\bar{Q}_t = \lambda_t(g)F,$$

thus $$[I - \hat{A}_t(g)(1+g)]\bar{B}_t(g)\bar{Q}_t = \lambda_t(g)F,$$

where $$\hat{A}_t(g) = \bar{A}_t(g)[\bar{B}_t(g)]^{-1}.$$

After simple manipulation:

(IX.II) $$\bar{B}_t(g)\bar{Q}_t = [I - \hat{A}_t(g)(1+g)]^{-1}\lambda_t(g)F.$$

Analogously,

(IX.III) $$[\bar{B}_t^*(g) - \bar{A}_t^*(g)(1+g)]\bar{Q}_t^* = \lambda_t^*(g)F$$

and

(IX.IV) $$\bar{B}_t^*(g)\bar{Q}_t^* = [I - \hat{A}_t^*(g)(1+g)]^{-1}\lambda_t^*(g)F.$$

The crucial point to observe is that $\hat{A}_t\,(g) \geq \hat{A}_t^*\,(g)$ because $\bar{D}_j\,(r) < T$, $j = 1, ..., n$. (IX.I) to (IX.IV) can then be used to obtain

$$\bar{A}_t\,(g)\,(1+g)\,\bar{Q}_t > A_t^*\,(g)\,(1+g)\,\bar{Q}_t^*$$

This implies that (VIII) always holds true if $\bar{P}_t = \bar{P}_t^*$.

To construct an example corresponding to the above situation, think of the particular case where all processes belonging to the same technique have the same input profile; starting from this case, it is now easy to construct an example where (VIII) holds, while $\bar{P}_t \leq P_t^*$[58].

[58] Suppose the consumption commodity is non-composite and let h be the process corresponding to the finished good which is consumed. Since economic life is a discrete variable in the present model it is always possible to choose $\varrho_0 > 0$ s.t. the optimal truncation at r of process h in the system of production in use at time t would be unaffected by a sufficiently small change of the input coefficients $A_h\,(u, t)$ $u = 0, ..., T$. Let $A\,(u, t)$, $B\,(u, t)$, $L\,(u, t)$ $u = 0, ..., T$ be s.t. $A_i\,(u, t) = A_j\,(u, t)$, $l_i\,(u, t) = l_j\,(u, t)$ $i, j = 1, ..., n$ $\quad u = 0, ..., T$, which implies: $\bar{D}_i\,(r) = \bar{D}_j\,(r)$ $\quad i, j = 1, ..., n$; $\bar{P}_t = \bar{P}_t^*$. Let us also assume that $A_i\,(u, t) = A_i\,(u+1, t)$, $l_i\,(u, t) = l_i\,(u+1, t)$ $\quad i = 1, ..., n$ $\quad u = 0, ..., T-1$, which, together with $\bar{D}_i\,(r) = \bar{D}_j\,(r)$ $\quad i, j = 1, ..., n$, implies:

(IX.VII.) $$\hat{a}_{ij,t}\,(r)\,\pi = \hat{a}^*_{ij,t} \qquad \hat{l}_{i,t}\,(r)\,\gamma = \hat{l}^*_{i,t}\,(r) \qquad i, j = 1, ..., n$$

where $0 < \gamma < \pi < 1$ since $\bar{D}_i\,(r, \varrho_0) < T$ by assumption.

Choose j s.t. $a_{jh}\,(u, t) > 0$; multiply $a_{jh}\,(u, t)$ $\quad u = 0, ..., d$ by the scalar $\sigma > 1$, multiply $a_{jh}\,(u, t)$ $u = d+1, ..., T$ by the scalar $\delta < 1$, where σ and δ are s.t., at the given r,

(IX.VIII.) $$\sum_{u=0}^{d}\,(\sigma - 1)\,a_{jh}\,(u, t)\,(1+r)^{\bar{D}_h(r,\varrho_0)-u} = \\ = \sum_{u=d+1}^{\bar{D}_h(r,\varrho_0)}\,(1-\delta)\,a_{jh}\,(u, t)\,(1+r)^{D_h(r,\varrho_0)-u}.$$

Indicating with $A'\,(u, t)$ $\quad u = 0, ..., T$ the modified material input matrixes, (IX.VIII) can be also expressed as $\hat{a}'_{jh,t}\,(r) = \hat{a}_{jh,t}\,(r)$. One is allowed to state that for σ and δ sufficiently close to 1 [while still meeting (IX.VIII.)] the vector of the optimal truncations at ϱ_0 would be unaffected by the above change in coefficients. At the same it would be, by construction:

(IX.IX.) $$\hat{a}'_{jh,t}\,(r)\,\pi > \hat{a}'^*_{jh,t}\,(r).$$

Compare the vectors

$$\bar{P}'_t = \hat{L}_t\,(r)\,\bar{w}_t\,(r)\,[I - \hat{A}'_t\,(r)\,(1+r)]^{-1} = \bar{P}_t,$$

$$\bar{P}'^*_t = \hat{L}'^*_t\,(r)\,\bar{w}'^*_t\,(r)\,[I - \hat{A}'^*_t\,(r)\,(1+r)]^{-1}, \gamma\,\hat{L}_t\,(r)\,\bar{\bar{w}}'_t\,(r)\,[I - \pi\,\hat{A}'_t\,(r)\,(1+r)]^{-1}$$

(where $\bar{\bar{w}}'_t = 1 / \{\hat{L}_t\,(r)\,[I - \pi\,\hat{A}'_t\,(r)\,(1+r)]^{-1}\}$), which are normalized to obtain:

$$\bar{P}'_t\,F = \bar{P}'^*_t\,F = \gamma\,\hat{L}_t\,(r)\,\bar{\bar{w}}'_t\,(r)\,[I - \pi\,\hat{A}'_t\,(r)\,(1+r)]^{-1}\,F = 1.$$

Recalling

$$\hat{A}'_{i,t}\,(r) = \hat{A}'_{j,t}\,(r), \hat{l}_{i,t}\,(r) = \hat{l}_{j,t}\,(r) \qquad i, j = 1, ..., n,$$

one can write:

$$\bar{P}'_t = \gamma\,\hat{L}_t\,(r)\,\bar{\bar{w}}'\,[I + \sum_{z=1}^{\infty}\,(\pi\,\hat{A}'_t\,(r)^z] \leq \bar{P}'^*_t.$$

Through an appropriate choice of σ and δ, $[\bar{P}'^*_t - \bar{P}'_t]$ can be made arbitrarily close to $[0, ..., 0]$ while still $\hat{A}'_t\,(r) \geq \hat{A}'^*_t\,(r)$. This shows how to construct an example where (VIII) holds true with $\bar{P}_t \leq \bar{P}_t^*$.

C.4. It is now assumed that in the system where $\varrho > 0$ a second technique, denoted β_t, is written at each date t in the book of blueprints. In full notation this technique is:

$$A^{\beta}(u, t), B^{\beta}(u, t), L^{\beta}(u, t), u = 0, 1, ..., T^{\beta}.$$

By contrast the technique considered in the previous paragraph and also written at time t in the book of blueprints is now referred to as α_t:

$$A^{\alpha}(u, t), B^{\alpha}(u, t), L^{\alpha}(u, t), u = 0, 1, ..., T^{\alpha}.$$

All the assumptions concerning technique α_t carry over to technique β_t. Each process of the latter, if compared to the process of the former which yields the same final output, satisfies the conditions defining a FCS technical change as specified in section B of the appendix. The latter is also assumed to undergo the same pattern of technical progress generating the sequence of best practice α techniques. Let us also define the system of production $\bar{B}_t$ (which is not necessarily cost minimizing at r) obtained if the *jth* process, $j = 1, ..., n$, of the system α_t is replaced by the process:

$$\bar{A}^{\beta}_{j,t}(u), \bar{B}^{\beta}_{j,t}(u), \bar{l}^{\beta}_{j,t}(u) \qquad u = 0, ..., T^{\beta};$$

as before:

$$(X) \quad \begin{array}{ll} \bar{A}^{\beta}_{j,t}(u) = A^{\beta}_{j}(u, t) & \\ \bar{B}^{\beta}_{j,t}(u) = B^{\beta}_{j}(u, t) & u = 0, ..., T^{\beta} \\ \bar{l}^{\beta}_{j,t}(u) = l^{\beta}_{j}(u, t) & u = 0, ..., d \\ \bar{l}^{\beta}_{j,t}(u) = l^{\beta}_{j}(u, t)(1 + \varrho)^{u} & u = d + 1, ..., T^{\beta} \text{ if } T^{\beta} > \bar{T}^{\beta} \\ \bar{l}^{\beta}_{j,t}(u) = l^{\beta}_{j}(u, t) & u = d + 1, ..., T^{\beta} \text{ if } T^{\beta} = \bar{T}^{\beta} \end{array}$$

Going back to the system where $\varrho = 0$ we may call α^* the technique available in such a system. By construction (see above, paragraph C.2.), for a properly chosen date t_0 it is: $\alpha^* = \alpha_{t_0}$; let us now introduce in that system a second technique β^* such that $\beta^* = \beta_{t_0}$. Since technical knowledge is given and machines have constant efficiency, if technique α^* is adopted it must be $\bar{\alpha}^* = \alpha^*$; alternatively, if β^* is adopted it must be $\bar{\beta}^* = \beta^*$.

C.5. Let us now call S_r the subset of $[0, R^{\alpha}]$ such that if $r \in S_r$, then α^* is at least as profitable as β^* at r, while, for a sufficiently high level of ϱ, $\bar{\beta}_t$ is more profitable than $\bar{\alpha}_t$ at r. The present paragraph specifies a sufficient condition such than, under the restricted definition of FCS technical change given in section B., the set S_r is not empty. Before proceeding in this direction it is worth adding a few remarks. A first remark has to do with the assumption of constant efficiency. The assumption implies that if $\varrho = 0$, untruncated processes would be in use, no matter what the level of r is. If however $\varrho > 0$, a non-decreasing relationship would hold between the optimum process life and the rate of profit r. A second remark has to do with the wage-profit functions $w_t^\alpha(r)$, $w_t^\beta(r)$ corresponding to techniques α_t and β_t, and defined on the closed domain $[0, R^\alpha]$, $[0, R^\beta]$, respectively. V. and VIII. of section B. imply: $R^\beta > R^\alpha$. Since it is assumed that r_1 exists, $0 \leqq r_1 < R^\alpha$, such that $w_t^\alpha(r_1) > w_t^\beta(r_1)$, then there exists r_0 in $(0, R^\alpha)$ such that $w_t^\alpha(r_0) = w_t^\beta(r_0)$. This follows from the continuity of $w(r)$ and from the fact that $R^\beta > R^\alpha$. Obviously the optimum truncation of a system of production depends on r and on ϱ; for this reason the optimally truncated system $\bar{\alpha}_t$ is now more correctly referred to as $\bar{\alpha}_t(\varrho, r)$. The corresponding life for process j is $\bar{D}_j^\alpha(\varrho, r)$. Since $r_0 < R^\alpha$ it is reasonable to assume that there exists a sufficiently high level of ϱ, let us call it ϱ_0, such that $\bar{D}_j(\varrho_0, r_0) < T^\alpha \quad j = 1, \ldots, n$. If one compares the time integrated unit labour coefficients of the systems $\bar{\alpha}_t(\varrho_0, r_0)$, $\bar{\beta}_t$ and of the techniques α_t, β_t one obtains that if T^β is sufficiently low[59], then:

(XI) $$[\hat{\bar{l}}^\beta_{j,t}(r_0) / \hat{l}^\beta_{j,t}(r_0)] < [\hat{\bar{l}}^\alpha_{j,t}(r_0) / \hat{l}^\alpha_{j,t}(r_0)] \quad j = 1, \ldots, n.$$

Thus, if C^α and C^β are two diagonal matrixes such that

$$\hat{L}_t^\alpha(r_0)\, C^\alpha = \hat{\bar{L}}_t^\alpha(r_0),\ \hat{L}_t^\beta(r_0)\, C^\beta = \hat{\bar{L}}_t^\beta(r_0),$$

(XI) implies:

(XII) $$C^\alpha > C^\beta$$

For definition of r_0,

$$\frac{1}{w_t^\alpha(r_0)} = \hat{L}_t^\alpha(r_0)\, [I - \hat{A}_t^\alpha(r_0)\,(1 + r_0)]^{-1} F =$$
$$= \hat{L}_t^\beta(r_0)\, [I - \hat{A}_t^\beta(r_0)\,(1 + r_0)]^{-1} F = \frac{1}{w_t^\beta(r_0)}.$$

Let us also assume, temporarily, $\underset{j}{\mathrm{Min}}\,(c_{jj}^\alpha) > \underset{j}{\mathrm{Max}}\,(c_{jj}^\beta)$. One can write:

$$\frac{1}{\bar{w}_t^\beta(r_0)} = \hat{L}_t^\beta(r_0)\, C^\beta\, [I - \hat{A}_t^\beta(r_0)\,(1 + r_0)]^{-1} F =$$

[59] (XI) below is of course necessarily true in the extreme case where $T^\beta = \bar{T}^\beta$ so that IX. of appendix B and the last equality of (X), paragraph C.4., apply. (XI) may well be true, however, also for $T^\beta > \bar{T}^\beta$.

$$= \hat{L}_t^\beta(r_0)\, C^\beta [I - \hat{A}_t^\beta(r_0)(1 + r_0)]^{-1} F \leqq$$
$$\underset{j}{\mathrm{Max}}\,(c_{jj}^\beta)\, \hat{L}_t^\beta(r_0)\, [I - \hat{A}_t^\beta(r_0)(1 + r_0)]^{-1} F =$$
$$= \underset{j}{\mathrm{Max}}\,(c_{jj}^\beta)\, \hat{L}_t^\alpha(r_0)\, [I - \hat{A}_t^\alpha(r_0)(1 + r_0)]^{-1} F <$$
$$\underset{j}{\mathrm{Min}}\,(c_{jj}^\alpha)\, \hat{L}_t^\alpha(r_0)\, [I - \hat{A}_t^\alpha(r_0)(1 + r_0)]^{-1} F <$$
$$< \hat{L}_t^\alpha(r_0)\, C^\alpha [I - \hat{A}_t^\alpha(r_0)(1 + r_0)]^{-1} F = \frac{1}{\bar{w}_t^\alpha(r_0)}$$

Now observe that, if $T^\beta = \bar{T}^\beta$ then $\underset{j}{\mathrm{Min}}\,(c_{jj}^\alpha) > \underset{j}{\mathrm{Max}}\,(c_{jj}^\beta)$ since $c_{jj}^\alpha > 1$, $c_{jj}^\beta = 1 \quad j = 1, ..., n$. If $T^\beta > \bar{T}^\beta$, as long as (XI) still holds true, one can easily state sufficient conditions for $\underset{j}{\mathrm{Min}}\,(c_{jj}^\alpha) > \underset{j}{\mathrm{Max}}\,(c_{jj}^\beta)$ such as:

(XIII.I.) $\qquad L_t(u) = L_t(u + 1) \qquad u = 0, 1, ..., T - 1;$

(XIII.II.) $\qquad \bar{D}_i(\varrho_0, r_0) = \bar{D}_j(\varrho_0, r_0) \qquad i, j = 1, ..., n.$

C.6. In the present paragraph it is shown that, in conditions of balanced growth at the rate g $(0 \leqq g < r)$, the gross investment ratio i_t^β for the system of production β_t may well be lower than the gross investment ratio $i_t^{\alpha *}$ for the system of production $\bar{\alpha}_t^*$. The proposition rests on the fact that if FCS technical change satisfies conditions VII. and VIII. (section B.) we must have:

(XIV) $$\hat{\bar{A}}_t^\beta(g) < \hat{\bar{A}}_t^{\alpha *}(g).$$

The same steps indicated in paragraph C.3. can now be followed to show that for $\lambda_t^\beta(g) = \lambda_t^{\alpha *}(g)$ one obtains:

(XV) $$\bar{A}_t^\beta(g)\,(1 + g)\,\bar{Q}_t^\beta < \bar{A}_t^{\alpha *}(g)\,(1 + g)\,\bar{Q}_t^{\alpha *}.$$

(XV) suggests that examples can easily be constructed, where $i_t^\beta < i_t^{\alpha *}$ even under moderate price effects acting in the opposite direction.