political economy *Studies in the Surplus Approach*

volume 4, number 2, 1988

Joint Production: Review of Some Studies on Sraffa's System

Enrico Levrini*

Introduction

The present paper is a review of some relevant issues about joint production; the text does not contain original results. We shall avoid a formal treatment of the subject. A basic knowledge of the single production system is assumed as given as well as concepts commonly used within that framework.

The paper is divided in two sections. In the former we shall examine the general joint production system, focusing on those results that seem to explore more fruitfully the framework itself,[1] while in the latter we shall give an account of a number of studies dealing with particular kinds of joint production systems — those systems where the only element of joint production is fixed capital.[2]

The structure of the paper makes it clear that we are going to deal only with the very basics of joint production. There are other extremely interesting fields of analysis that will not be examined in this paper. A typical example is the series of studies on the changes in methods of production in joint production systems. In this field of research it has been pointed out that, under certain conditions, it is not always possible to determine the most profitable among alternative systems of production, and that, even when it is possible to determine this system, it may not be unique. It is possible to collect a number of comments on these results, but not as many analyses of the effects and counter-effects that determine the overall result.

In our opinion the basic reason for this is that we still lack a full and

* I would like to thank the editorial committee of *Political Economy* and an anonymous referee of the journal for helpful comments on an earlier version of the paper; any errors and omissions that may remain are my responsability alone.

[1] The reader might wish to refer, for a more complete formal treatment, to G. Giorgi, U. Magnani, "Problemi aperti nella teoria dei modelli multisettoriali di produzione congiunta", *Rivista internazionale di scienze sociali*, LXXXVI, 1978, pp. 435-468.

[2] This theoretical framework will be referred to by an expression frequently used in the debate, *single production with fixed capital.*

clear intuitive image for the determining of prices in joint production, a view which, conversely, we consider essential for an effective study of this framework. This order of considerations led us to confine the paper to those basic studies of the system most closely related to the analysis of prices, whose results can be more effectively compared to those that derive from single production systems.

I. The general Joint production system

1. The basic hypotheses

A general joint production system can be written in the form:

$$(A_1p_a + B_1p_b + \ldots + K_1p_k)(1+r) + L_1w = \\ = A_{(1)}p_a + B_{(1)}p_b + \ldots + K_{(1)}p_k$$

$$(A_2p_a + B_2p_b + \ldots + K_2p_k)(1+r) + L_2w = \\ = A_{(2)}p_a + B_{(2)}p_b + \ldots + K_{(2)}p_k$$

$$\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots$$

$$(A_kp_a + B_kp_b + \ldots + K_kp_k)(1+r) + L_kw = \\ = A_{(k)}p_a + B_{(k)}p_b + \ldots + K_{(k)}p_k$$

where 'k' commodities are produced by 'k' industries.

We can now summarize the main basic hypotheses as follows:

1) neither direct means of production nor commodities produced can be negative; however, there might be some null coefficients for some of the 'k' commodities. A joint production — at least two commodities jointly produced — takes place in at least one industry. Moreover, if every other industry produces a single commodity, this will be a different commodity from those of the rest of the industries. At the same time, on the side of the means of production, the production of the 'k' commodities are linked in such a way that it is not possible to separate subgroups of them (by means of a new indexing of the industries). In other words, it is not possible to gather subgroups of industries formed by equations including — as means of production and produced commodities — nothing else but commodities belonging to that subgroup;
2) the direct quantity of labour is positive in every industry;
3) the 'k' industries must be linearly independent — no industry will exist which, when taken in a different proportion, can be shown to be equal to some of the remaining industries, or might even be the result of a linear combination of some (or all) of them. This last hypothesis means that each industry is substantially different from

the others in the system and that its inclusion is a substantial addition to the system itself;

4) the system can produce a net output.

It may be convenient to add a few more words to the general scheme. The system is taken as 'square', i.e. the number of industries is exactly equal to the number of commodities. Since the end of the seventies, this has attracted more and more attention and has come to be considered an assumption that needs further consideration.

In his book,[3] Sraffa writes that, if there were less equations than unknowns "the conditions would no longer be sufficient to determine the prices". "In these circumstances — Sraffa adds — there will be room" for parallel processes in which the commodities are produced by a different method and in different proportions.[4] The existence of such a process and hence the 'squareness' of the system is simply assumed.[5]

2. The main features of joint production systems

When we check for ways to extend the most important properties characterizing single production systems to joint production, we face many problems. In particular, prices can be decreasing as the rate of profit rises.[6]

Nowadays, we can list a number of papers that study the differences in the movement of prices and distributive rates between the two different single and joint production systems. However, these studies appear not to be very closely linked to one another and hence not very suitable for inclusion in a coordinated review. Moreover, the economic reasoning that leads from the premises to the formal results is often poor.

The present situation looks like a sort of collection of specific examples — undoubtedly important and deserving further study — in which it is possible to detect properties and results different from the ones that characterise the single production system.[7]

In the present part of the paper we will be dealing with basic commodities and with the developments that this central notion undergoes in this setting.

[3] P. SRAFFA, *Production of Commodities by Means of Commodities. Prelude to a Critique of Economic Theory*, Cambridge, At the University Press, 1960. References to this work are to paragraph numbers.

[4] See P. SRAFFA, *Production of Commodities, op. cit.*, n. 50.

[5] This is not the place to discuss whether such an assumption is legitimate or not. For an interesting discussion and a possible solution the reader might refer to B. SCHEFOLD, "On Counting Equations", *Zeitschrift für Nationalökonomie*, XXXVIII, 1978, pp. 253-285.

[6] In this paper we shall almost always express prices in terms of wage units. This measurement is particularly useful since at the same time it makes it possible to describe the behaviour of the wage rate as the profit rate varies.

[7] It is perhaps possible to trace a part of the actual uncertainty in the joint production framework back to this fragmented understanding.

During the analysis there will be a discussion of the peculiarities of the notion of a sub-system in this different economic structure. We shall conclude by focusing our attention on a particolar group — called *all-engaging* systems by B. Schefold[8] — and on an unusual version of the *reduction to quantities of labour.*

3. The notion of a sub-system in joint production

The possibility for an industry to produce more than one commodity — given the conditions of production that fix the relative proportions in which commodities are produced — implies important changes in the results obtained when applying the concept in the joint product case.

The system is assumed to be productive — i.e. able to produce a net output for at least one commodity. Differently from single production, it is not always possible for a commodity to be produced *separately*[9] from the others, due to the fixed proportions in which goods are jointly produced.

As an example, we may imagine a system producing commodities A and B, whose conditions of production are such to allow us — by means of the usual, and adequate, reproportioning of industries — to build a system producing *only* a net output of A (sub-system A), but not to proportion it to produce *only* a net output of B (sub-system B). In other words, the system considered cannot produce B *separately*.

If we wanted to consider sub-system B despite this, we would have to imagine a further step. Firstly, we would have to consider the system producing a net unit of B — jointly with an 'x' amount of A — and then the sub-system A. Finally, we would have to subtract 'x' times the corresponding industries of the latter from the former.[10]

It thus becomes easy to realise how this virtual procedure could result in a system including negative quantities of direct means of production, a situation simply unknown in single production and strictly deriving from the fact that it is assumed that B cannot be produced separately.

When developed by means of sub-systems and, therefore, in terms of means of production directly and indirectly needed to produce a given net product, the example sketched above helps us to show the extreme complications of using the same notion of a basic commodity proper to a single product framework.

In any case, it is rather doubtful whether there is any chance of finding a convincing economic interpretation to negative quantities of means of

[8] See B. SCHEFOLD, "Multiple Product Systems With Properties of Single Product Systems", *Zeitschrift für Nationalökonomie*, XXXVIII, 1978, pp. 29-53.

[9] In other words it is not always possible to produce as many net products consisting only of a single commodity as the number of commodities being produced in the system.

[10] In the text we assume that sub-system A produces *one* net unit of A.

production even in such a transparent construction as a sub-system.[11]

In the next paragraph we shall therefore recall the definition of a basic commodity proper to the new context.

4. The notion of a basic commodity in joint production

Sraffa defines *basic* commodities as the goods which, in a given system, cannot be eliminated either from the means of production side or from the output side by means of linear combinations among the industries that form the system.[12]

This definition has no simple and intuitive interpretation as it has in the single production case.[13] Nevertheless, the linear independence of basic goods seems to maintain a special rôle for this type of commodity in the different scheme.

In fact, by means of linear combinations of the industries in the system, it is possible, by definition, to eliminate non-basics from the resulting equations — the so-called *Basic equations*.[14] This set of equations, equivalent to the original one, allows us to see that the relation between the distributive variables lends itself to being expressed in terms only of basic goods, thus conserving an important feature for this type of commodity.

However, non-basic goods seem to gain more importance in joint production, by entering the very definition of the linear combinations which yield the *Basic equations*. This last aspect strictly derives from a specific, basic feature of the joint production scheme, i.e. the fact that in an industry the direct conditions of production of each commodity cannot be split from the conditions of production of the other jointly produced commodities, so that basics and non-basics retain a complex link.[15]

[11] See the brief remarks given in L. PASINETTI, "A Note on Basics, Non-Basics and Joint Production", p. 54, in L. PASINETTI (ed.), *Essays on the Theory of Joint Production*, Macmillan, London, 1980.

[12] See P. SRAFFA, *Production of Commodities*, *op. cit.*, nos. 60-61.

[13] When applied to a system formed of single product industries, the definition is equivalent to the one more commonly known.

[14] See P. SRAFFA, *Production of Commodities*, *op. cit.*, n. 62.

[15] During his discussion of basic and non-basic commodities, Sraffa writes that, when for instance a tax — a tithe — is imposed on a non-basic good jointly produced with some basic commodities, the tax itself, "will have no effect beyond the price of the taxed commodity and those of such other non-basics as may be linked with it. This is obvious if we consider that the transformed system of Basic equations, which by itself determines the rate of profits and the prices of basic products, cannot be affected by changes in the quantity or price of non-basics which are not part of the system" (P. SRAFFA, *Production of Commodities*, *op. cit.*, n. 65).

The example chosen by Sraffa raises an interesting point. The reduction in the quantity of the non-basic good produced — the effect of the tax — changes the *proportion* in which that commodity is produced jointly with the others. This implies that it is as if we had changed a method of production in the original system. This would imply that there would be the usual effects on prices and distribution.

The same fact can be seen from another point of view. Let us consider the system we obtain

5. *All-engaging* systems

In the theoretical framework we are dealing with, those particular groups of systems showing properties of single product systems are of great interest. Here we shall focus our attention on that particular set referred to as *all-engaging*.[16]

All-engaging systems are those systems for which it is possible to build sub-systems for each commodity using strictly positive multipliers for every industry in the system. In other words, in an *all-engaging* system, every industry is necessary to the production of each commodity — these systems will be formed of basic commodities only.

This specific set shows all the main properties of single product systems. Before we start describing a way to represent them, we will depict the way the notion of a sub-system can be applied in such a group.

5.1. *All-engaging* systems and the notion of a sub-system

In a joint production system, as we have already underlined, it is not possible in general to build sub-systems for each commodity including only non-negative coefficients.

In those systems where it is possible to build sub-systems for each commodity, it is also possible to represent any net output by combining sub-systems in the proper way or, what boils down to the same thing, by taking the industries of the original system in different proportions. In these systems, the total amounts of labour and means of production are then non-negative (with at least one positive).

Similarly to what it is possible to do in single production systems, the fact that every commodity is separately producible and that sub-systems will then exist for every commodity in the system shows that prices can also be considered by means of the *reduction to labour terms*; in this *reduction* only positive quantities of labour will appear.

after we have imposed a tax on the output of a particular non-basic commodity jointly produced with the basics. If we wanted to obtain the *Basic equations* from the new system we would have to carry out linear combinations different from the ones we had to perform with the original system.

The combinations will differ because if we wanted to eliminate the non-basic commodity we would have to choose different multipliers for that industry every time it is involved in a linear combination — and it will be involved at least once.

For a more complete treatment the reader should consult I. STEEDMAN, "Basics, Non-Basics and Joint Production", in L. PASINETTI (ed.), *Essays*, *op. cit.*, pp. 44-50, where the point was made for the first time. It is a pity that this last paper has not attracted much attention in the debate, since the problem surely deserves further analysis.

[16] See B. SCHEFOLD, "Multiple Product Systems", *op. cit.* These systems — Schefold writes — "are not only interesting in themselves but also because it can be shown that the finished goods produced by machines in basic fixed capital systems are separately producible while the processes using new machines are indispensable" (p. 38). Here, we shall however limit ourselves simply to giving an outline of these joint production systems.

5.2. The *reduction to quantities of labour* in *all-engaging* systems

If we consider an *all-engaging* system, when we reduce to labour the price of A, we will necessarily be dealing with non-negative quantities of labour and means of production. This implies that the wages paid to the direct and indirect labour used in the production of the net unit of A are a positive amount. Further, the means of production that assisted that labour are all non-negative quantities. This implies that, when we build the system with, as net product, precisely those amounts, we have to take the corresponding sub-systems that we know are made up of non-negative quantities in those non-negative proportions. Again, this implies that the total amount of direct and indirect labour necessary to produce those means of production, and, hence, the wages paid to that labour, are non-negative quantities.

Iterating this procedure, we obtain the expression:

$$p_a = L_0 + rL_1 + r^2L_2 + \ldots$$

where all L_i are non-negative.[17]

When the rate is null the price is equal to the direct and indirect quantity of labour needed to produce it; when the rate of profit rises from zero to its maximum, prices, in terms of wage units, monotonically increase, due to the profits matured on advanced wages;[18] when the wage rate is null and the rate of profit is at its maximum, the price tends to infinity.[19]

The relation between the wage and the profit rate is thus inverse.

6. A different way of checking for positive prices

We end this first section of the work by pointing out the results obtained in a study in which a less intuitive, but still interesting, approach is used to obtain the necessary and sufficient conditions for the set of prices to be positive in the general joint production framework.[20]

[17] It is possible to show that the series converge for $r < R$, where R represents the maximum rate of profit of the system.

[18] The statement implies that the wage rate decreases (or at least does not increase) whatever terms it is expressed in. While this is immediately evident if the *numéraire* is a single commodity, it is less evident for a composite one. However, it is possible to show the same result for composite units.

[19] This last statement becomes obvious if the expression on the right hand side of the equation is written p_a/w.

[20] C. FILIPPINI, "Positività dei prezzi e produzione congiunta", *Giornale degli Economisti*, XXXVI, 1977, pp. 91-99. Among the hypotheses at the basis of this approach we single out the one that assumes that the determinant of the matrix $(B - (1 + r)A)$ is different from zero for every value of the rate of profit between zero and its maximum (matrix B and A being respectively the matrix of output and input coefficients). When the rate of profit is null this condition has an immediate economic meaning: the net quantity of every commodity in each industry has to be linearly independent from the corresponding ones in the remaining industries. When the value of the rate of profit is different from zero, on the other hand, the condition is "somewhat arbitrary, nevertheless preserving a definite meaning" (C. FILIPPINI, "Positività dei prezzi", *op. cit.*, p. 94).

Before enunciating the statement, we need the following definitions: in a linear economic system, *technical dominance* will occur if, in at least one industry, its net output of every commodity is not less than the corresponding net quantities in the other industries, and greater in the case of at least one commodity. Analogously, *economic dominance* will exist if the same thing occurs with inputs multiplied by the factor $(1 + r)$ and hence with net outputs reduced accordingly.

The theorem states that the necessary and sufficient condition for a system to show positive prices for an admissible rate of profit is not to retain *economic dominance* (*technical dominance* when the profit rate is null).

In spite of the fact that the economic meaning of the notion of *economic dominance* is not immediate, it seems that the theorem helps at least to identifying the industries and commodities whose direct conditions of production are immediately responsible for the possible occurrence of negative prices for given values of the rate of profit.

II. Fixed capital systems

1. The basic case

Sraffa's treatment of fixed capital consists in considering what is left of each durable instrument of production at the end of the period as a joint product to be possibly utilised in the next period as a mean of production itself.

We start summarizing the basic outline of what is often called *single production with fixed capital*,[21] focusing attention on those problems arising in the situation when at most a single element of fixed capital is used in every industry.

For instance, if the industry producing commodity G is the only industry in the system to use equipment of type M among its means of production, the general system can be represented as follows:

$$(A_1p_a + B_1p_b + \ldots + K_1p_k)(1 + r) + L_1w = A_{(1)}p_a$$
$$(A_2p_a + B_2p_b + \ldots + K_2p_k)(1 + r) + L_2w = B_{(2)}p_b$$
$$\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots$$
$$(M_0p_{m0} + A_gp_a + B_gp_b + \ldots + K_gp_k)(1 + r) + L_gw = G_{(g)}p_g + M_{(1)}p_{m1}$$
$$(M_1p_{m1} + A_g^1p_a + B_g^1p_b + \ldots + K_g^1p_k)(1 + r) + L_g^1w = G_{(g)}^1p_g + M_{(2)}p_{m2}$$
$$\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots$$

[21] This term specifies a situation where the only joint production items existing in the model are used machines.

$$(M_{(n-1)}p_{m(n-1)} + A_g^{n-1}p_a + B_g^{n-1}p_b + \ldots + K_g^{n-1}p_k)(1+r) + L_g^{n-1}w = \\ = G_g^{n-1}p_g$$

. . . . . .

$$(A_k p_a + B_k p_b + \ldots + K_k p_k)(1+r) + L_k w = K_{(k)}p_k$$

where M has 'n' periods of variable physical efficiency.

To consider used machines as joint products in a system formed by 'k' industries and 'k' final goods (hence excluding new machines) causes the number of equations to rise to '$k+n$' ('$k-1$' equations corresponding to commodities produced only with circulating capital, the equation of commodity G — the equation where only G is on the side of the outputs — the one relating to the production of the new type of machines M, plus '$n-1$' equations for the joint production of the '$n-1$' used machines and G), while the number of the unknowns will accordingly rise to '$k+n+2$' ('k' commodity prices, 'n' prices for the new and used type of machines and the 2 distributive variables). It is then sufficient to fix a *numéraire* and, typically, the wage rate in order to make the number of unknowns equal to the number of the equations.

1.1. A numerical example

A simple numerical example will help to clarify the basic economic reasons for the main problem occurring in this kind of situation — the possible occurrence of negative prices for used machines.

To find a simple but effective ground for the reasoning that follows, we shall imagine that a system produces one unit of corn — half a unit per year — (corn being the only consumption good) by means of labour and a 2-year variable efficiency machine M, produced by labour only. We shall assume the total amount of corn produced by the system as physical measurement of the corn itself.

The system thus depicted can be represented as follows:

$$L_m = p_{m0}$$

$$p_{m0}(1+r) + L_0 = \frac{1}{2}p_g + p_{m1}$$

$$p_{m1}(1+r) + L_1 = \frac{1}{2}p_g$$

using wage units as the unit of value.[22]

[22] There are no basic goods in the system. This is equivalent to assuming *non circular* production. Nonetheless, the propositions derived in the text provide the correct principle also when basic commodities are admitted into the system.

The value of the used machine from the system above is shown to be:

$$p_{m1} = \frac{L_0 + L_m(1+r) - L_1}{(1+r)+1}.$$

The reader will notice that the value of the used machine can turn negative when the value — in labour commanded — of the wages paid to the labour serving the used machine in the production of corn (L_1), is greater than the value of the wages paid in the first year of production of the machine plus the wages for the production of the machine itself plus the profits paid on those wages ($L_0 + L_m(1+r)$).

That is:

$$p_{m1} > < 0 \quad \text{when} \quad L_1 < > L_0 + L_m(1+r).$$

1.2. The wage rate in the numerical example

We turn now to check what is happening to the wage rate. Its expression in terms of corn is:

$$\frac{w}{p_g} = \frac{(1+r)+1}{2(L_m(1+r)^2 + L_0(1+r) + L_1)}$$

as we obtain when considering the inverse of the price of corn.

When a machine is in use in a production process it is always possible to remove it from the process before the end of its productive life.

If the machine were to be discarded, the system would be rewritten as:

$$L_m = p_{m0}$$

$$p_{m0}(1+r) + L_0 = \frac{1}{2}p_g.$$

The wage rate in terms of corn in the system above is:

$$\frac{w}{p_g} = \frac{1}{2(L_m(1+r) + L_0)}.$$

We shall refer to the expression above with w_1, and use the symbol w_2 to denote the value obtained when the machine is used for two years.

The conditions for w_1 to be greater than w_2 can be obtained from the expressions given above — values being in terms of the same *numéraire*, corn.

It is easy to verify that:

$$w_1 > w_2 \quad \text{when} \quad L_1 > L_0 + L_m(1+r).$$

This result shows the proper economic meaning of the case of negative prices for used machines. The condition by which $w_1 > w_2$ harmonises

with the condition obtained in the previous paragraph for the price of the used machine to be negative. Concluding, we have:

$$p_{m1} < 0 \quad \text{if and only if} \quad w_1 > w_2,$$

i.e. when it is possible to obtain a higher wage rate by arresting the use of the machine after its first year of life.

2. The studies on negative prices

Let us start considering a system of the kind outlined at the beginning of this section, therefore using a single type of n-year variable efficiency machines. It is then possible to consider 'n' different cases according to the 'n' possible assumptions on the use of the machine in production. In the first 'productive configuration'[23] the machine is used only for the first year of its physical life, in the second two years, ..., in the nth 'productive configuration' 'n' years, i.e. to the end of its life.

The necessary and sufficient condition for a 'productive configuration' to yield positive prices for used machines is that the matching wage is the greatest among all the possible wages corresponding to those 'productive configurations' that assume a shorter use of the machine; the analytical rule is therefore the same as the one we found above for the simplest case.

The possibility that used machine prices can be negative "simply indicates that the productive system, or, rather, the number of periods of utilisation of the machine, is inefficient, in the sense that, if the number of periods were reduced then the system, with the same rate of profit, could allow higher wages".[24]

Moreover, this condition for optimum *truncation* is necessary and sufficient for the system to show non-negative prices for used machines and also for final goods, provided that the rate of profit is not greater than the maximum feasible.[25]

If, as usual, we tried to analyze prices by means of the *reduction to quantities of labour*, we would realise that in general this is not possible.[26]

[23] The expression is used in S. BALDONE, "Fixed Capital in Sraffa's Theoretical Scheme", in L. PASINETTI (ed.), *Essays*, *op. cit.*, pp. 88-137. Among the studies that deal with this topic, it seems possible to single out two papers that have most satisfactorily analyzed the problems outlined in the preceding paragraph — the paper by Baldone just cited, and the study by P. VARRI, "Prices, Rate of Profit and Life of Machines in Sraffa's Fixed-Capital Model", in L. PASINETTI (ed.), *Essays*, *op. cit.*, pp. 55-87.

[24] P. VARRI, "Prices, Rate of Profit", *op. cit.*, p. 81.

[25] We note some differences in the treatment of the problem in the two papers by Varri e Baldone, mainly due to the different notion of 'productivity' (or viability) in the system. In fact, Baldone assumes the system to be viable in its nth 'productive configuration', while Varri demonstrates its theorems for a system which is 'productive' in at least one of its 'n' possible states.

[26] We refer the reader to P. SRAFFA, *Production of Commodities*, *op. cit.*, n. 79, where the difficulties in carrying on that *reduction* are clearly shown and explained for used machines.

Furthermore, we would have to deal with the problem of *truncations* in such a situation.

If a *reduction* is not possible in the usual sense in a *single production system with fixed capital*, it is nevertheless possible to consider a special kind of *reduction* carried out on the same systems when treated in a particular way. The approach, proposed in a study by Schefold,[27] is conceived especially to investigate particular cases of changes in methods of production. Since, as we said, we are not dealing with this topic here, we limit ourselves simply to outlining the basics of the proposal.

3. The *reduction to dated quantities of labour* in *single product systems with fixed capital*

Let us suppose that, in a system like the one outlined at the beginning of this section, we have determined the optimum *truncations* for every feasible rate of profit. In this way we have determined the most profitable system of production for every rate of profit in the relevant range. Each of these systems can be reduced to a form analogous to that of a single production system, where used machines have disappeared, by means of a specific combination among all the methods that produce G.[28] We shall refer to the 'n' systems in this form with the locution 'combined' systems.

If we imagine a virtual system whose coefficients vary with the rate of profit in such a way as to become equal to the analogous coefficient of the most profitable 'combined' system, we would have built the system Schefold refers to as the *centre*. In this virtual system prices rise monotonically as the profit rate rises. By construction, its wage-profit curve is the 'external' envelope of the set of 'n' wage-rate curves corresponding to the 'n' different 'combined' systems. Prices determined in the *centre* are, of course, the same as the ones determined in the original system, provided optimum *truncations* are carried out.

The coefficients of the *centre* depend on the profit rate. Apart from this extremely important feature, the resemblance to a single product system is complete, so that it is possible to perform the *reduction* with the usual procedure. The output of this operation will display an equation of the form:

$$p_a = L_0[r] + (1 + r)L_1[r] + (1 + r)^2L_2[r] + \ldots$$

where each term $L_i[r]$, as we have seen, is a function of the profit rate. Each $L_i[r]$ can be demonstrated to be a non-decreasing function of the rate of profit. The *reduction* so obtained can hardly be defined as a *reduction* in the usual sense — nor does Schefold intend the reader to think that

[27] B. SCHEFOLD, "Reduction to Dated Quantities of Labour, Roundabout Processes, and Switches of Technique in Fixed Capital Systems", *Metroeconomica*, XXVIII, 1976, pp. 1-15.

[28] The procedure is outlined in P. SRAFFA, *Production of Commodities*, *op. cit.*, n. 76.

it is. In any case, it constitutes the image closest to the *reduction to dated quantities of labour* valid in single production.

4. An attempt to generalise the fixed capital scheme

4.1. Introduction

In his *Production of Commodities by Means of Commodities*, Sraffa examines the changes in the single product system brought about by the introduction of fixed capital under two main hypotheses: a) every industry limits its use of fixed capital to one type of machinery; b) each piece of equipment has constant efficiency.

As we have seen in paragraph 2, there have been studies that have removed the second of these assumptions. A few other papers, on the other hand, have tried to analyse the effects of removing both.

We shall report briefly on these studies, first by outlining the main problem arising in the new context as it was depicted in a 1971 paper by Roncaglia. We shall then review the solutions proposed by the Authors who took part in the discussion.[29]

4.2. The problem defined

In his 1971 paper Roncaglia attempted to investigate the analytical consequences brought about by the simultaneous use of two or more types of machinery in the same industry. In particular, attention was focused on the analysis of the *determinacy* of the system when the physical efficiency of each type of equipment *depends* on the age and on the characteristic of the machines that are simultaneously used in the same industry.

[29] The discussion consisted in the following studies: 1) A. RONCAGLIA, "Il capitale fisso in uno schema di produzione circolare", *Studi Economici*, XXVI, 1971, pp. 232-45; 2) P. VARRI, "Prices, Rate of Profit", *op. cit.*; 3) A. RONCAGLIA, "Sulle macchine utilizzate congiuntamente", *Studi Economici*, XXXI, 1976, pp. 127-132; 4) P. VARRI, "Sulle macchine utilizzate congiuntamente: risposta ad Alessandro Roncaglia", *Studi Economici*, XXXI, 1976, pp. 133-137; 5) N. SALVADORI, "Sulle macchine utilizzate congiuntamente: note ad un dibattito", *Studi Economici*, XXXII, 1977, pp. 151-167; 6) P. VARRI, "Sull'aggregazione di macchine usate congiuntamente nella produzione: risposta a Neri Salvadori", *Studi Economici*, XXXIV, 1979, pp. 95-102; 7) N. SALVADORI, "Sulle macchine utilizzate congiuntamente: una replica", *Studi Economici*, XXXIV, 1979, pp. 75-85; 8) N. SALVADORI, "Il capitale fisso come 'specie' del 'genere' produzione congiunta", *Economia Politica*, a. III, n. 1, 1986, pp. 21-38; 9) S. BALDONE, "Il capitale fisso come 'specie' del 'genere' produzione congiunta: un commento a Neri Salvadori", *Economia Politica*, a. IV, n. 2, 1987, pp. 247-258; 10) P. VARRI, "Il capitale fisso come 'specie' del 'genere' produzione congiunta: alcune precisazioni", *Economia Politica*, a. IV, n. 2, 1987, pp. 259-263; 11) N. SALVADORI, "Il capitale fisso come 'specie' del 'genere' produzione congiunta. Ulteriori precisazioni ed una risposta", *Economia Politica*, a. IV, n. 2, 1987, pp. 265-275.

Papers nos. 1, 3, 4, 5, 6, 7 and the Italian, original version of n. 2, have been gathered in N. SALVADORI (ed.), *Esperimenti intellettuali ed economia politica*, Franco Angeli, Milano, 1981. Future quotations from the studies above will refer to this last collection.

Then, if the industry producing G uses two different types of machine, each with a physical duration of two years, "we shall have as many different prices, for a given type of machine at a given point of its productive life, as its possible 'past lives'. Conversely, we shall have as many equations as unknowns, corresponding to as many possible productive processes as the possible combinations of the type of machines, considering as different machines also those machines of a given type, in the same year of their productive life, but with a different 'past life'".[30] But the theoretically possible 'past lives' are not a finite number.

A new machine of the second type, or an old one, could have been associated with the machine of the first type in its first year of life in the example used in this paragraph. In the case of an old one, we would have to consider the conditions under which the old machine worked — in other words, whether it produced its output side by side with a new machine of the first kind or an old one, and so on. It is then clear that the possible prices for the old machines of the first type can be infinite. Essentially, this is the point made by Roncaglia.

4.3. The proposed solutions

The problem sketched above gave rise to a significant debate. In the discussion, different solutions were proposed. We shall briefly illustrate them, and then move to consider their limits and the further problems arising after each of them.

The first proposal suggests assuming that "the efficiency of the machines of a certain type is not influenced by the age of other types of machine that they work side by side with".[31]

The second solution presupposes the formulation of the concept of a *plant*.[32] The use of this notion leads us to consider the different types of machines used in an industry as a single machine — indeed, a *plant* — whose period of physical efficiency is the least common multiplier of the periods

[30] A. RONCAGLIA, "Il capitale fisso", *op. cit.*, p. 35.

[31] In this way, if we assume that there are machines of type M and N lasting respectively t_m and t_n periods, we have to consider prices associated with the various periods of life for the N machines for a given period of life of the machines of type M and *vice versa*, obtaining $t_n + t_m - 1$ equations for $t_n + t_m - 2$ prices of used machines plus the price of the final good G. When both the equations corresponding to the industries producing the new machines as well as the associated unknowns (the prices of the machines) are introduced, we will obtain $t_n + t_m + 1$ equations and $t_n + t_m + 1$ unknowns. Adding this last set of equations to the other part of the system, we finally obtain $k + t_n + t_m + 1$ equations for $k + t_n + t_m + 3$ unknowns. As usual, taking a distributive variable and a unit of measure as fixed, the system is determined. Furthermore, as Roncaglia claims, "all the other equations corresponding to all the other possible combinations — of the age of the different types of machine — not yet considered" would depend on the $t_n + t_m - 1$ equations (A. RONCAGLIA, "Il capitale fisso", *op. cit.*, p. 36).

[32] As the reader may notice, the debate eventually focused on the theoretical applicability and pertinence of this notion in this context.

of the different types of machine that constitute it. Thus the *plant* "will have a life equal to the period within which all the individual machines are simultaneously eliminated, while the substitution of the individual machines of which it is composed will appear as simple operations of repair and maintenance of the *plant*".[33]

The general framework has its object of study "in every self-replacing system producing a certain net amount of every commodity".[34] The problem to be solved implies the existence of an infinite number of methods, not all equally necessary to the definition (composition) of a self-replacing system. The third proposal then indicates that it is sufficient to associate only a certain number of methods to form a system — even if nothing guarantees against the existence of infinite alternative systems.

Nevertheless, this is not a problem for the theory, since, at a given rate of profit, the system would adopt the most profitable among the available alternative systems — the system with the highest wage rate. The resulting system would consist of an equal and finite number of equations and unknowns. The determinacy problem would thus be solved without any restrictive hypothesis.[35]

4.4. Remarks on the proposed solutions

The first proposal tends to isolate and mark the bottom limit of the possibilities of developing of the theory. The hypothesis to be assumed — in the words of the Author whose proposal this is — has the "advantage of allowing a clear and unique treatment of the problem under consideration" and shows "under which kind of hypothesis [the problem] (...) can be brought back to the problem of the use of a single type of machine in every industry".[36]

It is possible to make various criticisms of the second proposal, in particular of the notion of a *plant*. Deliberately leaving aside the hypothesis about the dynamic of the systems[37] — which seems improper, we might recall the following:

— the notion of a *plant* does not permit the determination of the price of the different machines jointly used in it, but only of the *plant*

[33] P. VARRI, "Prices, Rate of Profit", *op. cit.*, p. 86.

[34] N. SALVADORI, "Sulle macchine utilizzate congiuntamente", *op. cit.*, p. 161.

[35] N. SALVADORI, "Sulle macchine: note", *op. cit.*, p. 161. The Author judges the notion of a *plant* as restrictive.

[36] A. RONCAGLIA, "Sulle macchine", *op. cit.*, p. 151.

[37] The proposal implies a hypothesis of stationary state for the system. This assumption is also present, in Varri's opinion, in the work by Sraffa (P. VARRI, "Prices, Rate of Profit", *op. cit.*, p. 85). It might be useful to remember that such an hypothesis is not formulated at all in *Production of Commodities*. On the contrary, Sraffa assumes the system to be in a self-replacing state, without any implication as regards the dynamic.

itself. It does not seem possible to accept this limitation for a theory whose main aim is to determine the distribution and the production prices of commodities;[38]

— the possibility of applying the notion itself is doubtful. In fact, it is sufficient to assume that the industry under consideration can purchase used machines belonging to other industries in order to have a theoretically infinite number of equations.[39] It would then be necessary to exclude this event from our analysis to have the chance to use the notion;[40]

— if it is not possible to sort the processes in the system into a sequence that can be interpreted as a *plant*, neither will it be possible to use the notion itself;[41]

— finally, a note on methodology. The use of the notion of a *plant* does change the inner nature of the original problem, independently of the opinion we may have of the different assumptions necessary to apply it. If the original problem is to determine the prices of the system as it is given, it would be rather surprising if we accepted as a solution something that concludes the analysis with no more than a partial solution of the problem itself. In other words, the solution of the original problem cannot aim to eliminate a subset of the unknowns. The specification of the unknowns cannot change during the analysis.

The notion of a *plant* is sometimes seen as a possible way to remain in that situation known as *single production with fixed capital* even with multiple types of machines in each industry.

The analytical cost of the assumption seems to imply a renunciation of the analysis of the situation known as *joint production with fixed capital*, i.e. of the situation in which elements of joint production — beyond fixed capital elements — remains irremovable.

The third solution seems to provide a principle that can help to find a possible way to solve the general problem. Nevertheless, the chance that — with alternative systems — the most profitable system may not be determined for a given rate of profit, is a serious challenge.[42]

[38] N. SALVADORI, "Sulle macchine: note", *op. cit.*, p. 162.

[39] The reader will find the statement in A. RONCAGLIA, "Sulle macchine", *op. cit.*, p. 149.

[40] Varri does not agree with this criticism. He points out that the transferability of machines among industries was ruled out by definition in his analysis, which deals with *single production with fixed capital*. It is obvious, the Author continues, that when transferability is allowed, no transformation of the system can be performed so as to obtain a unique *single product* equation, not even in the simplest case of a single type of machine per industry. It is therefore useless, Varri concludes, to formulate *ad hoc* assumptions to treat the more elaborate case pointed out by Roncaglia, since the problem itself would remain unsolved even in the simplest case.

[41] N. SALVADORI, "Sulle macchine: note", *op. cit.*, p. 166.

[42] The reader can consult the problems pointed out in N. SALVADORI, "Switching in Methods of Production and Joint Production", *The Manchester School*, vol. 53, 1985, pp. 156-178.

Conclusion

The theorems effectively demonstrated in the studies quoted in the second part of this work establish several properties of the value magnitudes in the system. Notwithstanding, an intuitive representation of the price structure is still missing. A solid, intuitive view of this structure would be of the greatest help for the analysis. Any possible progress in this direction is therefore highly desirable in order to solve those problems in the fields not considered in this paper, especially the difficulties that arise when changes in methods of production are allowed for.

As regards the studies we dealt with in the last part of section two, the different analyses seem to have been developed on a basic misunderstanding. It seems to us that every single participant in the discussion has assumed a slightly different theoretical framework from the others. For instance, Baldone writes that "the device of the *plant* allows an effective treatment of the case of the joint use of different types of machines" in the same idustry, preserving "Sraffa's characteristic attitude", and reaffirming *single production with fixed capital* as the basic framework.[43] Thus Baldone shows that he assumes, incorrectly in our opinion, that Sraffa's conception of fixed capital has necessarily to imply a 'structure' of fixed capital starting from the new machine and ending with its last year of life, 'structure' providing the opportunity "to eliminate the joint production element from the system by means of a procedure of 'temporal integration'".[44]

If we were to share this attitude we should have to conclude that the debate "seems to have arrived at a conclusion that definitely downgrades the interest of that particular case of joint production that is fixed capital".[45]

*Banca d'Italia, Roma**

[43] S. Baldone, "Il capitale fisso come 'specie'", *op. cit.*, p. 248 and p. 256-257.
[44] S. Baldone, "Il capitale fisso come 'specie'", *op. cit.*, p. 248.
[45] N. Salvadori, "Il capitale fisso", *op. cit.*, p. 22.

* The opinions expressed in the text are those of the author and are not necessarily shared by the Bank of Italy.