political economy *Studies in the Surplus Approach*

volume 4, number 2, 1988

Some Considerations on Joint Production

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I. Some fresh results and an old dubious point

Many of the results obtained for single product systems have been extended by Bertram Schefold to the joint production case in a series of recent contributions.[1] He has in fact proved that a dominant technique exists and (in almost all cases) is unique and square, if demand is given in fixed proportions and if the rate of growth is less than the rate of profits. A square dominant technique means that the number of commodities with positive prices are always equal to the number of processes actually in use to satisfy final demand. This result justifies Sraffa's idea of handling square systems also in joint production. The wage curve associated with the dominant technique — which shows the real wage rate as a function of the rate of profits — is essentially monotonically falling until a finite maximum rate of profits is reached. Obviously the methods and the commodities which form the dominant technique are not always the same at all rates of profits. Some processes can be more profitable than others at different rates of profits. The wage curve corresponding to the dominant technique is analytically identical with the curve that relates consumption per head and the rate of growth.

These results are obtained by selecting the dominant technique by a process of global optimization which assumes free disposal in adapting the supply of commodities to demand and a competitive environment. As Schefold himself points out, there would be some trouble if we had to explain how the dominant technique can be attained through a decentralized process of choice, starting from a situation in which other methods are in use and other prices rule. In single production the emergence of the dominant technique and of the corresponding production prices can be seen as the result of a decentralized process of choice, where entrepreneurs, starting from a casually given price vector, fix their prices periodically so that these

[1] See particularly the two following papers: "On counting equations", *Zeitschrift für Nationalökonomie*, vol. 36, 1978 and "The dominant technique in joint production systems", *Cambridge Journal of Economics*, vol. 12, 1988.

can give a required rate of profits and select those methods which allow their object to be pursued at the lowest cost and therefore at the minimum price. With regard to a selected technique (A, B, a_0), the adjustment process can be written as follows:

$$p_{t+1}B = p_t'A + wa_0 + r^*p_t'A \qquad (1)$$

where r^* is the uniform required rate of profits[2] and w is the given level of the money wage rate. In single production the solution of system (1) converges to production prices at rate r^* if the technique is productive. If the selected technique is not the dominant technique at $r = r^*$, then some of the processes will sooner or later be replaced by other cheaper methods belonging to superior techniques. This process will converge towards the dominant technique and the corresponding production prices. Unfortunately this property generally does not hold in joint production. First of all, process (1) in general does not converge even for a given technique and, moreover, there can be more than one technique which minimizes prices.[3]

From this standpoint we might think there may be some truth in the old idea, supported by J. S. Mill and Jevons, according to which, when we deal with joint outputs, the cost of production can do nothing else than decide their joint value; it can have nothing to do with deciding the relative value of the associated commodities. It is logically impossible to share out the total cost of production among the different outputs and to fix their prices on pure technological bases when we refer to the classical example of wheat and straw. This does not mean that it is impossible to define prices, but rather that every process of price determination in joint production includes elements which are to some extent arbitrary and often unrelated to production costs. It is in a sense surprising that the price of a joint product has to be zero, even if it comes out of a process showing positive costs when it is overproduced and free disposal is assumed.

A solution to this problem which more than others follows the spirit of the production-price theory is to consider some other production methods that separately produce a sufficient number of the joint outputs for their prices to be fixed on the basis of the corresponding production costs. It is thus possible to determine residually the price of the last commodity by imposing a uniform price on this commodity, even if it is produced by different methods. It is the same as supposing that commodities are produced in the same way, both in the joint production process and with the single production methods.

[2] The existence of a uniform level for the required rate of profits can be justified from a long-run standpoint also in the presence of price-makers, if we assume that new entrepreneurs can come into the market. This also justifies the assumption that processes are chosen in order to minimize costs.

[3] The presence of two cost-minimizing techniques, A and B, means that a method of A which is not in B yields losses in terms of prices of B, and a method of B which is not in A yields losses in terms of prices of A.

II. The multiple production case

Although this solution cannot always be feasible (there may be no other process available), there are some cases in which this device is always praticable. Among these we can mention production with fixed capital in its pure form and multiple production where outputs derive from a single method, but temporally are not joint. These are cases of "weak" joint production since "joining" is due only to the presence of some common costs (fixed costs), which must be shared out among several production periods and/or among several commodities which can be produced in an alternate sequence.

In fixed capital used machines are the only joint products. If we suppose they are not transferable from one sector to another, then the temporal integration of the activities, into which the mechanized process is subdivided, allows the durable instrument of production to be formally eliminated. The finished good becomes the sole output and the determination of prices is therefore brought back to the forms of single production. The book prices of the used machines are then determined starting from the prices of the finished goods.

The case of multiple production is still more suitable to underline the logical problem that is inherent in the determination of prices in joint production and to illustrate the solution we have suggested above. Since each commodity can be produced severally by the same plant, then we can depict it, at least in abstract terms, as if it were specialized in producing each commodity in the class of products which it is technically feasible to produce. It is thus possible to identify a set of productive processes which is sufficient to determine independently the prices of the feasible commodity set, except one which is determined by the original multiple production process.

The following example may help to clarify the point. Consider a two-sector economy where the first sector produces a durable instrument of production (commodity 1), using circulating capital (commodities 2 and 3) and labour. The machine can be utilized in the second sector for two periods to produce two commodities — commodity 2 and commodity 3 — using circulating capital and labour. These two commodities are obtained by specializing production for one of the two periods in the corresponding output. Since there are two processes and three finished goods (commodities 1, 2, 3), the corresponding price system is under-determined. One further equation can however be provided by considering that the machine could be used to produce only one of the two commodities, for instance commodity 3, during the two periods of its utilization. This alternative will be denoted as the «virtual» sector 3. If we suppose that in multiple product industry, commodity 3 is produced in the first period and commodity 2 in the second period of machine utilization, the «virtual» production technique can be represented as follows:

$$A = \begin{bmatrix} 0 & 1 & 0 & 1 & 0 \\ a_{211} & a_{221} & a_{222} & a_{231} & a_{232} \\ a_{311} & a_{321} & a_{322} & a_{331} & a_{332} \\ 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 \end{bmatrix} \qquad B = \begin{bmatrix} 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 \\ 0 & 1 & 0 & 1 & 1 \\ 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 \end{bmatrix} \qquad (2)$$

$$a_0 = \begin{bmatrix} a_{011} & a_{021} & a_{022} & a_{031} & a_{032} \end{bmatrix}$$

where a_{ijk} and a_{0jk} measure respectively the quantity of commodity i and of labour which are used in the k th activity of industry j to produce the outputs measured by the corresponding elements of matrix B.

If we denote the factor of profits by $R = 1 + r$, technique (2) can be rewritten in temporally integrated forms as follows:

$$A(R) = \begin{bmatrix} 0 & R & R \\ a_{211} & a_{221}R + a_{222} & a_{231}R + a_{232} \\ a_{311} & a_{321}R + a_{322} & a_{331}R + a_{332} \end{bmatrix} \qquad B(R) = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & R & R+1 \end{bmatrix} \qquad (3)$$

$$a_0(R) = \begin{bmatrix} a_{011} & a_{021}R + a_{022} & a_{331}R + a_{032} \end{bmatrix}$$

Technique (3) is equivalent to the following "single" product system:

$$A(R)B^{-1}(R) =$$

$$\begin{bmatrix} 0 & \frac{R}{R+1} & \frac{R}{R+1} \\ a_{211} & a_{221}R + a_{222} - \frac{R}{R+1}(a_{231}R + a_{232}) & \frac{R}{R+1}(a_{231}R + a_{232}) \\ a_{311} & a_{321}R + a_{322} - \frac{R}{R+1}(a_{331}R + a_{332}) & \frac{R}{R+1}(a_{331}R + a_{332}) \end{bmatrix} \qquad (4)$$

$$a_0(R)B^{-1}(R) =$$

$$\begin{bmatrix} a_{011} & a_{021}R + a_{022} - \frac{R}{R+1}(a_{031}R + a_{032}) & \frac{R}{R+1}(a_{031}R + a_{032}) \end{bmatrix}$$

As is immediately clear, processes 1 and 3 in (4) are the same as in (3), only they are expressed at a "unit" level of production. On the contrary, process 2 is obtained from the original method by subtracting the intermediate consumptions corresponding to a production of "R" units of commodity 3. In doing this, it is assumed that this commodity is produced with the same technical characteristics that it shows in sector 3. As we have seen, this reproduces sector 2 when it is specialized in producing commodity 3 only.

It should be noted, finally, that there are reasonable assumptions ensuring that technical coefficients of process 2 in (4) are non-negative. That is the case when machines show constant efficiency, so that inputs per unit of output do not change for each commodity as time goes by. Non-negativity is obviously strengthened if we assume, as is reasonable, that changes in product design involve some set-up costs for the manufacturing line. In spite of the presence of set-up costs, it may be profitable to start the multiple product process, rather than the specialized processes, if this allows a sufficient level of economies of scope to be realized. Their effect would affect the commodity, the price of which is determined residually by the multiple product process — commodity 2 in our example.

Analogous results can also be obtained when machines show a decreasing efficiency. It must be remembered that multiple product firms — especially if production is sufficiently standardized — try to plan production by repeating elementary plans which present the same or a similar sequence of outputs in order to simplify organization and to improve reliability. If the number of elementary plans carried out during the working life of the machines is high, the average efficiency of the machines will be very similar among the different productions and also similar to the degree of efficiency the machines would show if they were specialized in single productions. It follows that the average coefficients of production of each good (set-up costs aside) are fairly similar both in multiple and in specialized production, as they are when efficiency is constant.

This form of joint production is interesting because it allows some present trends of technological change to be described. The advances in and the continuous diffusion of automation and, in general, of computer-integrated manufacturing technology have, in fact, allowed changeover times and costs to be drastically reduced since the task of machine set-up involves little more than reading a computer program. Consequently, this has made even small production runs profitable, so that it may be cheaper to make replacements to order than to warehouse them. Therefore, a frequent change in productive specialization has been made possible and the same machinery can produce a wide range of pieces and products. The search for greater efficiency in production seems therefore to shift from economies of scale, resting on a great volume of production which allows for the use of expensive special-purpose equipment, to economies of scope, resting not on quantity but on variety in production. This is, therefore, a characteristic of technical change which assumes, from an analytical standpoint, the forms of a tendential passage from single to joint production.[4]

III. Some finl remarks

The two cases of joint production we have here underlined not only allow the problem of price determination to be solved in line with the logic of single production, but also seem to behave "regularly" from the point of view of production. Fixed capital production, in the above-mentioned forms, can, in fact, fit any structure of final demand. As far as multiple production is concerned, it must be noted that output structure can be modified by specializing production either in one commodity or in another for periods of convenient length. If production of each single activity into which the productive process is subdivided is small in relation to the whole production of each commodity, output structure can be fitted sufficiently closely to demand structure. Going back to our example, it must be noted that process 3 is only a virtual reference term in determining prices and it is not necessary to activate it. In this case the quantity system would be non-square.

As we have shown, there are other cases of joint production besides fixed capital which can depict remarkable characteristics of modern industrial production and which retain many of the main characteristics of single production both in the analytical representation of technology and in the properties of the corresponding prices of production.

Obviously some properties are attenuated also in these cases of weak joint production. As we have shown, a particular sequential structure of outputs is, in general, necessary in multiple production to adapt the supply of commodities to the demand. This implies a particular characterization of the production technique, so that prices in general will be affected, showing an indirect linkage between demand and prices.

It is obvious that multiple production is the extreme situation in a class of cases in which joint products are producible in proportions that can be changed within a given range. Variability in proportions is a less unusual situation than we might suppose, both in the short and in the long run, where technical change can manifest itself simply by modifying the proportions of the jointly produced outputs, whenever the demand structure requires a change of this kind. As Marshall wrote "... if straw were valueless,

[4] On some analytical aspects of similar forms of technical change, see S. Baldone, "Analisi interindustriale e mutamento tecnico", *Atti della XXIX Riunione Scientifica della Società Italiana degli Economisti*, Roma, 1988.

farmers would exert themselves more than they do to make the ear bear as large a proportion as possible to the stalk. Again, the importation of foreign wool has caused English sheep to be adapted by judicious crossing and selection so as to develop heavy weights of good meat at an early age, even at the expense of some deterioration of their wool".[5]

In both cases there is an endogenous change from demand to technology which allows, on the one hand, supply to be adjusted to demand and, on the other, the problem of price determination to be solved either by eliminating some joint commodities or by fixing the price of some of them at the maximum level consistent with the price of the most competitive products.

We can, therefore, see that the indeterminancy of prices and the possibility of adapting the supply of commodities to the demand are problems which seem less worrying if they are faced from a long-run point of view. This is, however, a situation which will become actual in time with the emergence of techniques which are socially important in relation to given trends in final demand and in income distribution. Referring to a random technology in this framework is not a very meaningful way of stating the problem of price determination.

As we have seen, production-price theory, from a certain logical point of view, needs to bring back joint production to the forms of single production. Such a need is especially evident if we intend to give micro-economic bases to production prices like those we have considered above.

Joint production analysis should therefore be further developed bearing this characteristic in mind. This can be done by pointing out further cases of joint production which can be read in that way and, for the other cases, by re-proposing a reading closer to the spirit of the theory of production prices.

The results that can be obtained will most probably show to some extent less general properties than in single production. Anyway, some recent anomalous results, mainly within the Marxian theory of value,[6] should neither surprise nor worry us too much. A satisfactory explanation of these results has been given and, at the same time, alternative interpretations have been provided in terms of optimal choices or of "individual" and "market" values which exclude them. It is not joint production which put the Marxian theory of value in a critical position; it was already deeply shaken in single production by the production-price theory.

[5] See A. Marshall, *Principles of Economics*, London, Macmillan, 1952, p. 323.

[6] The roots of the problem can be found in I. Steedman, "Positive profits with negative surplus value", *Economic Journal*, vol. 85, 1975 and in the comments by M. Morishima, "Positive profits with negative surplus value: A comment", *Economic Journal*, vol. 86, 1976 and by E. Wolfstetter, "Positive profits with negative surplus value: A comment", *Economic Journal*, vol. 86, 1976.

If the solution to the problems of joint production were only in terms of global optimization under the assumption of free disposal, which is a sort of by-product of the law of supply and demand, then production prices most probably would become only a linear variety — parametric in demand — of neoclassical prices.

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