political economy Studies in the Surplus Approach

volume 4, number 2, 1988

Joint Production. A Further Assessment

Bertram Schefold

Κυάμων ἀπέχου —
abstain from beans
(*Pythagoras*, ca. 500 B. C.)

1. Introduction

As is well-known, the classical theory of value would still be stuck in an impasse without Sraffa's theory of joint production, because the authors of the 19th and early 20th century saw no possibility of providing a general explanation for the pricing of the outputs of multiple product industries without having recourse to the subjective theory of preferences, as the early mariginalists did. The solution was very elegant and agreed with the new approach, which superseded the Ricardian and Marxian idea of explaining prices of production as modified labour values, by having *direct* recourse to the structure of production and consumption. The structure was expressed in the form of an input-output system, with given output levels, so that prices could be calculated for single product industry systems, given the rate of profit, as being equal to the normal cost of production. The extension to joint production systems then followed from the simple consideration that the production of a given vector of commodities as (gross or net) outputs (the "requirements for use") is generically possible — if there is some variability in the scale at which processes may be run — only if there are at least as many processes used as there are commodities to be produced. Hence there will be enough processes in use to determine prices, and if the rate of profit is to be uniform, the number of processes cannot exceed that of commodities; otherwise, prices would be overdetermined. The system therefore is "square".

Only after this proposition has become clear, does one realize that it represents a generalization of the rules of pricing used by the classical economists who were in the habit of distinguishing between the main product and by-products of an industry. The price of the by-product was determined by that of a close substitute produced by a parallel single-product industry, so that the price of the main product of the first industry was equal to the cost of production minus the value of the by-product; thus, there were two processes to determine two prices. Sraffa's generalization also allows, among other things, for a determination of the price of the by-product on the input side of a second industry, the value of the output of which is determined in a third process, so that a price may be imputed to the by-product as an input and, indirectly, to the output of the first process.

If only one machine is used in each process for the production of a given commodity, the treatment of fixed capital as a joint product easily fits into this framework, since one process is added to the system whenever a machine grows one year older. Land may be treated as a joint product which leaves a process in the same state as it had entered it; the land-price so determined is equal to the capitalized rent. If the production of one commodity is then extended to different qualities of land, the counting of equations proceeds as in the one-machine case. But, in general, the rule of counting of equations imposes a specialisation in that all qualities of land are often technically suitable for the production of a given small number of commodities, yet — if there are more qualities of land than there are products — most of them will produce only one crop because the number of processes which may be run is equal to the number of prices and rents to be determined. This is a kind of enforced specialisation which is relevant to the theory of international trade.

2. Puzzles and more serious problems

The formal properties of such joint production systems have now largely been clarified, at least for the case of constant returns to scale: if a list of methods of production and a given vector of consumption goods demanded are given, the processes to be used may, under very general assumptions, be chosen at a given rate of profit and a — usually lower or zero — rate of growth, such that unprofitable methods are not used and overproduced goods receive zero prices.[1] The solution is (generically)[2] square — the number of positive activity levels equals that of positive prices — though not necessarily unique. It is cost-minimizing in that all processes not in the solution show losses if evaluated in terms of the prices pertaining

[1] M. Lippi, *I prezzi di produzione. Un saggio sulla teoria di Sraffa*, Bologna, Il Mulino, 1979.

[2] A sufficient condition for squareness is that the coefficients of the relevant truncations are regular in the sense of B. Schefold, "On Counting Equations", *Zeitschrift für Nationalökonomie*, vol. 42, 1978, pp. 253-285. Regular systems will be obtained with probability one, if non-negative coefficients are chosen at random, under the condition of self-reproduction. But the coefficients will then also be positive with probability one. One therefore must add the assumption that some coefficients are zero, and that the *p*-feasible and *q*-feasible truncations which appear as solutions are regular, despite the presence of zeroes. This implies e. g. that the input matrix of any solution must not vanish.

to the solution. The real wage, expressed in terms of the demand vector, can be expected to fall, as the rate of profit rises, on the basis of plausibility arguments (here, there are exceptions which are not flukes). The wage curve cannot be extended to infinity for basic systems. It is continuous (except only for flukes) if the growth rate equals the rate of profit. Since the solutions are square, they are in general not affected by small variations of demand; one obtains a — *sit venia verbo* — local non-substitution theorem. At intersections of wage curves, there may be switches of methods, of goods and truncations, among other things.[3] (Truncations involve the deletion of one process and one commodity from a square system so that a system of lower dimension is obtained, which overproduces the deleted commodity.[4]

These results suffice for an extension of the "macro" characteristics of single product systems to joint production, concerning the rationality of the choice of technique, the existence of the wage curve, its use for capital theory etc., although not all concepts can be transferred. The standard commodity, for instance, does not always exist in joint production systems, but that is not important if the standard commodity is interpreted as a didactic device to facilitate the exposition of the problems surrounding the concept of the quantity of capital and the transformation of values into prices.[5] And various conundrums appear if one of the assumptions is violated. For instance, if truncation is not allowed for, a cost-minimizing system may not exist. An example has been provided with three processes and two commodities, such that two square systems with positive prices may be formed by combining either processes 1 and 2 or processes 1 and 3. It turns out that neither system is cost-minimizing because the process not used in each system shows surplus profits, so that the image is created of an economy switching endlessly back and forth between two systems in the competitive process.[6] But if truncation is admitted, there is a cost-

[3] If truncation is not admitted, satisfactory solutions are not always obtained, as results from the example in Salvadori (N. SALVADORI, "Switching in methods of production and joint production", *The Manchester School*, vol. 53, 1925, p. 164) where three processes, producing two goods, can be used to form three square systems of two processes and two goods. Of these three systems, one shows lower costs than the two others, but it involves a negative price in some range of the rate of profit. This system therefore is not the cost-minimizing solution. To find it, truncation has to be admitted. The wage-curves of all the three systems and the six possible truncations are shown in diagram 1. Inspection shows that there are no anomalies or difficulties if truncation is admitted, except insofar as some wage curves are horizontal because they contain no basic.

[4] For proofs of these assertions, see B. SCHEFOLD, "The Dominant Technique in Joint Production Systems", *Cambridge Journal of Economics*, vol. 12, 1988, pp. 97-123.

[5] See B. SCHEFOLD, *Nachworte*, in PIERO SRAFFA, "Warenproduktion mittels Waren", Frankfurt, Suhrkamp, 1976; and by the same author, "The Standard Commodity as a Tool of Economic Analysis - A Reply to Flaschel", *Zeitschrift für die gesamte Staatswissenschaft*, vol. 142, 1987, pp. 603-622.

[6] See N. SALVADORI, "Switching in Methods", *op. cit.*

minimizing system of order one, which dominates all other systems, and the paradox disappears.[7]

Or there is the problem of self-reproducing means of production. They cause a difficulty in the single product case, if they are non-basic and such that their rate of reproduction is lower than the maximum rate of profit of the basic system. Their price then diverges to infinity, when the rate of profit attains the value of the rate of reproduction. A famous example is provided by Sraffa's "beans". His solution was to postulate that the peripheral non-basic was to be discarded.

The seemingly more difficult analogue in joint production is as follows: Suppose that synthetic rubber is produced in the first ("industrial") process of a basic single-product system with a maximum rate of profit R. R is higher (35%, say) than the rate of reproduction r_T (3%, say) of caoutchouc-producing trees which may be used, together with labour and raw materials in smaller amounts than in the first process, to produce caoutchouc as a substitute for rubber by means of an alternative "natural" joint production method of number zero. Rubber (synthetic or caoutchouc) is not consumed. The caoutchouc-trees are neither consumed nor used in any other process.

The assumptions imply that the "natural" process would be used at rates of growth lower than r_T, with the rate of profit being anywhere between zero and the maximum. The price of the trees would be zero, since they would be overproduced. But if the rate of profit r and of growth g are both higher than r_T (say 15% and 4% respectively), two curious effects are observed. First, the raising of g would lead to a discontinuous drop in the real wage at $g = r_T$, because it would now suddenly become necessary to use the "industrial" process with its higher costs in labour and materials. Second, with $r_T < g \leq r < R$, the price of the trees could not fall to zero, so that they would be "phantom commodities". To this extent, the system

[7] The wage curves of Salvadori's ("Switching in Methods", *op. cit.*, p. 165) example are shown in diagram 2, based on Salvadori's table 3. Again, there are no anomalies in the solutions, if truncation is admitted, except for the irregularity of horizontal wage curves, of which two happen to coincide. Truncation (1, 2) is the cost-minimizing solution in the interval where Salvadori seems to imply that there is no long-period position (N. SALVADORI, "Switching in Methods", *op. cit.*, p. 166).

Salvadori (who was only comparing subsystems of order two) noted that the systems made up of processes [1, 2], and [1, 3] both produce the requirements for use at $g = 0$ in stationary conditions, but neither was p-feasible for $1 < r < 9/5$. Since subsystem [2, 3] is not q-feasible at $g = 0$, he did not observe that [2, 3] is p-feasible, and q-feasible to boot, for g and r in that range, so that [2, 3], there, is the golden rule solution, while the truncation [1, 2] is the solution, if $g = 0$ and $1 < r$. A limit to the rate of profit is here missing because [1, 2] contains no basic.

The irregularities of Salvadori's examples are not essential to the argument. In the construction of numerical examples, one easily hits irregular systems because numbers are not chosen at random, but in such a way that computation is easy. A regular truncation of order one necessarily has a positive input coefficient because, for regular systems, it is assumed that the input matrix is not singular. Because of this assumption, pure consumption goods require a separate consideration. The zero-input in [1, 2] is responsible for the lack of a limit to the rate of profit.

is not "square". For if the trees were valued at zero, the "natural" process would show surplus profits in terms of the prices of the system using the industrial process, although it could not actually be used, because the rate of growth exceeded the rate of reproduction of the trees, whereas, if a sufficiently high price were ascribed to the trees, "virtual" losses in the "natural" process would reflect the fact that the "natural" process was not viable. The lowest price which could be ascribed to the trees would be equal to the price obtained in the system consisting of all the processes, including both the industrial and the natural process with the trees, with prices based on equalities for all the methods.[8]

The curious consequences of the existence of non-basic growth-limiting means of production (here the caoutchouc trees) deserve to be discussed, yet I have dealt with them only in passing in previous publications, and this has been criticised by Takeda.[9] My formal reason is that they lead to irregular systems because a right-hand eigenvector[10] is orthogonal to d, the demand vector, and irregular systems are of measure zero in the set of regular systems. As I have also emphasized, certain irregular systems may be of interest, nevertheless, because they exhibit a specific structure (e. g. systems with a uniform capital intensity).

The substantial reason is this: Limiting means of production which are basic are important. A basic limiting means of production could be defined as a commodity which is basic and which has its own rates of reproduction,

[8] Formally, we have $n + 1$ methods and goods, with labour $(l_0, l_1, ..., l_n) > 0$. Matrix (a_{ij}); $i, j = 1, ..., n$; and matrix (a_{ij}); $i = 0, 2, ..., n$; $j = 1, ..., n$; are both semi-positive, productive and indecomposable, and we have semi-positive prices such that

$$p_i \leq l_i + (1 + r)(a_{i1} p_1 + ... + a_{in} p_n);\ i = 1, ..., n;$$
$$p_0 + p_1 \leq l_0 + (1 + r)(a_{00} p_0 + a_{02} p_2 + ... + a_{0n} p_n),$$

with 1 being the "industrial", 0 the "natural" method, with $(l_0, 0, a_{02}, ..., a_{0n}) \leq (l_1, a_{11}, ..., a_{1n})$ and $r_T = 1/a_{00} - 1 < R$, where R is the maximum rate of profit of the system consisting of the methods and commodities 1, ..., n. Semipositive activity levels are such that a semi-positive demand vector $d = (0, 0, d_2, ..., d_n)$ is produced or overproduced, with prices of overproduced goods (here trees, if $0 < g < r_T$) being zero and activity levels q_i of methods not used (method 1 for $0 \leq g < r_T$ and method 0 for $r_T \geq g < R$) being zero. One finds that r_T is a pathological point (see B. SCHEFOLD, "The Dominant Technique", *op. cit.*) because p_0 tends to infinity as r is lowered to r_T. The wage remains positive all the same, because trees are not consumed. The "natural" system consisting of the price *equations* $i = 0, 2, ..., n$ with $p_0 = 0$ has the wage curve w_N. It is q-feasible for $0 \leq g < r_T$ and p-feasible for $0 \leq r < R_n$, where R_n, $R < R_n < \infty$, is the finite maximum to which the rate of profit may tend in the "natural" system. The "industrial" system consists of price *equations* 1, ...,n and of the relationship for process 0 with sign " $\leq$ ", and with $q_0 = 0$. The corresponding real wage is w_I. The industrial system is q-feasible for $0 \leq g < R$ and p-feasible for $r_T < r < R$ (excepting for the phantom price). We have $w_N(r) > w_I(r)$, $0 \leq r < R$. The wage curves are shown in diagram 3.

[9] See B. SCHEFOLD, "On Counting Equations", *op. cit.*, p. 282 and by the same author, "The Dominant Technique...", *op. cit.*, p. 108: see also S. TAKEDA, "Joint Production and a Discontinuous Switch on the Wage-Profit Frontier", *Economic Studies Quarterly*, vol. 37, 1984, pp. 54-66.

[10] In the example, it is a $(n + 1)$-column eigen-vector, with a non-zero element as its first and zeroes as its other components, which is associated with r_T.

the highest of which is an upper limit for the reproduction of the system as a whole. For instance, a corn-using system cannot grow faster than corn on the land which is most fertile in terms of the reproduction of corn, if corn is basic. Non-basic limiting means of production like our caoutchouc-trees are technically conceivable, but economically it is strange to have non-basics which are not consumed. And, technically, as in the case of Sraffa's "beans", the limit to the rates of profit and of growth of the system should, in normal cases, not be thought to depend on individual processes but, rather, on the basic system as a whole.[11]

It may be that more important special cases of irregular systems will be uncovered, but I do not think that research in the area of joint production should be mainly oriented in this direction, nor do I expect that such cases could lead to a substantial modification of the idea that the important features of joint production systems without land[12] are understood by concentrating on the regular case. At any rate, while the elegance of Sraffa's work continues to afford opportunities to extend and refine the analysis by formal means, there are diminishing returns to that activity, and the reconstruction of classical theory also requires other intellectual inputs.

3. Important areas of research

We still lack a complete and systematic morphology of joint production systems. Different types of non-basics may be distinguished according to economic effects: whether basics may be produced, even if non-basics are not, whether non-basics may be taxed without affecting the prices of basics, etc. This much is known, and many distinctions have been made to differentiate joint production systems, which are somehow "close" to single production, from the general case, especially with regard to distribution.

But changes of quantities are, at least empirically, more interesting than changes in the rate of profit in their different effects on different systems. As is well-known, the reduction of prices in terms of some fixed standard at a given rate of profits is *not* an unambiguous indicator of efficiency, if joint production processes are present. The cheapening of a method of production in use may lead to the increase of some prices at a given rate

[11] See Sraffa's reply to Newman in K. Bharadwaj, "On the Maximum Number of Switches Between Two Production Systems", *Schweizerische Zeitschrift für Volkswirtschaft und Statistik*, vol. 106, 1970, pp. 424-428.

[12] It has not yet been worked out satisfactorily how land should be inserted into this framework. It may be regarded as a limiting mean of production with a rate of reproduction of zero. In fact, $r = 0$ is a highly pathological point where land prices as capitalized rents diverge to infinity. Hence one important cause for non-proportional growth: production on fully utilized lands is stagnant and expansion proceeds on no-rent lands or through intensification where the more land-using method must *contract* to make room for the less land-using one.

of profit, because the lowering of the prices of some products may mean that other prices have to be raised in order to achieve a higher profit contribution in processes not directly affected by the technical advance, except in that some output prices are falling. If, for instance, hot springs are discovered which render steam cheaper, a cogeneration plant will have to charge more for electricity, because the profit contribution of steam will be lower, steam being lower in price. Or, in a more complicated case, if the steam price, as determined by direct combustion, is fixed, but households need less of it because of better insulation, electricity from cogeneration plants will also rise, because a lesser proportion of the waste heat can be sold and the cogeneration plant is forced to move to a less energy-efficient method of producing electricity (the profit contribution of steam is reduced, steam being lower in quantity). The latter case represents a long-period equilibrium only if the incomplete use of the potential of waste heat is, as in many places, socially necessary because of institutional barriers which prevent a more extensive sale of steam.

I have examined various such cases[13] and there have been other contributions towards a morphology of joint production, but a complete picture has not yet emerged. The main result has been to understand that properties by which joint production systems differ from single product systems are not to be regarded as anomalies. On the contrary, in the explanation of the "anomalies" consists perhaps the most important "microeconomic" contribution of the various models connected with Sraffa's theory.

Fixed capital and rent have been fairly thoroughly explored from the formal point of view, and intermediate cases in between one-machine worlds or differential rent of the first kind on the one hand, and the intricacies of joint production systems in general on the other, have been analysed in several papers.[14] What is missing is the appropriate link with applied work. For example, the theory predicts constant amortisation, a falling capital charge and hence rising (progressive) depreciation for a machine of constant efficiency and a finite life-time. But progressive depreciation is illegal according to German tax laws, and degressive depreciation is generally recommended. The discrepancy between theory and application is easily explained in this case: degressive depreciation or constant depreciation with a falling amortisation quota are what one should expect beause of increasing maintenance costs in the use of a given machine and in the presence of technical progress. But only a few authors have ventured into such comparisons. Similarly, applications of the theory of rent are

[13] See, B. SCHEFOLD, "Multiple Product Techniques With Properties of Single Product Systems", *Zeitschrift für Nationalökonomie*, vol. 38, 1978, pp. 29-53.

[14] See, e. g., N. SALVADORI, "Fixed Capital within a Von Neumann-Morishima Model of Growth and Distribution", *International Economic Review*, vol. 29, 1988, pp. 341-352.

notable for their absence. For instance, the theory predicts a high degree of specialization in the use of different lands, but specialization does not seem to go that far. I expect to find the most challenging discrepancies between theory and empirical evidence in this area, and that the temptation will be greatest to seek refuge in subjectivist explanations.

The theory — at least in my interpretation — predicts that the competitive process should lead to square Sraffa systems. It is interesting to note that this is a postulate rather than a conclusion of those analysts of empirical input-output systems who are not satisfied anymore with making joint production disappear by means of simple aggregation. As I have emphasized in my Palgrave entry,[15] different input-output procedures dealing with joint production have this same postulate in common. But there is no complete input-output table with joint production so far, and it is clear that there would be theoretical, not only applied, problems involved in its construction for any country. For the theory also predicts that rectangular, not square systems will be observed during processes of transition. One expects "too many" processes insofar as different methods compete to emerge as the socially necessary technique for the production of a given number of commodities, and one often observes "too few" processes in typical multiproduct industries where the processes are hidden which might make the system square. The main hidden processes arise from two causes: one does not easily observe how outputs might be varied and one does not "see" the different household processes for the satisfaction of given needs, which may be necessary in order to arrive at a sufficient number of processes in the counting of equations. What, then, are the operational rules for isolating the socially necessary technique, relative to which advanced methods yield surplus profits, and backward ones losses? How are these to be distinguished from temporary inequalities of profit rates, due to changes of supply and demand, and from those due to market power?

So far, gravitation has been an axiom rather than a result in classical theory, for if we had a general theory of gravitation, it would supersede the theory of prices of production. In fact, there are very diverse approaches to "gravitation": market prices are variously defined as prices somehow given in a market (but why are prices uniform?), and profitability is calculated to determine the direction of investment. Or market prices are calculated from supply and demand, somehow specified, or they result from charges on costs. Or market prices are averages, resulting from mixed technologies, etc. The surplus profits and losses, observed in transitions between techniques (my "Surplus profits and losses mechanism" or Bidard's

[15] B. SCHEFOLD, "Joint Production in Linear Models", in J. EATWELL, M. MILGATE, P. NEWMAN (eds.), *The New Palgrave - A Dictionary of Economics*, London, Macmillan, 1987, pp. 1030-1034.

market algorithm)[16] are a preliminary consideration. The gravitation models yield different results, and they must be distinguished further in their effects on joint production. I do not regard it as likely that theoretical models for the gravitation of particular forms of market prices towards prices of production will replace the axiom of gravitation with its intuitive appeal. This is not said in order to deny the importance of theoretical and applied work in this area, especially in relation to an analysis of the short run.

It may well be that the level of abstraction has to be changed when applied problems have to be dealt with, but it would not do simply to juxtapose the theory of prices of production and, for instance, a Keynesian aggregate model for the analysis of effective demand, because it has to be shown why different approaches can be thought to be compatible. And in some cases, such as the theory of exhaustible resources, a classical model can be adapted to the question at hand, without a substantial modification of its familiar structure.[17]

There have been many attempts to extend the classical theory by developing new approaches to the theory of distribution and in order to reconcile it with the Keynesian theory of effective demand, or by introducing hypotheses about technical progress and about changes in the composition of output and of needs. All these areas are to a greater or lesser degree connected with problems of joint production. I do not think that those theoretical endeavours will be successful, however, if they are not accompanied by important contributions to economic policy in traditional fields, such as the regulation of economic activity, and more recent ones such as environmental economics. Outside observers of classical theory are right to ask not only what it is, but also what purpose it may serve. I hope to have provided some suggestions for answering the four important questions raised by the editors of this journal.

Institut für Markt und Plan, Johann Wolfgang Goethe-Universität, Frankfurt am Main

[16] C. BIDARD, "An Algorithmic Theory of the Choice of Techniques", 1987, mimeo.

[17] B. SCHEFOLD, "Une digression sur les ressources épuisables: Existe-t-il une théorie classique des resources épuisables?", in C. BIDARD (ed.), *La Rente. Actualité de l'approche classique*, Paris, Economica, 1987, pp. 83-97.

Appendix

This appendix contains the diagrams for Salvadori's (1985) example discussed in footnotes to the text. The input and output coefficients are shown on his table 2, p. 164, for diagram 1 and on his table 3, p. 165, for diagram 2. The wage curve $[i, j]$ is the wage curve for a subsystem of order two, consisting of processes i and j. The wage curve (i, j) is the wage curve for a subsystem (truncation) of order 1, consisting of process i, with good j being a commodity which is produced in the precise quantity required. As usual, the wage at rate of profit r is equal to consumption per head at the rate of balanced growth g. In those ranges of a rate of growth where a subsystem produces or overproduces the requirements for use at non-negative activity levels, the wage curve is drawn as a dotted line (q-feasible). In those ranges of the rate of profit where a subsystem yields non-negative prices, such that no process shows excess profits, the wage curve is drawn as a broken line (p-feasible). The golden rule curve is drawn as a both dotted and broken line. All other wage curves are drawn as continuous lines.

The last diagram (3) shows the effect of a limiting mean of production in a dispensible process (the caoutchouc trees of footnote 8). The "natural" process is the dominant technique, yielding the highest real wage, for low rates of growth and high rates of profit.

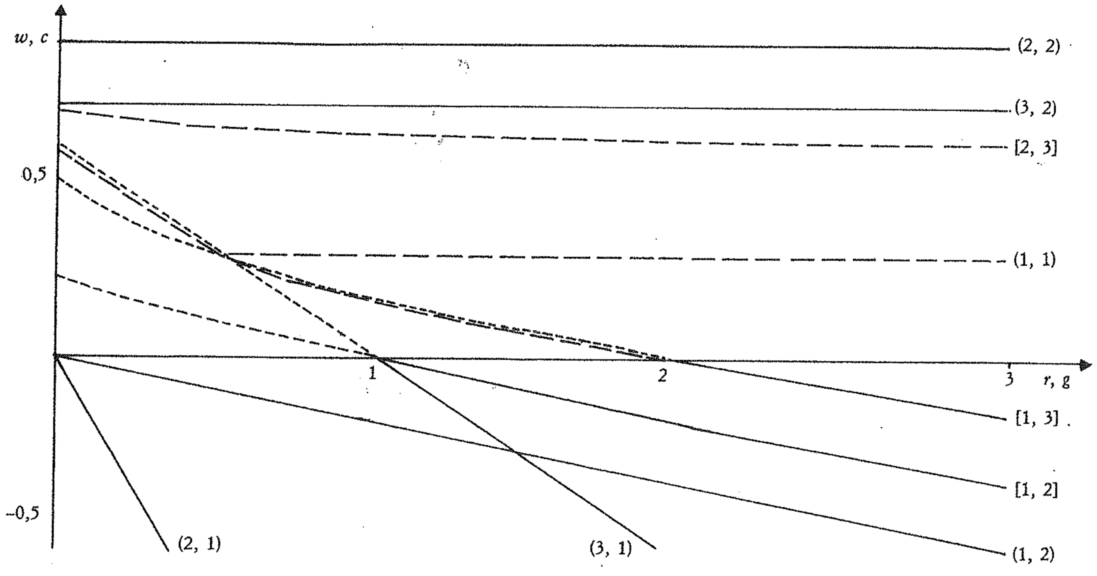

Diagram 1

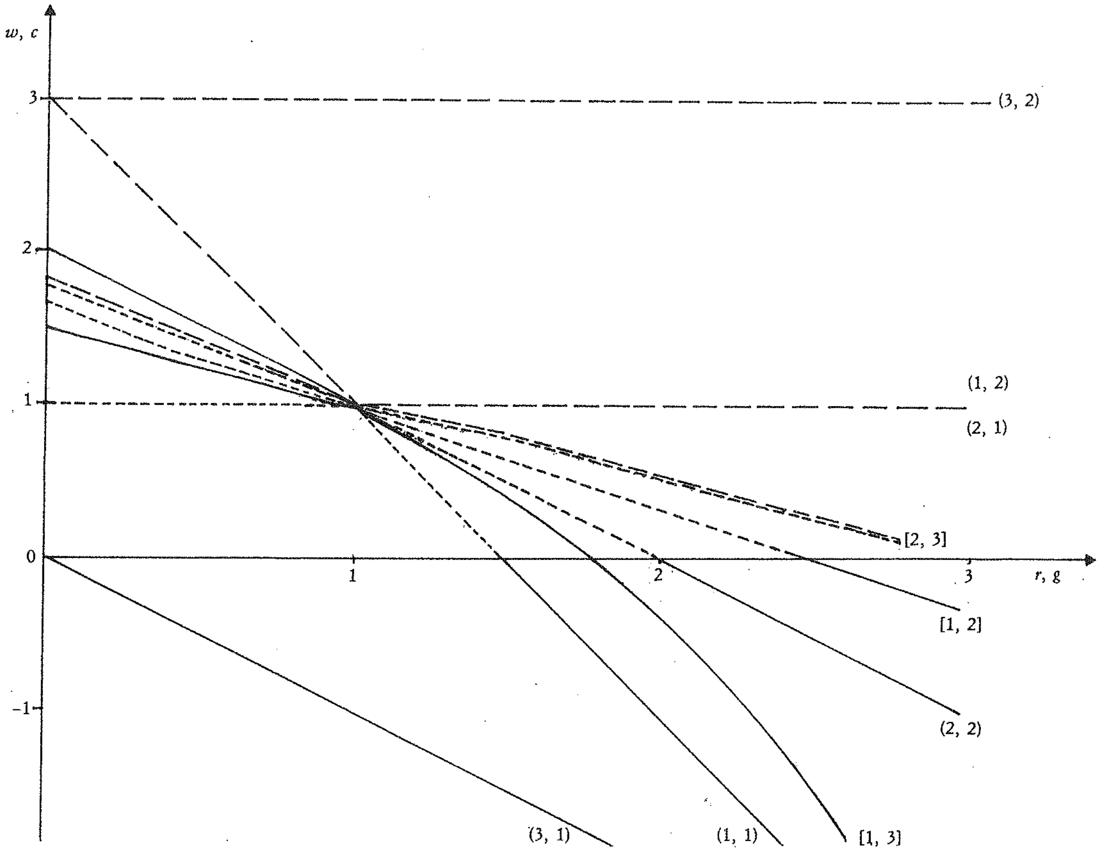

Diagram 2

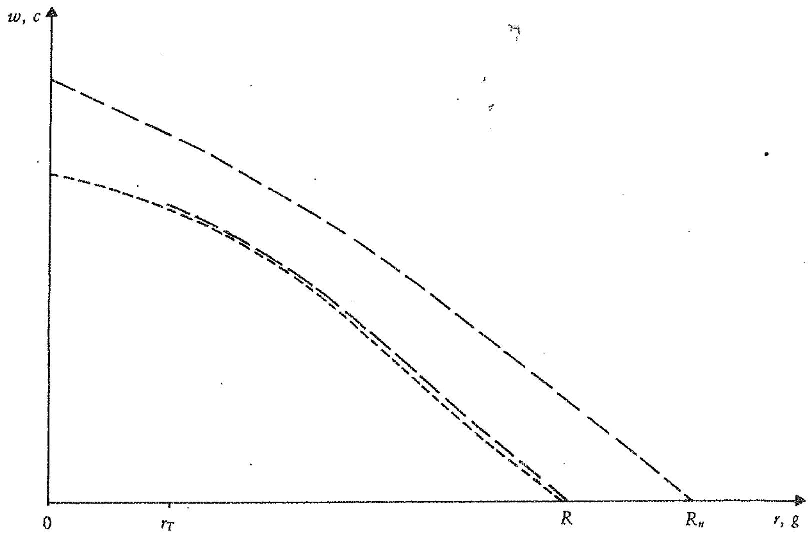

Diagram 3