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Single and Joint Production Systems: a Search for Fundamental Analogies

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Theoretical problems of joint production have had a peculiar history so far in economic literature. They have been relegated among the complications by the theories that try to explain the working of the economic system on the basis of the production conditions of commodities; otherwise they have been almost totally ignored, as in the case of marginalism which, at its own level of abstraction, does not take into account whether commodities are single or joint products, and might even determine prices in a pure exchange economy with no production at all.

The problems of joint production are therefore a minor detail for marginalism and, on the other hand, they are a much more relevant question not only for Classical theories and their modern derivations, but also for input-output analysis and mark-up theories, to mention only the best-known developments.

Recently, interest and concern for the analytical complications of joint production seem to have increased and some authors even suggest¹ that the unsolved problems of joint production are responsible for the decline of the classical approach to the problems of political economy. As a matter of fact, I think that, at the time when the marginalist revolution took place, the analytical difficulties of the labour theory of value were already large enough to determine its abandonment, even without taking into consideration the fact that labour values of joint products might be undetermined. Of course this conclusion takes it for granted that at the roots of the marginalist revolution there are analytical reasons, a view that many historians of economic analysis do not share at all.

Nevertheless, I agree with the fact that at present, since the difficulties of the labour theory of value have been cleared up, and the relations between the conditions of production, the distributive variables and the prices of

¹ See H. D. KURZ, "Classical and Early Neoclassical Economists on Joint Production", *Metroeconomica*, vol. 38, February 1986, p. 2.

the commodities have been rigorously defined,² the analytical difficulties of joint production have become more relevant and it is important to understand them at the broadest level of common persuasion.

I. THE CONDITIONS OF PRODUCTION AND THE DETERMINATION OF PRICES

The modern theory of price determination, based on the conditions of commodity production, is a direct extension of the original intuition of Smith and Ricardo that the relative price of two commodities is equal to the ratio of the quantities of labour required for their production. This intuition has acquired logical coherence and rigour, and has become a general theory of value, when it has taken into account the fact that all commodities are at the same time products and means of production and that there is a relation between prices and distributive variables. As a consequence, it has been possible to determine the price of the commodities produced in the whole economic system by means of a unique system of simultaneous equations.³ It represents a kind of general accounting framework for the entire economy.

If, for simplicity's sake, we overlook the necessary analytical details,⁴ we may remember that in the simple case of single production systems:

- (1) the solutions for prices are economically meaningful for the full range of variation of the uniform profit rate;
- (2) the variations of activity levels under constant returns to scale do not imply any variation in the relative prices and in the distributive variables;
- (3) the comparison of different methods for the production of the same commodity may be made without taking into account activity levels, but it involves the price of all the commodities produced.

Many other analytical exercises may be done by utilizing the same mechanism of price determination: it is possible to introduce non-uniform profit rates in each sector, and different wage rates; it is also possible to introduce more than one method for the production of the same commodity with different levels of efficiency that give rise to non-uniform profit rates, and so on. All these alternative applications show that the problem of price determination has the instrumental nature of an accounting method. Since Ricardo's times, the theory of value is the analytical tool that enables

² See P. SRAFFA, *Production of Commodities by means of Commodities*, Cambridge, Cambridge University Press, 1960.

³ See P. SRAFFA, *op. cit.*, Ch. 2.

⁴ They may be found, for instance, in L. L. PASINETTI, *Lectures in the Theory of Production*, Macmillan, London, 1977.

economists to discuss the problems of income distribution, economic growth, employment, competition etc...

These remarks also make it clear that prices may be determined without taking into account individual behaviour, either as the outcome of consumers' choices or of entrepreneurs' decisions.

II. EXTENSIONS OF THE ANALYSIS

The description of the conditions of commodities production only in terms of labour and circulating capital is clearly not sufficient. In order to acquire the necessary generality the mechanism of price determination considered so far should take into account explicitly:

- natural resources (i.e. non-produced means of production such as land);
- fixed capital (i.e. durable means of production like machines).

In this way the trilogy of Labour, Capital and Land that the economists have always used to describe, in general terms, the physical production of commodities and services would be complete.

To achieve this purpose, theoretical analysis may be developed in two different directions:

- one specific to each particular extension (the use of land, machines, plants, the introduction of different turnover periods etc...);
- the other one at the higher level of abstraction of a general scheme of joint production that potentially includes all possible specific extensions.

Of course, the choice between one or the other of these alternatives has different consequences. While in the first case it has always been possible to give an economic interpretation to the analytical results obtained without any modification of the mechanism of price determination, in the second case various difficulties have appeared. They have soon induced some authors to modify the original mechanism of price determination.

If, for the sake of brevity, we once again disregard the analytical details, the fundamental difficulties are:

- the possibility that the price equation system will not be determined if the scheme is not square;
- the possibility of negative solutions for some price.

To overcome both these difficulties, it has been suggested that the problem of price determination should be solved together with the problem

of the choice of the most efficient technique and with the problem of the determination of the activity levels of each process of production.⁵

In this way it has been possible to obtain a formal analogy of results both from single production and from joint production schemes, but the different theoretical features of this alternative mechanism of price determination cannot be ignored.

The great generality of the accounting mechanism previously underlined is lost as a consequence of the many additional hypotheses that the consolidation of the three above-mentioned analytical problems requires. To realize this it is sufficient to list them: constant returns to scale, rationality in the choice of techniques, the singling out of a rule of choice, the assumption of perfect competition implied in uniform profit rates and wage rates, the assumption (not the result!) of non-negative prices and activity levels (free disposal).

All these assumptions would not be necessary if the determination of prices maintained its own accounting nature also in the presence of joint production.

For this purpose it is necessary to look closer at the two previous difficulties. We may start from the obvious consideration that, if the number of commodities produced is not equal to the number of the processes of production, the price system will be either under or overdetermined. Of course we are not able to say if it is "realistic" to assume that the system is square. But we may notice that a number of processes smaller than the number of the commodities would make it impossible to modify the structure of final goods and this suggests the existence of collateral processes that enable the system to achieve the required levels of production.⁶ In other words, it does not seem, that the price of wheat and straw, as joint products, should remain undetermined, because it is very unlikely that the proportion in which they are produced will coincide with the proportion in which they are utilized in the economic system. This makes it logically reasonable to suppose that in the system of production there is a process capable of integrating the production of one of the two commodities or to dispose of the overproduction of the other one.

On the other hand, the presence of more processes than the number of commodities produced is clearly inconsistent with the often required uniformity of distributive variables.

⁵ This analytical treatment of joint production is based on the misleading view that joint production schemes necessarily imply a problem of choice of techniques. This approach has gained wide approval. See among others: B. SCHEFOLD, "On Counting Equations", *Zeitschrift für Nationalökonomie*, vol. 38, 1978; B. SCHEFOLD, "Von Neumann and Sraffa: Mathematical Equivalence and Conceptual Difference", *The Economic Journal*, vol. 90, 1980; N. SALVADORI, "Existence of Cost-Minimizing Systems within the Sraffa Framework", *Zeitschrift für Nationalökonomie*, vol. 42, 1982; and very recently B. SCHEFOLD, "The Dominant Technique in Joint-Production Systems", *The Cambridge Journal of Economics*, vol. 12, 1988.

⁶ This is the same argument as that used by SRAFFA, *op. cit.*, p. 47.

We might, of course, look for particular explanations of the productive structure of the economic system, but within the limits of a theory of value that regards the effective conditions of production of the commodities as exogenous data, I think that these remarks are sufficient to justify the assumption of square systems.⁷

Non-square systems of production should be specifically justified in order to represent a relevant limitation of the theoretical approach discussed so far, and the justification should regard the whole system and not the single processes of production.

The second difficulty that must be overcome is about negative prices. Recent analytical results help us to understand why negative prices may arise. They necessarily imply something that was originally discovered only for fixed capital systems: the inefficiency of one or more processes of production, in the sense of being physically dominated, at the ruling rate of profit, by other processes in the economic system yielding a greater net output.⁸

Of course, the fact that given conditions of production, in the form of a given set of technical coefficients, give positive prices at a certain level of the profit rate, does not imply that the same prices remain positive at another, equally "feasible", level of the rate of profit. This is an obvious consequence of the fact that the efficiency of the production processes depends on the level of the distributive variables. Nevertheless, while in single production systems positive prices are in any case able to distribute the profitability of each process of production in the entire system in any way we like (i.e. we might calculate those prices that allow the concentration of all profits in one single sector for a given uniform wage rate and obtain positive solutions), with joint production this occurs only to a much more limited extent.

It seems to me that the requirement that, if prices are positive at a given distributive configuration, they should hence remain positive also in a large interval thereof as in single production, is in no way essential. A different distribution of income usually implies a different composition of net product and a different choice of the technique of production both in single and in joint production.

The fact that in single production the same technique gives economically meaningful solutions for all prices along the entire range of variation of the rate of profit is therefore only an analytical fluke and, in my opinion, it is in no way an essential property of the theory of value.

⁷ A more complete and very interesting review of arguments about the problems of overdetermination and underdetermination in Sraffa's joint production systems is contained in B. SCHEFOLD, "Sraffa and Applied Economics: Joint Production", *Political Economy*, vol. 1, n. 1, 1985.

⁸ See C. FILIPPINI, "Positività dei prezzi e produzione congiunta", *Giornale degli Economisti ed Annali di Economia*, vol. 36, N. S., 1977, pp. 91-99.

But there is another analytical result of general validity that is relevant in this context: the possibility of giving an economic interpretation to the fact (possible in joint production) that even if prices are positive, the relation between the wage rate and the rate of profit may be increasing. This implies that the system of production may be divided into two parts which, at their activity levels, provide non-uniform returns in the sense that one can pay at least the same wage bill as the other while using capital goods of lower value, and which therefore are mutually inconsistent.⁹ Therefore, also in joint production systems, the relation may be decreasing, even if to a much smaller extent than the one we would have, so to speak, in single production, and it does not necessarily include a zero rate of profit.

III. FINAL REMARKS

The implications that may be derived from the analysis of the theoretical problems of joint production are thus manifold.

First of all it is necessary to remark that, contrary to single production, there is no simple rule to establish if any scheme of joint production, given arbitrarily, is economically meaningful.

However, all anomalous results have a plausible economic interpretation, and therefore there is no need and justification for modifying the accounting approach originally adopted to determine prices in single production situations. The flows of commodities measured in physical terms, are the exogenous data of the problem.

They allow us to calculate the technical coefficients of the economy, be it a single or a joint production system.

The selection of technical coefficients from a set of production possibilities by means of a rule of optimization, even if it allows us to obtain a square system and non-negative solutions for prices, cannot become the analytical presupposition of the theory of prices for at least two different fundamental reasons. First of all, because the assumption that technology is a "book of blueprints" is too simple and has been heavily criticized for its lack of realism, and secondly, because the ways by which, in real economies, the problem of overproduction or underproduction of particular commodities, with respect to their utilization, is solved, is certainly more complex than the rule of free disposal.

It turns out to be rather similar to the dynamics of technical progress

⁹ For further details see C. FILIPPINI and L. FILIPPINI, "La relazione fra saggio di salario e saggio di profitto", in P. VARRI (ed.), *I prodotti congiunti: aspetti controversi di teoria della produzione*, Vita e Pensiero, Milano, Università Cattolica S. C., 1982; and also C. FILIPPINI and L. FILIPPINI, "Two Theorems on Joint Production", *The Economic Journal*, vol. 92, 1982, pp. 386-390.

and quite different from the choice, in static conditions, of the optimal technique out of a given, and already known, production possibility set.

For these reasons in my opinion the assumption that the system is square is more general than the assumption that the squaring is the outcome of a particular process of optimization. To assume from the start that the system is square means leaving the door open to many possible explanations of its structure and it is not an ad hoc hypothesis.

Only empirical data on the structure of the system of production, or a theory of production capable of providing an explanation of this structure, might call into question the wisdom of assuming that the system is square from the beginning.

The general conclusion that I would draw is that from an economic point of view joint production schemes are not the generalization of single production schemes. They are only a uniform way to formalize analytically some economically meaningful extensions of the theory of value: in particular, the extension of the simple case of production by means of labour and circulating capital to the general case of production with fixed capital and non-produced natural resources.

Single cases of joint production are certainly relevant as a matter of fact, and they may be economically meaningful, but there is no reason to believe that any abstract theoretical scheme of joint production is economically relevant and may represent a test for the theory of value. The opposite, seems more likely to be true, i.e. that the cases of joint production, which, in the absence of empirical data, we should consider economically meaningful, are the only ones able to give meaningful results.

IV. OPEN QUESTIONS

The utilization of fixed capital and land in productive activities certainly does not exhaust the relevance of joint production schemes in economic analysis even if they are perhaps the most important phenomena.

The outcome of scrap residual as by-products of industrial processing and, in more general terms, the so-called external effects of productive activity are examples of real phenomena that may usefully be described in terms of joint production schemes.¹⁰

Moreover, it might be possible to analyse the characteristics of particular processes of joint production inside single production schemes. Finally, in an "empirical" perspective, it would be interesting to analyse inter-industrial

¹⁰ For an extensive description of real situations of joint production, see I. STEEDMAN, "L'importance empirique de la production jointe", in C. BIDARD (ed.), *La production jointe*, Paris, Economica, 1984.

tables with multiple products as they are reckoned by statistical offices before being singularized.

But there is an important feed-back effect that originates from joint production and goes towards general economic theory.

Real economic systems are in a situation of continuous change, and the representation that we may give of them, in a given period of time, is necessarily biased with respect to the theoretical need of being in stationary conditions. It is in the nature of fixed capital and of land, to show their productive effects in the course of many periods of time. This means that, in a joint production scheme, what is usually implied is a multi-period dynamic dimension. Our stationary mechanisms for determining the price and the quantity of each commodity should be adapted to this dynamic dimension. I think that this dynamic extension is the most important and difficult problem of economic theory that the analysis of joint production puts before us.¹¹

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¹¹ Many useful hints for this dynamic extension may be found in L. L. PASINETTI, *Structural Change and Economic Growth*, Cambridge, Cambridge University Press, 1981.