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The Existence of the Standard System: Sraffa's Constructive Proof

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> "Piero Sraffa may make peculiar assumptions, but he never makes a mistake". (With apologies to Amartya Sen.)

1. Introduction

> "This is one example of the inadequacy of several of Sraffa's proofs. Since most of his theorems are essentially correct, it is an open question whether Sraffa in fact (perhaps with the help of *Besicovitch, Ramsey et al.*) has more adequate proofs up his sleeve, proofs which he did not include for fear of making the book 'too mathematical' ".[1]

In spite of widespread disagreements on the nature and significance of Sraffa's '*Production of Commodities by Means of Commodities*' reviewers and commentators seem to have agreed on at least one point: the inadequacy or incompleteness of the proofs of the stated propositions or theorems. From sympathetic reviewers like Peter Newman and Carlo Felice Manara all the way to less than friendly remarks by Richard Quandt the issue of unsatisfactory proofs has been the one, and almost the only, theme on which agreement seems to have been quite unanimous. To remedy the defects, gaps etc., in the proofs, the same unanimity has been forthcoming: the suggestion being an appropriate application of some variant of the Perron — Frobenius theorems or a fix-point argument — the latter especially in the case of existence proofs. Thus, for example, Quandt notes:

* My greatest indebtedness, in the preparation of this paper, is to Professor Guglielmo Chiodi of the University of Perugia. In the last few years my former Modena students, now scattered all over Northwestern Europe, Andrea Brandolini, Giorgio Gobbi and Stefano Zambelli have been instrumental in enabling me to sustain this particular interpretation by means of penetrating discussions, observations and comments. Finally, lectures in the University of Modena in the autumn of 1986, in Professor Gianni Ricci's seminar series, gave me the chance to test these ideas against a critical audience where the friendly, but sharp, criticisms of Professors Marco Lippi and Fernando Vianello did much to clarify some of the remaining cobwebs. Alas none of these worthies are even remotely responsible for what remains here — especially the errors, omissions and obscurities.

[1] P. Newman, "Production of Commodities by Means of Commodities": A review article. *Schweizerische Zeitschrift für Volkswirtschaft und Statistik*, XCVIII, March, 1962, pp. 72-3, fn. 1.

"One feels that the existence proof would, under somewhat different assumptions, be amenable to a fix point argument".[2]

Not very long ago wearing my 'formalist' hat, I explored, together with Lionello Punzo, these algebric and topological lines of attack on Sraffian themes.[3] On the other hand, in another 'incarnation', now almost ten years ago, wearing my 'constructivist' cloak I remarked:

"There is a crucial distinction between the methodology followed by Pasinetti and that followed by Sraffa in proving the important propositions. ...
... . The distinction seems to be that Sraffa, whenever he gives an explicit proof, invariably gives us a constructive proof, whereas all the proofs Pasinetti (and almost everyone else who has attempted to formalise and generalise Sraffa) gives, follow the method of the formalist mathematicians".[4]

I should, of course, have been a little more specific by referring to existence proofs and their constructive underpinnings. In the notes put together here, an attempt is made to make it clear, by an example, that Sraffa's existence proofs are perfectly satisfactory and mathematically adequate, when interpreted within the framework of the *constructivist* program. In the next section, to substantiate this claim, Sraffa's one detailed constructive proof of the existence of the standard system and the determination of the standard ratio in the case of single-product industries is reinterpreted and recast in the form of an algorithm. Note will be made of some of the other existence proofs with brief remarks on the constructivist methodology and literature. Further elementary mathematical and logical questions pertaining to Sraffa's presentation are discussed in the brief concluding section.

I should like to add one precautionary note before proceeding: on the question of whether or not an assumption of constant returns to scale is involved in the ensuing analysis I refer to Peter Newman's highly plausible interpretation[5] with which I totally agree; to this I would also like to add that the notion of viability in the Sraffa system as interpreted by Chiodi[6] seems to me to be reasonable and very much in the same spirit as this note — at least in the single-product case. This latter remark is made in view of the fact that I have to assume viability in the discussions below. Apart from these two issues I shall not have any reason to enter any of the conceptual or capital theoretic debates that have come about as a by-product of alternative interpretations of Sraffa's highly condensed work. On the other hand I do consider it a work of deep philosophical and mathematical significance.

[2] R. QUANDT, Review of P. Sraffa's book "Production of Commodities by Means of Commodities", *Journal of Political Economy*, vol. LXIX, March, 1961, p. 500.

[3] Cf. L. PUNZO, K. VELUPILLAI, "Multisectoral Models and Joint Production" in: F. Van der Ploeg (ed.), *Mathematical Methods in Economics*, New York, John Wiley & Sons, 1984.

[4] K. VELUPILLAI, Review of L.L. Pasinetti's book "Lectures on the Theory of Production", *Journal of Economic Studies*, New Series, vol. VII, 1980, pp. 64-5; Italics in the original.

[5] Cf. P. NEWMAN, *op. cit.*, pp. 70-1.

[6] Cf. G. CHIODI, "On Sraffa's Notion of Viability" Mimeographed, Perugia, December 1988.

2. Existence as construction

> "In ordinary mathematics we often make use of what are sometimes called 'intuitive' arguments, for example in proving the binomial theorem or summing an arithmetical series. It is usually said that these arguments are either fallacious, or mere approximations to the real proofs, which proceed by induction. But these proofs reach the right answer, and they convince everybody; they are called intuitive because no one has given a rationale of them".[7]

No one — with one possible exception[8] — hostile or not has seriously questioned the correctness of the propositions in Sraffa's book. The question has been how to give (for the proofs) a 'rationale of them'. As mentioned above the rationale has been proposed via wellknown algebraic and topological formulations. The point I wish to make here is that the 'rationale of them' has always been there, directly to be perceived, provided the proofs are read as *constructive existence proofs.*

To distil a working definition of *constructivism* for the purposes of this note let me first cite a couple of the definitions from two specialist monographs on 'Constructive Mathematics'.[9]

> "... the constructive mathematician must be presented with an algorithm that constructs the object x before he will recognize that x esists.
> What do we mean by an *algorithm*? We may think of an algorithm as a specification of a step-by-step computation, such as a program in some computer language, which can be performed, at least in principle, by a human being or a computer in a finite period of time; moreover, the passage from one step to another should be deterministic".[10]

[7] A. G. D. Watson, "Mathematics and its Foundations", *Mind*, vol. XLVII, 1937, p. 447.

[8] I have in mind Frank Hahn's cryptic observation: "*Sraffa's book contains no formal propositions which I consider to be wrong although here and there it contains remarks which I think to be false*". (F.H. Hahn, "The neo-Ricardians", *Cambridge Journal of Economics*, vol. VI, December, 1982, p. 353.

[9] Time was, and it was not long ago, when there were hardly any "text books" on "Constructive Mathematics" — except for Bishop's classic (E. Bishop, *Foundations of Constructive Analysis*, New York, McGraw-Hill, 1967) and a few survey articles in the foundational literature (e. g., some of the essays in A. Heyting (ed.), *Constructivity in Mathematics*, Amsterdam, North-Holland, 1957); Bourbaki and *formalism* ruled. Today the situation is quite different. The rapid advances in computability and complexity theory have given a fresh and exciting impetus to constructive mathematics and excellent expository works have appeared and continue to appear. The texts by Beeson and Bridges & Richman (see above, footnotes 10 and 11) are accessible to most reasonably mathematically minded economists. The interested sceptic can be referred to the above two excellent monographs and to M. Dummet, *Elements of Intuitionism* (Oxford, Clarendon Press, 1977) for deeper probes into the philosophical foundations (at least of a variety of constructive mathematics — the Brouwerian variant). Excellent descriptive and discursive outlines on the relationship between formalism, constructivism — especially in its intuitive variants — and logicism and their underlying philosophies can be found, for example, in M. Kline, *Mathematics: The Loss of Certainty*, Oxford, Oxford University Press, 1980 and P. J. Davis, R. Hersh, *The Mathematical Experience*, Brighton, The Harvester Press, 1981.

[10] D. Bridges, F. Richman, *Varieties of Constructive Mathematics*, London Mathematical Society Lecture Note Series, n. 97, Cambridge, Cambridge University Press, 1987; Italics in the original.

"In constructive mathematics, we count a problem as solved only if we can explicitly produce the solution. That is, 'there is an x such that P(x)' *means* we can explicitly produce an x such that P(x). If the solution to the problem depends on some parameters, we must be able to produce the solution explicitly by some *algorithm* or *rule* when given values of the parameters. That is, 'for every x there is a y such that P(x,y)' *means* that we possess an explicit method of finding y from x such that P(x,y). Thus, we are immediately led to consider what it means to be explicitly given various mathematical objects".[11]

From these two definitions we can adapt, as a working definition of a constructive existence proof, the following:

A constructive existence proof *is* an algorithm thie implementation of which, in real time, results in the execution of a well specified program which leaves no room for intuition or ambiguity in each successive step of the algorithm and terminates, in principle, with the construction of the (mathematical) object whose existence is postulated.

'No room for intuition' here must not be confused with the important role of intuition for the foundations of constructive mathematics in one variant of it — the Brouwerian version. I think I am correct in saying that my wordy definition is consistent with any of the various schools of constructivism where, in turn, the main distinctions are either due to differences in the primitives or due to the underlying philosophical foundations. These details, in any case, need not detain us any further.

Sraffa gives a complete constructive procedure only for the case of the construction of the standard system in an economic system of single-product industries. We shall follow him and take the example as paradigmatic.

The following two assumptions are explicit in the definition of the 'mathematical object':

(a) The economic system is viable.[12]
(b) Only the basic industries of the economic system come under consideration.[13]

Finally, although not an assumption, it must be remembered — and cannot be emphasized too strongly — that the various numerical exercises we shall indulge in the immediate sequel have nothing to do with assumptions about returns to scale.[14] In the exact sense in which an algorithm

[11] M. J. Beeson, *Foundations of Constructive Mathematics*, Heidelberg, Springer-Verlag, 1985, p. 3; Italics in the original.

[12] Cf. P. Sraffa, *Production of Commodities by Means of Commodities*, Cambridge, Cambridge University Press, 1960, p. 5, n. 1.

[13] Cf. *ibid.*, p. 26; Italics added.

[14] Peter Newman's perceptive remark is absolutely to the point on both counts — returns to scale and 'thought experiment': "*One could argue in defense, ... that this trick has merely been a computing device to enable us to find the appropriate [multipliers]*". P. Newman, *op. cit.*, p. 70. And again: "*We are still dealing only with a hilfskonstrktion, the standard system, and are not committed to the assertion that if we actually* changed levels by a fraction λ_i, *we would observe output to be changed by the same fraction* λ_i". (*ibid., p. 71; Italics in the original*).

need only be feasible *in principle* the computational procedures which Sraffa enunciates are *thought experiments*.[15] Indeed it is explicitly stated:

> "That any actual economic system of the type we have been considering can always be transformed into a Standard system may be shown by an *imaginary experiment*".[16]

(For the definitions of the relevant economic terms I refer to appropriate sections in Sraffa's book). Sraffa's algorithm for the construction of the Standard system is the recursive application of the following two-step program:

Step 1: "... start by adjusting the proportions of the industries of the system in such a way that of each basic commodity a larger quantity is produced than is strictly necessary for replacement" (Sraffa, op. cit., p. 26, #37).

Step 2: "... reduce by means of ... proportionate cuts the product of *all* the industries, *without interfering with the quantities of labour and means of production* that they employ. ... [till] ... the cuts reduce the production of any one commodity to the minimum level required for replacement ...".[18]

Repeat Step. 1.

Stopping Rule: Terminate the program when "... the products have been reduced to such an extent that all-round replacement is *just possible* without leaving anything as surplus product"[19]

Result: "The proportions attained by the industries are the proportions of the standard system".[20]

Let us apply this algorithm to the numerical example discussed by Sraffa in his chapter IV:[21]

	90 t.iron +	120 t.coal +	60 qr.wheat +	3/16 labour	→ 180 t.iron
	50 t.iron +	125 t.coal +	150 qr.wheat +	5/16 labour	→ 450 t.coal
	40 t.iron +	40 t.coal +	200 qr.wheat +	8/16 labour	→ 480 qr.wheat
Totals:	180	285	410	1	

The only pity is that Newman did not explore the 'computational' metaphor and implication to its mathematical and logical conclusions which, if he had, would have stopped, once and forever, any further doubts about the unsatisfactoriness of Sraffa's proofs.

[15] I should like to add that the sense in which I interpret Sraffa's exercise as a "thought experiment" is based on Kuhn's essay in honour of Alexandre Koyre: "A Function for Thought Experiments", in T.S. Kuhn, *The Essential Tension: Selected Studies in Scientific Tradition and Change*, The University of Chicago Press, Chicago and London, 1977, ch. 10, pp. 240-65.

In particular, a point put forward and then almost rejected by Kuhn himself has been my own guiding light in understanding the role of 'thought experiments' in theoretical discussions: "*... the new understanding produced by thought experiments is not an understanding of nature but rather of the scientist's conceptual apparatus*". (*ibid., p. 242; Italics in the original*). Kuhn does not think such an interpretation of the function for thought experiments is 'quite right'. (*ibid.*, p. 242). Surely, however, a 'new understanding' of the 'scientist's conceptual apparatus' ought to lead to a new understanding of nature!

[16] P. Sraffa, *op. cit.*, p. 26; Italics added.

[17] *Ibid.*, p. 26.

[18] *Ibid.*, p. 26; Italics added.

[19] *Ibid.*, p. 27; Italics added.

[20] *Ibid.*, p. 27.

[21] *Ibid.*, p. 19.

In this viable [assumption (a)] basic economic system [assumption (b)] the iron industry produces its output 'in a quantity just sufficient for replacement'.[22] We implement step 1 of the algorithm by adjusting, for example, the proportions of the wheat industry by 'chipping off'[23] en eighth of it:

	90 t.iron +	120 t.coal +	60 qr.wheat +	3/16 labour	→ 180 t.iron
	50 t.iron +	125 t.coal +	150 qr.wheat +	5/16 labour	→ 450 t.coal
	35 t.iron +	35 t.coal +	175 qr.wheat +	7/16 labour	→ 420 qr.wheat
Totals:	175	280	385	15/16	

Now we can move on to step 2 and uniformly reduce output levels till any one industry becomes exactly self-replacing. There is, of course, a simple rule for the choice of the amount by which contraction can proceed: it is determined by the industry with the lowest surplus ratio (5/180, 170/450, 35/420). In this case it is the iron industry an thus we can reduce to 0.9722 (175/180) of the previous output levels which gives:

	90 t.iron +	120 t.coal +	60 qr.wheat +	3/16 labour	→ 175 t.iron
	50 t.iron +	125 t.coal +	150 qr.wheat +	5/16 labour	→ 437.5 t.coal
	35 t.iron +	35 t.coal +	175 qr.wheat +	7/16 labour	→ 408.3 qr.wheat
Totals:	175	280	385	15/16	

Now we have to return to step 1 because the iron industry is, once again, in a self-replacing state. We can start this iteration by chipping off a tenth of the wheat industry which results in:

	90 t.iron +	120 t.coal +	60 qr.wheat +	3/16 labour	→ 175 t.iron
	50 t.iron +	125 t.coal +	150 qr.wheat +	5/16 labour	→ 437.5 t.coal
	31.5 t.iron +	31.5 t.coal +	157.5 qr.wheat +	6.3/16 labour	→ 367.5 qr.wheat
Totals:	171.5	276.5	367.5	14.3/16	

Since we have the wheat industry at levels of exact replacement we cannot return to step 2 (second time) without further proportionate changes. Let us try chipping off a fifth of the coal industry:

	90 t.iron +	120 t.coal +	60 qr.wheat +	3/16 labour	→ 175 t.iron
	40 t.iron +	100 t.coal +	120 qr.wheat +	4/16 labour	→ 350 t.coal
	31.5 t.iron +	31.5 t.coal +	157.5 qr.wheat +	6.3/16 labour	→ 367.5 qr.wheat
Totals:	161.5	251.5	337.5	13.3/16	

Now we can return to step 2 for the second time. Once again the surplus rate in the iron industry is dominated by the other two. Thus, uniform contraction to 161.5/175 = 0.9228571 of the previous levels leaves us at:

[22] *Ibid.*, p. 19.
[23] *Ibid.*, p. 20.

	90 t.iron +	120 t.coal +	60 qr.wheat +	3/16 labour	→	161.5 t.iron
	40 t.iron +	100 t.coal +	120 qr.wheat +	4/16 labour	→	323 t.coal
	31.5 t.iron +	31.5 t.coal +	157.5 qr.wheat +	6.3/16 labour	→	339.15 qr.wheat
Totals:	161.5	251.5	337.5	13.3/16		

We can return, for the third time, to step 1. Set us try taking 0.952381 of the wheat industry and 0.750 of the coal industry.[24] We get:

	90 t.iron +	120 t.coal +	60 qr.wheat +	3/16 labour	→	161.5 t.iron
	30 t.iron +	75 t.coal +	90 qr.wheat +	3/16 labour	→	242.25 t.coal
	30 t.iron +	30 t.coal +	150 qr.wheat +	6/16 labour	→	322.99 qr.wheat
Totals:	150	225	300	12/16		

It is easy to see that the three surplus rates are equal (to 0.9287926) and a uniform contraction by this common rate brings output levels to:

150 t.iron; 225 t.coal; and 300 qr.wheat

which brings us to self-replacement for all the industries simultaneously and also provides the answer for the value of the standard ratio in one fell swoop: 20% — the % by which output levels must be increased to bring us back to the original conditions of production.

Three remarks are in order at this point:

(a) In the above example, in the sequence of adjustments, I have not observed Sraffa's 'normalization rule' (cf. above Step.2, the italicised part). This is a minor point and easy to include. I have omitted that part of the step simply to keep the process as simple as possible to make the essential points clear.

(b) The trained 'programmer' would observe immediately that Step 1 of the procedure is not uniquely defined. If the rule for '*adjusting the proportions of the industries of the system...*' is not precisely defined, some appeal to ingenuity etc., will be necessary — which is, of course, to be avoided in constructive existence proofs. This, again, is easy to remedy. Some definite rule based on the ratio of aggregate means of production to gross production is, perhaps, the simplest way out to meet this objection.

(c) The '*Stopping Rule*' should be fleshed out by some constraints on the two steps such that appropriate decreasing sequences of outputs are obtained. This will ensure convergence to the stopping rule. It must be remembered that convergence to the '*Stopping Rule*', even in constructive mathematics, is not necessary in a finite number of steps. What is essential is that preassigned precision of any preassigned step in the process should be constructively defined and shown to be approaching a well defined limit. The '*Stopping Rule*' can then be activated when an appropriate level of precision has been attained.

[24] The reader might be perplexed by the choice of such a precise number at this step which, moreover, leads us immediately to the "solution". Appearances are always dangerous. The discerning and sympathetic reader would have realized that I have worked backwards from the known solution. This is simply to terminate the monotonous procedure - which is characteristic of all algorithms. The procedure, in conjunction with concessions to point (c) in the text, is complete in itself as far as a constructive proof is concerned.

Non we may ask some of the standard mathematical questions to check whether Sraffa's procedure is satisfactorily complete or whether there are redundant assumptions or even insufficient hypotheses; in other words, have 'intuitive' factors been let in through the back door, so to speak. The immediate mathematical questions pertain to necessity and sufficiency — but, of what? To get to the answer to this question let me indulge in a homely and quite 'constructive' example.

Imagine an artisan, say a carpenter, who has been given a block of wood of a certain quality and a design to be fashioned out of the block. He has also access to some tools (in which is included a work-bench). The analogous questions are:

(a) Is the design complete or is the carpenter supposed to use some ingenuity, intuition etc., at some junctures?
(b) Is the wood 'malleable' enough relative to the available tools (and the given design)?
(c) Are the tools necessary — and sufficient, given the particular block of wood and the design?

In our definition of a constructive existence proof we have assumed away any role for ingenuity, intuition etc. In other words the design is complete in all its details. This of course means that, included in the design, are also specific instructions as to the order in which the wood is to be fashioned to produce the final object. Thus, the Standard system is explicitly defined and the algorithm is clearly specified. Implementation of the algorithm by means of the two 'tools' of proportionate variations and uniform contraction is to be on the 'block of wood' — the economic system. How 'malleable' is it relative to the tools: proportionate variation and uniform contraction? 'Malleability', in this case, is circumscribed by the two assumed characteristics of the economic system: that it is viable and that it has been brought into a system of basic industries.

Now we can ask specific questions about the necessity and sufficiency of the hypotheses and the constructive process itself; for, *the existence proof is the whole process of construction* encompassing all of the above three elements (a), (b), and (c). Thus, are there superfluous or insufficient assumptions at any stage in the construction? Of course the question cannot be answered separately for the three different stages because the tools are used on the block of wood to fashion a given design — all interconnected. The question, therefore, is whether for this particular set of tools and for the given design is the exercise feasible on the given block of wood? Therefore:

i. Are proportionate variations necessary for the given economic system to construct the standard system and determine the standard ratio? Are they sufficient?
ii. Are uniform contractions analogously necessary and sufficient?

This is what the question of necessity and sufficiency boils down to in the case of this particular constructive existence proof. The answer is immediate — or so it seems! Let us take it in turn. First, proportionate variations. Are they necessary? The answer is immediate except for trivial cases where the economic system is, from the very outset, in a self-replacing state or already a standard system — i.e., with a common surplus rate in all the industries. Are they sufficient? Again the answer *can* be affirmative (even in terms of Sraffa's own example which we took up for illustration above) — but should *not* be! To expand on such a paradoxical statement let us take the second 'tool', uniform contraction.

Is it necessary? Obviously not. *Ultimately* the standard system is simply a reproportioned actual system and, therefore, proportional variation is sufficient. Is uniform contraction sufficient? Again very clearly not except for the trivial cases noted above. Then why has Sraffa been so explicit about the '*two types of alternating steps*'?

This is where the answer requires the full force of assumptions characterizing constructive existence proofs; and, above all, a proper understanding of the definition of 'algorithm'. A key ingredient in the definition is the requirement that the process of construction be 'unambiguously' recursively defined: each step determines the next step in the construction process without any need for special ingenuity etc. Without any guidance as to the direction in which proportionate variations are to proceed, it will be almost impossible to construct the Standard system, except by fluke and the use of ingenuity — especially for sufficiently large economic systems (even of the order of four industries). This particular step in the algorithm for the constructive existence proof is *necessary* (but not sufficient) in a wider sense than if mathematics was viewed simply through the glasses worn by the formalists; its necessity is most aptly illustrated by approaching a study of laws and processes with a tempered view:

> "... the laws of nature, Dirac says, control a substratum of which we cannot form a mental picture without irrelevancies. ... Mental pictures, the little models we construct in our minds, are oversimplified and padded with irrelevancies".[25]

Where even 'mental pictures' cannot be formed 'without irrelevancies' it would be foolish to pretend that 'mental constructions' can be achieved 'without irrelevancies'. Even without this appeal to a wider epistomology the case for the mathematical necessity of steps akin to 'uniform contraction' can be made by an appeal to the logical implications of computability theory which has as its foundation, in any case, the notion of recursive functions.

It is eminently clear, therefore, that Sraffa's system is made up of just the right combination of assumptions at all levels to enable the Standard system to be constructed and to determine the standard ratio. However, to appreciate its adequacies it is neither fair nor necessary to view it from

[25] L. Young, *Mathematicians and Their Times*, Amsterdam, North-Holland, 1981, p. 299.

the point of view of the conventional mathematics of formalism and the standard technique of existence proofs where noncontradiction is the sole criterion.

3. Concluding notes

> "To the philosopher or to the anthropologist, but not to the mathematician, belongs the task of investigating why certain systems of symbolic logic rather than others may be effectively projected upon nature. Not to the mathematician, but to the psychologist, belongs the task of explaining why we believe in certain systems of symbolic logic and not in others, in particular why we are averse to the so-called contradictory systems in which the negative as well as the positive of certain propositions are valid".[26]

Much, if not all, of the mathematical mode of thinking in economic analysis is dominated by the methods of the formalists. These methods are deep and powerful. However the one-dimensional training in the methods of the formalists leads, naturally, to a distortion in perspective. Such distortions have been instrumental in perpetuating an unfair interpretation of the logic of Sraffa's mathematical formulations. I have attempted to redress the balance by offering the alternative perspective of constructive mathematics. I believe all existence proofs in a work like Sraffa's should be interpreted constructively; I also believe the whole exercise is a classic 'thought experiment'.

A reading of *Production of Commodities by Means of Commodities* from the dual methodological perspectives of constructivism and 'thought experiments' would be interesting in itself — quite apart from the additional insights one is able to obtain into the economic assumptions and their implications. This must be especially true in the case of the problem of reducing to dated quantities of labour systems of multiple-product industries and, of course, the construction of the Standard system with joint products.

Wheter Sraffa himself intended an interpretation in the way indicated in this paper is, perhaps, besides the point. I may be permitted to cite, in conclusion, the dedication written by Sraffa in my copy of *Production of Commodities by Means of Commodities*:

> "This book has the advantage of being compact"

signed: Piero Sraffa; dated: 20 Dec. 1980.

Some, including myself, will surely opt for the disadvantages.

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[26] L. E. J. Brouwer, "Intuition and Formalism", *Bulletin of the American Mathematical Society*, vol. XX, 1913, p. 84.