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The Objective Theory of Prices^{*}

Massimo Egidi and Giorgio Gilibert

“There is something that should matter more than Marx’s words, even to an orthodox Marxist: it is mathematics. To hold a proposition based on wrong premises, to persist in building a whole theory on it, means to devote oneself to a hopeless fancy” (G. Von Charasoff).

I. A PREMISE AND TWO WARNINGS

Georg von Charasoff was born in Tiflis, Georgia, in 1877. At eighteen years of age he enrolled in the Faculty of Medicine at the University of Moscow, but was shortly afterwards obliged to leave Russia following the student protests of 1896. He re-enrolled in Germany, this time in the Faculty of Mathematics and Natural Sciences at Heidelberg, taking his degree in Mathematics in 1901 and publishing his thesis in the year following. Thereafter he devoted himself to the study of Marxist economic theory, with the aim of reformulating the theory on logically unimpeachable foundations. Of a projected trilogy of works on the subject, only the first two volumes appeared: *Karl Marx* of 1909 and *Das System des Marxismus* of 1910, both published in Berlin.¹

The present article modestly proposes to expound, as concisely as possible, the theory of prices put forward by von Charasoff in his first work on economics and developed in the second; and in so doing to adhere as faithfully as possible to the original (including the notation). Two elements motivated the writing of the article. On the one hand the obvious interest of the argument in the context of the history of economic doctrine. On the other, its theoretical relevance for the present time; this lies in its technique of exposition as well as in the results that von Charasoff obtained. With regard to technique, that employed by von Charasoff appears highly versatile and quite unusual; the only point of reference in contemporary analysis would seem to be a theorem of Hukukane Nikaido — one which,

^{*} This is the English version of the paper “La teoria oggettiva dei prezzi”, published in *Economia Politica*, vol. I, 1984, pp. 43-61.

¹ G. VON CHARASOFF, *Karl Marx über Menschliche und Kapitalistische Wirtschaft*, Berlin, H. Bondy, 1909; *Das System des Marxismus*, Berlin, H. Bondy, 1910.

however, does not appear to have made much impression on readers of economic literature, at least up to now.²

As to the results, a brief hint can be given here. In dealing with the social product von Charasoff proposes returning — so to speak — to its origin; that is to say, he enquires which set of inputs (including workers' subsistences) must be consumed in order to produce it, and which other set must be consumed in order to produce those inputs in turn, and so on *ad infinitum*. Which at the time was a not unusual scheme.

Each element in the series thus obtained is a sum of well identified addenda, each of which represents a set of inputs destined in the first instance, second instance... n^{th} instance, to produce the single commodities present in the actual social product. As the series proceeds, it emerges that these addenda tend to be composed of the same commodities in the same proportions: the system of prices "of production" is thus immediately given by the proportions in which these addenda, physically comparable, inhere in an element of sufficiently high degree. The peculiarity is that the prices are thus determined independently of any knowledge of the rate of profit.

As the series proceeds, two subsequent elements in turn tend to be composed of the same goods in the same proportions. The rate of profit can thus be obtained simply by relating one element of sufficiently high degree with the subsequent one (its means of production), two sets which at this point are physically comparable.

These two elements of sufficiently high degree give rise to the peculiar situation of a product (destined for productive re-use) in which the various commodities are related in the same proportions that can be found in its own inputs — the situation, i.e., of a capital produced by means of itself. Von Charasoff calls it original capital, and the rate of profit can thus be interpreted as the rate of growth of this hypothetical original capital.

In dealing with an argument with which the reader will presumably have scant familiarity, it will be as well to add a word or two of caution. This article will discuss solely the theory of prices, the nub of the argument, disregarding the general debate to which von Charasoff intended to contribute. The argument would deserve a separate treatment. Nor do we wish to depict von Charasoff as a "great but neglected figure". Neglected he certainly was, or better, misunderstood. But labels of this kind should be avoided, since they divert attention from historically relevant problems. However isolated an author may be, he is nevertheless the product of a precise cultural milieu, and that of von Charasoff is clearly recognisable even though it has received singularly little attention. It could be described as a genuine Russo-German school of mathematical economics.³ But this

² See H. NIKAIDO, *Convex Structures and Economic Theory*, New York, Academic Press, 1968. An exception to this neglect is represented by M. Egidi, "Stabilità e instabilità negli schemi sraffiani", *Economia Internazionale*, 1-2, 1975.

³ Cf. G. GILBERT, "La trasformazione, vera storia di un falso problema", *Working Papers del Laboratorio di Economia Politica*, 8, 1982. Note also, in parenthesis, to what extent the culture

topic — as has been said — must be reserved for treatment elsewhere.

Lastly, we must avoid, as far as possible, a perspective of anticipations and priorities. Other writers after von Charasoff tackled the same problems and reached results that were similar from a formal point of view. But setting the findings of different authors side by side may prove pointless at analytical level, while at the historical level it is legitimate only if the authors in question can be shown to have been in contact with each other.

Although von Charasoff's reasoning does not make use of particularly advanced mathematical techniques, it is of an abstract nature and the reader may easily find himself at sea. The only concession made is the frequent recourse to a simple numerical example. We shall proceed by re-expounding the logical heads of the argument, illustrating it with graphs based on the numbers in the original example. The general proof of the main results will be supplied by using the language of modern analysis. This will often necessitate actual "translation" of von Charasoff's propositions. *These interventions, and the relative demonstrations will sometimes appear in the text in italics*, and can be disregarded by the reader who is not concerned with the formal aspects of the argument.

2. THE PRODUCTIVE SERIES

Consider an economy in which (i) there are constant returns to scale, (ii) there are no joint products (and no fixed capital is employed) and (iii) the actual consumptions by workers are given and assimilated to every other productive consumption.

Let X be the annual production vector in the space of n dimensions of commodities, and $a, b, c, \dots$ the vectors representing the production of the first, second, third... sector. Given the absence of joint products, we shall get $a = (x_1 \ 0 \ 0 \dots \ 0)$, $b = (0 \ x_2 \ 0 \dots \ 0)$, $c = (0 \ 0 \ x_3 \dots \ 0)$ etc., where $x_1, x_2, x_3, \dots$ are the quantities produced of the first, second, third... commodity. X can thus be represented as the vectorial sum of the productions of the various sectors: $X = a + b + c + \dots = (x_1 \ x_2 \ x_3 \dots)$.

According to the hypothesis of constant returns, we can represent the productive consumption with the following system of vector equations

$$\begin{aligned} a' &= la + mb + nc + \dots \\ b' &= qa + rb + sc + \dots \\ c' &= ua + vb + wc + \dots \\ \dots & \quad \dots \quad \dots \end{aligned}$$

of the 'historian' may influence the labelling of this or that important economist as "forgotten": whereas Dmitrieff was almost unknown in French-, English- and Italian-speaking cultures, he could hardly be considered as "forgotten in the German-speaking world".

where $a', b', c' \dots$ represent the capitals productively consumed in the various sectors in order to produce $a, b, c \dots$

These equalities feature coefficients of a particular type. They indicate the amount of annual production of a commodity that is employed by a certain sector: for example, m indicates the amount of production b destined for use in the sector producing a . In order to distinguish them from their better known confrères, we may call them "share coefficients".⁴

The way of writing here adopted, which has the virtue of respecting von Charasoff's notation, is not very customary, but can easily be returned to the more familiar modern notation. Thus, let A be the matrix of the technical coefficients (a_{ij} represents the quantity of commodity i used to produce one unit of commodity j) and T the matrix of the share coefficients ($t_{ij} = a_{ij}(x_i/x_j)$); and let Q be the matrix whose lines are constituted by the vectors $a, b, c \dots$

The productive consumption can be compactly expressed in two ways:

by using the share coefficients, in the form $Q' = TQ$
and by using the technical coefficients in the form $Q' = QA$

We shall thus have

$$Q^{-1}TQ = A$$

which shows how T and A are similar matrices and thus endowed with the same spectral properties.

The productive consumption, hitherto distinguished on the basis of the sectors to which it is destined, can be expressed in aggregate form by summing the lines of matrix Q' , that is, by premultiplying by vector $u = (1 \ 1 \dots 1)$.

$$uQ' = uQA = uTQ$$

or, by recalling that $uQ = X$,

$$X' = uTQ = XA$$

Let us now consider the capital X' as a product: by X'' we can indicate the capital consumed in order to produce X' , a capital which will be termed "of second order". We shall then have the following equivalences:

$$a'' = la' + mb' + nc' + \dots$$

$$b'' = qa' + rb' + sc' + \dots$$

$$c'' = ua' + vb' + wc' + \dots$$

⁴ "And lastly the sharing out of the product, its utilization in the various points of the circular process—can be represented with a coefficient similar to that of cost; it could be called share coefficient. The units of the product destined for a specific use are placed in relation to their overall amount. If the elements in question (6) are divided such that 2 go into one process and 4 into another, the corresponding share coefficients will be $1/3$ and $2/3$ " (W. Leontieff, "Die Wirtschaft als Kreislauf", *Archiv für Sozialwissenschaft und Sozialpolitik*, 3, 1928, p. 585).

or, in aggregate terms and using the technical coefficients,

$$X'' = X' A$$

Thereafter, turning to the capital of third, fourth... kth order, we can write a general system of equations of the type

$$a^{(k+1)} = la^{(k)} + mb^{(k)} + nc^{(k)} + \dots$$

or, in aggregate terms,

$$X^{(k+1)} = X^{(k)} A$$

which defines a chain of relations between the capitals of contiguous order as k varies.

"Thus we arrive at a productive series $X \sim X' \sim X'' \sim X''' \sim \dots X^k \sim$, which possesses the noteworthy property according to which in each of its rings the product of the one following is the capital of the one preceding; investigation of this series is indispensable to the study of all the theoretical questions of political economy".⁵

Of course, von Charasoff uses the relation $X^{(k)} \sim X^{(k+1)}$ to indicate, with a different symbol, the vector expressions we have just arrived at.

As K increases X tends to zero; this can easily be seen. But our interest focuses on the reciprocal behaviour of its various components; we are interested, that is, in discovering whether their ratios converge towards determinate values.

To this end, it will be as well to normalize the successive values of the capitals $X^{(k)}$; in particular, we can divide the various components $x_1^{(k)}, x_2^{(k)} \dots x_n^{(k)}$ by a weighted average, where the weights — interpretable as prices — are arbitrary: in our example, the simplest, all these weights are unitary. We shall thus have:

$$Y^{(k)} = X^{(k)} / X^{(k)} u$$

In this way $Y^{(k)}$, unlike $X^{(k)}$, does not tend towards zero as k increases; but the ratios between the components of $Y^{(k)}$ are the same as those existing between the components of $X^{(k)}$.

As can easily be seen, we can also write

$$Y^{(k)} = Y^{(k-1)} A Y^{(k-1)} A u$$

Let us now suppose that as k increases Y^ converges — as it does, at conditions that shall be later examined — on a limit vector Y^* . We can now write*

$$Y^* = Y^* A / Y^* A u$$

i.e., having set $\lambda = Y^ A u$*

$$Y^* A = \lambda Y^*$$

⁵ G. VON CHARASOFF, *Das System...*, op. cit., p. 120.

Since the matrix A possesses at least one column (relating to the wage-commodity) with all its components positive, Y^* , if it exists, is semipositive; and $\lambda = Y^*Au$ is positive.

Here we acknowledge the result obtained by Perron in 1907 and generalized by Frobenius in the years following.

3. BASIC PRODUCTS

Let us now consider the elements $X, X', X'' \dots$ of the productive series. Let us suppose that X contains luxury products that by definition are not employed in any production. The components of X' relating to those products will obviously be nil. Similarly, in passing from X' to X'' the means employed solely in the production of luxuries will disappear; in passing from X'' to X''' the second-order means of luxury production will disappear; and so on.

"However, since the productive series can by its very nature progress to infinity, there will clearly be a finite point beyond which there will be no need to progress to any further exclusion, and where all the remaining elements of the productive series are always composed of the same means of production, which are indispensable, in the last analysis, for the production of all possible products, and which we therefore call basic products..."

The entire problem thus boils down to the formation of the prices of these basis products. Given that they exist, from them we can derive the price of the means of luxury production and, ultimately, also those of luxury products".⁶

There is a gap in the reasoning here, but it is worthy of note how the definition of basic products remains valid all the same. For we can imagine non-basic products being used in the production of themselves, a case that von Charasoff does not take into consideration: on the basis of his argument, these products do not seem destined to disappear the further we return along the productive series. It can, however, be shown that, whereas the ratios between the components relating to basic commodities tend to have positive values, those between the component relating to these non-basic commodities and the components relating to basic commodities tend towards zero.⁷

4. THE THEORY OF PRICES

The problem of relative prices arises from the need to make comparable quantities of different, qualitatively distinct and incommensurable products.

⁶ *Ibid.*, pp. 120-1.

⁷ On the (economically obvious) conditions that guarantee this result, cf. M. EGIDI, *op. cit.*, p. 5.

But what make the problem hard to solve is not so much the heterogeneity of the commodities as that of the capitals destined for their production.

Let us return to capital X' (confining our attention, for the reasons just stated, to basic products alone) in its disaggregated form: $a' + b' + c' + \dots$. These various sectorial capitals are, in turn, made up of the different means of production. If they all happened to have the same composition — if, i.e., $l:m:n\dots = q:r:s\dots = u:v:w\dots$ etc. — they would be qualitatively indistinguishable: that is to say, the one would turn out to be a multiple of the other. In which case, the problem of prices could quickly be solved, since the prices would perforce be proportional to the quantities of homogeneous capital employed in the various productions.

Ordinarily, however, the capitals $a', b', c' \dots$ will be heterogeneous and we shall therefore be unable to make any reference to the “quantities of capital” present in the various sectors. For “two capitals of different type are on principle absolutely incommensurable, therefore, for example, it is impossible to say which capital represents the greater magnitude”.⁸

We now come to the second element of the productive series: $X'' = a'' + b'' + c'' \dots$. Considering this second order capital in its disaggregated form, according to the sectors for which is destined, von Charasoff remarks a fundamental property; whatever the composition of the sectorial capital at the outset, that of the second order is relatively less heterogeneous. For the composition of each second order capital is given by a weighted average of the compositions of all the starting capitals. This progressive convergence among the compositions occurs each time a capital of a certain order passes to the next order (X''' being defined in a perfectly analogous way to X'' , and so on).

Thus it will always be possible to reach a sufficiently high order, say X^* , such as to guarantee, with the desired approximation, that all the various partial capitals $a^*, b^*, c^* \dots$ have the same composition.

The capitals, that appear in the productive series, disaggregated according to sector of destination, may be written in matrix form

$$Q' = TQ, \quad Q'' = TQ' \dots$$

or, as will be recalled,

$$Q' = QA, \quad Q'' = Q'A \dots$$

from which, by iterative resolution, we get $Q^{(k)} = T^{(k)}Q$ or that $Q^{(k)} = QA^k$.

It must now be demonstrated that as k increases, the lines of the matrix T (or A) tend to structure themselves in such a way that the ratios between the respective components are equal: that is to say, for example, that $t_{i1}^{(k)} : t_{i2}^{(k)} = t_{j1}^{(k)} : t_{j2}^{(k)}$.

⁸ G. VON CHARASOFF, *op. cit.*, p. 122.

For the sake of custom, let us now use only the matrix of technical coefficients and return to the vectorial expression of the capital of order k . We shall have

$$X^{(k)} = XA^k$$

Since, as we said, matrix A has at least one column entirely positive, we know, by the above-quoted theorem of Nikaido,⁹ that the series $X^{(k)}$, duly normalized (i.e. as we did in the paragraph on productive series), converges to a finite limit, which we have called Y^* .

But if YA^k converges on Y^* , whatever may be the initial proportions of Y , we can make the latter successively equal to the vectors e^i and e^j , characterized by having all their components nil, with the exception, respectively, of the i th and the j th assumed to be unitary.

Thus constructed, the series will represent the trend of the i th and j th lines of the matrix A^k : since both the series must converge on Y^* , the two lines tend to be equal to one another.

We can now return to our equation $Y^*A = \lambda Y^*$ and, remembering that $Y^{(k)} = X^{(k)}/X^{(k)}u$, we can rewrite it in the following form

$$X^*A = \lambda X^*$$

where the constant X^*u that appears in both members of the equivalence has been eliminated.

The equation has a striking economic interpretation: X^* is the capital of sufficiently high order in the series and X^*A is the capital contiguous to it (the inputs consumed in its production). The equation tells us that two contiguous capitals of sufficiently high order are proportional according to a λ factor: thus they have the same composition, and $1/\lambda$ can be interpreted as a growth factor.

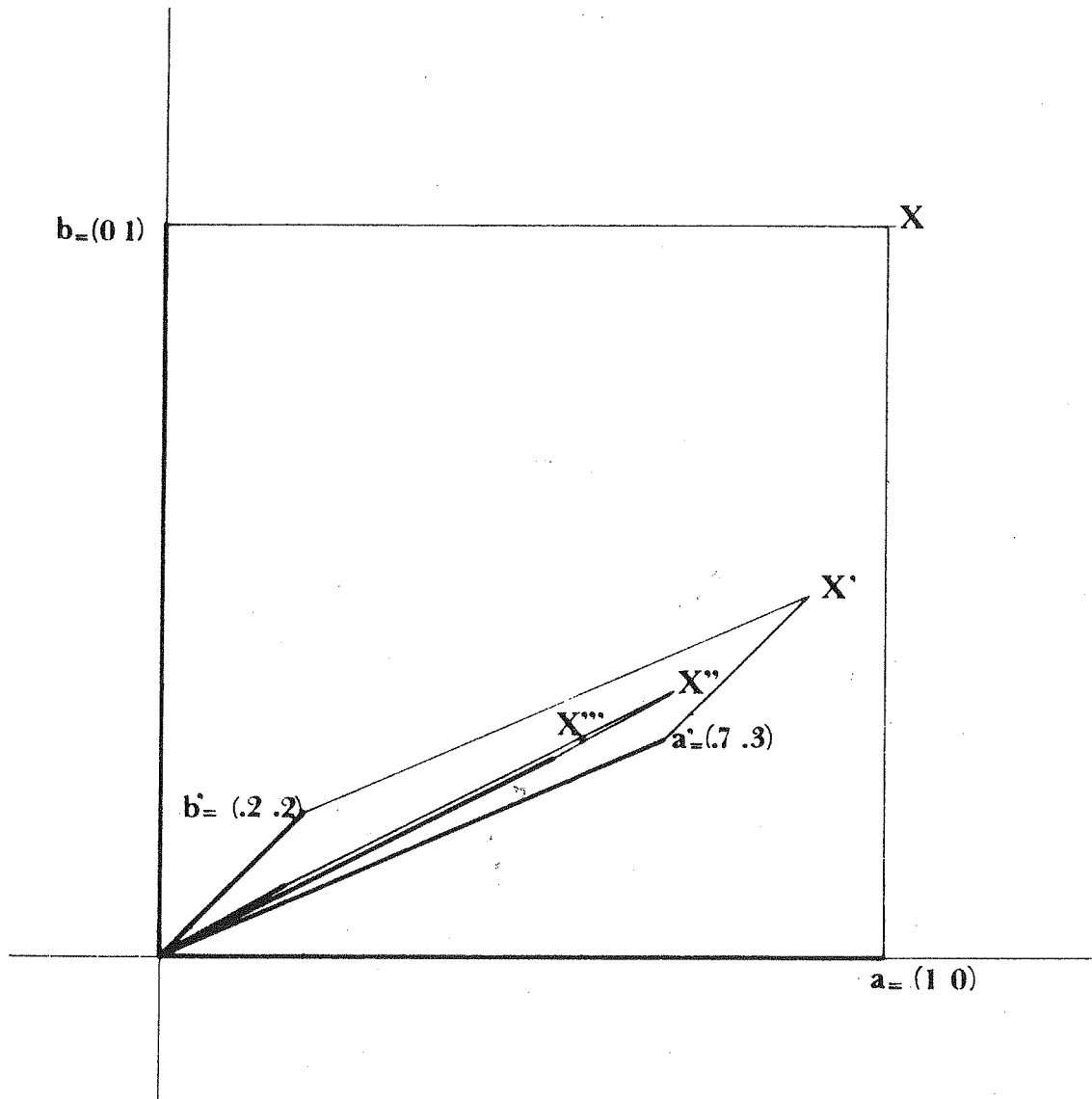
The problem of prices is now definitively solved (independently of whether or not the rate of profit is known): prices must be proportional to the quantities a^*, b^*, c^* ... of homogeneous capital necessary — so to speak — in the final instance to produce the first, second, third... commodity. A brief reflection will suffice to make it plain that this occurs for the same reason by which they should have been proportional to a', b', c' ... if the sectorial capitals had been homogeneous from the beginning.

X^* can be considered as homogeneous capital, in the sense that its component parts (a^*, b^*, c^* ...) all have the same composition; the said capital has been produced by employing capital of the next degree, necessarily of the same nature, $X^{*'}.$ The rate of profit will therefore be given by the simple formula

$$1 + R = X^*/X^{*'}$$

⁹ "Let A be an undecomposable non-negative matrix and $\lambda = \lambda(A)$, the $\lim_{t \rightarrow \infty} (A/\lambda)^t$ exists if, and only if, there is a positive integer k such that $A^k > 0$ " (H. NIKAIDO, *op. cit.*, p. 110).

At this point it may be useful to visualize the argument with a simple geometrical figure based on von Charasoff's example. This considers only two goods and can be illustrated by a two-dimensional graph. All our vectors $X, X' \dots X^*, a, a' \dots a^*$ etc., can be represented by points on the plane.



We indicate on the coordinate axes the quantities of the two commodities produced; the assumption of unitary levels of production is unimportant. X is equal to $a + b$. a' and b' represent the vector of the inputs necessary to produce a and b respectively: they are averages of a and b weighted respectively according to the weights l, m and q, r . The composition of a' and b' is represented by their slope. The lack of alignment indicates the heterogeneity of the sector capitals. X' is equal to $a' + b'$.

Let us now construct $a'' + b''$ (and X'' as their sum): they are the averages of a' and b' weighted according the same weights as before. Their slope will be comprised between those of a' and b' . In other words, the parallelogram having its acute angle at X'' will be flatter as compared to the one having its point at X' .

The imperfection inherent in representation by graph reduces the process of convergence to a few passages. Already by the third stage inputs and product (a''' , b''' and X''') are practically aligned with X'' : if we judge the approximation to be sufficient, the price of the first commodity in terms of the second will be given by the ratio $a'''/b''' = 3$, and $1 + R = X''/X''' = 1 + 1/4$.

5. ORIGINAL CAPITAL

When, as we go back along the productive series, we reach element X^* , we find ourselves before a phenomenon that is doubly worthy of note. On the one hand, the overall capital of the economy appears as a homogeneous quantity, in the sense that its various component parts (a^* , b^* , c^* ...) all have the same composition, namely that of the overall capital. On the other hand, since the proportions of the capital X^* have been reached, these will appear unchanged (or, better, with ever greater precision) in all the stages following: "we have represented in its pure state the idea of a capital that grows by means of itself, and indeed its rate of growth P^* appears equal to the capitalistic rate of profit R^* ".¹⁰

The proportions in which the various means of production stand within X^* define the composition of what von Charasoff calls *Urkapital*, "original capital". This "is none other than the basic production when the branches assume determinate dimensions. And precisely in order that the basic production shall appear as original capital, the decisive requirement is that the gross production have the same composition as the overall capital".¹¹

We can develop the argument by using slightly different terms that may be more familiar nowadays. The problem of prices can be solved thanks to the fundamental property of a productive series — that of tending towards a capital whose component parts all have the same composition. Formally speaking, as k grows, $X^{(k)}$, duly normalized in order to prevent its components from vanishing, tends to X^* ; since $X^{(k)} = XA^k$, all the lines of A^k tend to become proportional among themselves. And the factors of proportionality are the relative prices.

By inverting the argument it can be seen that the columns are also proportional among themselves, according to the factors that indicate the composition of the original capital X^* . We have thus attained an important

¹⁰ G. VON CHARASOFF, *op. cit.*, p. 112.

¹¹ *Ibid.*, p. 126.

result: the matrix A^* , to which A^k tends as k grows (duly normalized to prevent its elements from vanishing) contains in itself—so to speak—all the important properties of the system. Each column of A^* is proportioned to p^* , the vector of the relative prices; and each line is proportional to the original capital X^* .

Von Charasoff sets the concept of original capital at the centre of his analysis. This is not a concrete productive configuration — otherwise it would represent a special case, even though an interesting one. Nor is it a purely intellectual construction, aimed at solving abstract problems (determination of prices and rate of profit).

On the contrary, we are dealing here with the real nucleus that characterizes every economic system and whose form is determined by three basic factors: technology in use, wage level and length of the working day.¹² So real is it, indeed, that by solving the problem of prices von Charasoff claimed to have shown how for capitalists the whole law of value boils down to saving original capital: "They do it without knowing it".¹³

The original capital therefore exists and guarantees the solution of concrete problems, like that of adjusting capitalistic production to demand. It is usually supposed that variations in demand bring about alterations in the relative profitability of the various industries and that these alterations lead to variations in the production levels obtained by shifting capital from one industry to another. At first glance, however, this process would seem to be impossible, since the capitals of the various industries are not fungible; a problem which can only be overcome by recalling the existence, in the beginning, of a single capital infinitely versatile and malleable and thus freely removable from one production for the benefit of another. "It is as though one had a field that was formerly used to produce wheat and is now given over to rye".¹⁴

Although original capital exists, it is normally imperceptible to direct observation. We have, however, touched on an improbable but interesting case, where it appears, so to speak, at the surface. In a competitive economy, in which all profits are entirely invested, *Urkapital* is produced, as is now well known.¹⁵

¹² Given the way in which the wage is dealt with—as a set of actual consumptions, assimilated to the other productive inputs—it is hard to consider variations in distribution. The working day can, however, be varied: by assuming it becomes shorter, techniques being equal, "the composition of original capital varies and the rate of profit falls steadily, reaching zero when the surplus labour is annulled" (*ibid.*, p. 113).

¹³ *Ibid.*, p. 112.

¹⁴ *Ibid.*, p. 134.

¹⁵ "The process of accumulation must then be generally proportionate in such a way that each firm grows in unison each year and each private entrepreneur earns on his own capital a proportionate profit utilized in its entirety in the expansion of the firm. On the basis of this assumption, the social capital will be modelled in the original type and the rate of profit will clearly assume the significance that our research has shown to be due to it: it will be given by the growth rate of the original capital in the process of annual production" (*ibid.*, pp. 126-7).

"The productive series of a commodity is, so to speak, its family tree".¹⁶ And each tree leads back infallibly to the common stock, the original capital. Thus each product appears in the final instance as the fruit of successive alterations brought to bear first on the original capital and subsequently on its derivatives (of course the sequence should not be understood in a historical sense: see appendix I). "We have a complete analogy with the doctrine of heredity", comments von Charasoff.

6. THE MECHANISM OF COMPETITION

Let us suppose that the system of prices in force in the economy at a given moment is totally arbitrary and, anyhow, such as not to guarantee a uniform rate of profit in the various industries: these prices will of course serve for evaluating both final products and means of production. Let us further suppose that the effect of competition on the prices be reduced to its most elementary form: from time to time the prices of commodities will be modified so as to be proportional to the current values of the capitals employed in their production.

Let us now consider the situation at the start. Let the prices of products be $p_1, p_2, p_3, \dots$; the corresponding values of capital employed in their production $p'_1, p'_2, p'_3, \dots$; the values of the second order capitals $p''_1, p''_2, p''_3, \dots$; etc.

At first the prices of products will be modified so as to render them proportional to $p'_1, p'_2, p'_3, \dots$; but at the same time these latter will have to change in order to be proportional to $p''_1, p''_2, p''_3, \dots$. Subsequently, therefore, the prices of products will become proportional to $p''_1, p''_2, p''_3, \dots$ while those of their capitals will themselves too become proportional to $p'''_1, p'''_2, p'''_3, \dots$ and so on.

Represented by graph, the process shows a clear convergence of the starting prices towards the "production" prices. Let us trace the arbitrary vector of prices P . On the basis of this, the value of the means of production employed in the two industries p'_1 and p'_2 is given by the projection on P of the respective vectors of the unitary inputs, i.e. by AB and CD . Let us now construct P' proportional to p'_1 and p'_2 , that is with a slope equal to AB/CD . The ratio between the values of the means of production employed in the two industries is now EF/GH , a ratio already very close to the slope of P^* , the known vector of the prices of production. Note that throughout the procedure the rate of profit has played no explicit part.

Von Charasoff poses the following problem: what guarantees the convergence of the process just described? Let us consider, for example, two products that are different but equivalent on the basis of prices of

¹⁶ *Ibid.*, p. 128.

can only be due to the progressive physical identification of capital goods thereafter involved.

Once more, let us go deeper into the argument, in more formal terms. To return to matrix A^* : the perfect symmetry of the results that can be obtained enables us to invert the previous reasoning. Instead of considering the initial production X taken at pleasure, let us consider the initial system of prices P taken at pleasure; instead of working "backwards" with the sequence $X = XA$, $X'' = X'A...$, we work "forwards" generating the sequence $P' = AP$, $P'' = AP'...$, where at each step the prices are reportioned to the costs, according to the working of the competition as described by von Charasoff.

It is immediately found that $P^{(k)} = A^k P$ and thus, as k increases, the system of prices converges on A^*P . Since each column of A^* is proportional to the system of prices of production, A^*P will be likewise, being the combination of the columns of A^* through the components of P . In conclusion, competition makes any arbitrary system of prices converge on the system of prices of production; and this happens thanks to the structure of A^* , i.e. of the original capital (in disaggregated form).

"In this way the analysis of capitalistic competition leads us back to the theorem of original capital. If this original capital did not exist, a uniform rate of profit could not be achieved. Thus it can be seen that all those who speak of normal prices of production or of uniform rate of profit unconsciously affirm the existence of the aforesaid original capital. They do so without knowing it.

Note how the theory here advanced for capitalistic competition is none other than an extension of Marx's. For Marx begins by supposing that the starting prices of commodities are equal to their values, and then alters them to amounts proportionals to the value (i.e. to the starting price) of the capitals, explicitly referring to the capitalistic tendency to form a uniform rate of profit. At this point, however, Marx halts his transformation of values into prices, and this is the first imperfection in his theory — an imperfection that his critics have never tired of blaming him for, instead of overcoming it by dialectically developing his basic idea. A second imperfection is the following: Marx wished at all costs to start from the values of the commodities. But this is absolutely inessential for the theory of prices as such. The starting prices can be arbitrary".¹⁸

APPENDIX I

The elements of the productive series $X' \sim X'' \sim ... X^* \sim ...$ can be intuitively represented both as a temporal sequence and as components of the overall capital that society must continually renew in order to guarantee the production of X . In other words, by annually reproducing the capital $X' + X'' + ... X^* + ...$ society is able annually to utilize the product X for final purposes. Let us define this as "reproductive capital" of the product

¹⁸ G. VON CHARASOFF, *Das System...*, op. cit., p. 138.

X. The reproductive capital resembles a tree that annually yields the product X as a ripe fruit, while retaining its own existence and fertility.

Nothing prevents us from assuming that a single commodity can be produced net. We shall thus have the reproductive capital of that commodity, viz:

“The basic production, whose sectors are assumed in proportions such that the entire surplus assumes the pure form of product X... In general, in order to produce this article society must activate the various industrial sectors but only to the end of producing good X. The remainder that is produced is used in that society for purposes of production (and for the necessary consumption of the working class)”.¹⁹

Reproductive capital depends on the level of wages and the length of the working day. We can imagine the formation of a capital like this taking account, however, only of the technical inputs and not of the consumption of the workers: von Charasoff defines this capital as “reproductive base”. Reproductive capital and base coincide only in the extreme hypothesis of nil wages.

“Let R be the reproductive base of a certain product X , and a the quantity of human labour expended annually with this reproductive base in order to obtain product X ; then the annual reproduction of X will cost society exactly the quantity of labour a ”.²⁰

APPENDIX II

In Russia and Germany at the beginning of the twentieth century — as Schumpeter remarks — “theory minded economists had hardly any choice but to turn to Marx”. As we have seen, von Charasoff was no exception and approached economic theory through Marxism; and the problem of prices presented itself to him in the typical guise of the transformation of labour-values. In those same years the operation had for the first time been carried through in a satisfactory manner: by Tugan-Baranowsky, with a numerical example, in 1905; and two years later by von Bortkiewicz, in more general terms. The starting point had been the well known schemes of reproduction of capital.

Von Charasoff's peculiar approach is especially clear in the brief text we quote here: this is the appendix added to chapter VIII, entitled “Why critics do not understand Marx's theory of prices”, in the book *Karl Marx* of 1909.

“In my second book I shall offer extensive demonstration of how an absolutely exact theory of prices can be developed on the basis of the law of value. Here

¹⁹ *Ibid.*, p. 141.

²⁰ *Ibid.*, p. 147.

I wish only to communicate the findings of my further research to readers well versed in mathematics. Let the value w of the commodity be equal to $k + m$, and let $k' + m'$ be the value of capital k — where k' indicates the value of that capital that is utilized in the production of k , capital that will be termed “second order” — and thus $w = k' + m' + m$. I shall show that the price of the second order capital diverges from its k' value less than does the price of first order capital with respect to its k value. If we made the price of second order capital equal to its k' value, we should make a smaller error. From this the expression $k'(1 + p)$ for the price of first order capital would follow, and for that of the commodity the expression $k'(1 + p)^2$. The k' labour is saved while the labour $m' + m$ that enters equally into the value of the commodity and represents *unpaid surplus labour* is subtracted from the capitalistic accounting of the costs.

One can proceed in this fashion as far as one desires. For example, introduce the concept of a third order capital and let $k' = k'' + m''$, and so $w = k'' + m'' + m' + m$: thus we obtain an even more precise expression of the price $k''(1 + p)^3$, and so on. Likewise p , the average rate of profit, will be given from series of ever more precise numerical values.

$$m/k' \quad m'/k'' \quad m''/k'''$$

The law of capitalist economy consists, therefore, in the saving of only the labour embodied in n th order capital (for a sufficiently large n). Let us take two commodities, of value w and W , equivalent on the capitalist market, having i.e. an identical price. The following equivalences will then hold good, with ever increasing exactness:

$$\begin{aligned} w &= W \\ k &= K \\ k' &= K' \\ k'' &= K'' \end{aligned}$$

Thus it comes about that the n th order capital of two equivalent commodities has an identical value. I shall also show that these capitals $k^{(n)}$ and $K^{(n)}$ are *absolutely identical*; and that therefore capitalist economy consists in the saving of capital of a well determined type, which we shall call *original*. Let K be an original type and M the surplus labour lent during its use; it will thus come about, first, that the surplus product obtainable by means of capital K is once again an original type and second, that the general rate of profit is given absolutely exactly by the formula $p = M/K$. Only after giving definitive proof of these results shall we be on solid ground to offer a critique of capitalism and to make comparison between its principle of saving and a “human” economy aimed at saving the entire amount of labour expended”.

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