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Part III

Joint Production, Intertemporal Preferences and Long-Period Equilibrium. A Comment on Bidard

Bertram Schefold

1. PERMANENT AND TRANSIENT STATES

Bidard's stimulating paper raises a number of important questions on two levels: 1. the scope and extent of Sraffa's critique of neoclassical theory, 2. the further development of classical theory. It had always been clear that research at both these levels would influence each other because of the common concern with long period equilibrium. Intertemporal models seemed to represent a different 'Walrasian' world — neither truly short-term nor long-term in the Marshallian sense. Bidard now declares that he belongs to the crowd of those who proclaim that Sraffa's critique does not apply to the intertemporal models, and that "classical dynamics" should be used to overcome the "steady state" character of classical analysis.

However, a new perspective has been added recently because of the turnpike results for intertemporal equilibria models of the Arrow-Debreu type. To the best students of neoclassical theory it had always been clear that intertemporal models must, if in a constellation there are underlying forces permitting a long period equilibrium, somehow tend to or oscillate around it — or else it ought to be possible to identify the causes why the long-period equilibrium is not reached.

Thus, consider Samuelson's Reply (Samuelson, 1990b) to my Comment on his contribution in "Essays on Piero Sraffa" (the Sraffa-Conference volume). I had said about Sraffa's critique of Hayek: "The rate of profit is in general not uniform... in intertemporal general equilibrium models, but, to the extent that they are not due to permanent scarcities, the inequalities of different own rates of interst of different commodities... tend to disappear as a result of a special form of a competitive process, and a unique uniform rate of profit emerges as the time horizon is shifted towards infinity" (Schefold 1990c, 302). And: "The long-run equilibrium... is not only a special case of an intertemporal equilibrium that happens to have endowments in 'correct' proportions, but also the centre of gravitation of an intertemporal equilibrium, if that is formulated as one with a distant time horizon..., in the special case of permanent market clearing, full employment and perfect foresight during the process". (Schefold 1990c, 304). Samuelson replied: "Schefold's first comment deals at some length with the undisputed difference between *steady state* regimes and transient regimes when relative prices foreseeably are altered by evolving scarcities of produced-goods endowments (possibly on a rendezvous course with an asymptotic steady state)". And he went on to say that his critiques "were directed to Schefold's present case of *permanent* rather than transient scarcities" (Samuelson 1990b, 324).

Samuelson, therefore, agrees that the real issue concerns the differences between the different properties which classical and neoclassical economists ascribe to the long-period equilibria themselves, while there are other differences concerning the mechanisms of approaching such equilibria by means of the gravitation processes or in intertemporal models. Samuelson's position thus indicates that a criticism of "an older version" of neoclassical capital theory does not have "only an archeological value", as Bidard writes in section 3 of his paper. On the contrary, the question still is how the long period position is to be explained.

I first want to discuss some aspects of the "permanent" state, in particular joint production, before I turn to the question of whether the Arrow-Debreu formalization is really immune against the capital critique.

2. DEMAND AND DISTRIBUTION

The representation of the relationship between demand and distribution in classical theory has been the subject of some controversy. In a model with self-employed direct labour alone, it would be simple to choose the methods of production to satisfy the given demand for commodities: one would select the most productive method of production in each sector (assuming single production) by examining the productivity of labour in each sector separately. Then, the levels of activity culd be determined in order to satisfy demand. With the labour vector and activity levels so determined, the level of employment would be fixed; the problem of distribution would not arise.

If alternative methods of production in basic single product systems with homogeneous labour and capitalistic production are given, it is necessary to determine first distribution in order to choose the methods of production which minimize costs and, at the same time, maximize extra profits in transitions from sub-optimal techniques to optimal ones. Given these methods of production, activity levels could be determined to produce a basket of goods in final demand of any composition (assuming constant returns). A change of demand would not require a change of technique, but it would influence employment.

With joint production, technical choice depends on the composition of demand, not only on distribution. The neoclassical solution is to attempt the simultaneous determination of commodity prices and factor prices, activity levels and methods of production under the condition of full employment. This implies that the sequential character of decision taking in economics is not reproduced. In classical theory, there is — corresponding, it is claimed, to the sequence of primary causal influences — one chain of reasoning which leads from effective demand to employment, another which leads from distribution to the choice of technique and prices. The arguments in both chains of reasoning are usually presented at different levels of abstraction. For instance, monetary arguments play an important role in the theory of employment and possibly also in the theory of distribution. But, once distribution is given, monetary factors usually need not be discussed when we address the theory of relative prices. It is clear that it would be desirable to maintain such methodological separations as long as possible. Even neoclassical general equilibrium theorists, fully aware of the fact that everything depends on everything else, usually exclude money when they discuss general equilibrium theory or simplify general equilibrium, before they introduce money, or imperfect competition, or international trade. Their difficulty is that money affects employment so that the key initial assumption — the full employment of factors — eventually has to be withdrawn.

Prior to Sraffa, it had been thought that the classical method would entirely fail in the presence of joint production, because it seemed utterly impossible to assign costs of production (including profit) to the several outputs of one multiple product industry without having recourse to demand — and "demand" came to be identified with the neoclassical formalization of demand.

As is well known, Sraffa provided arguments why a joint production system would be square, *i.e.* why it would contain as many processes with positive activity levels as commodities with positive prices, given a vector of "requirements for use". This implied a non-neoclassical, formally primitive representation of demand in the shape of a given vector of commodities to be produced. "Too many" processes would imply an overdetermination of prices such as occurs in a situation in which the cost-minimizing technique has not yet been found and competing processes run side by side — the solution then is, if prices are uniform, that the rate of profit cannot be uniform; differentials in rates of profit should lead to the finding of the optimum technique. "Too few" methods would in general not allow production of the required output in the appropriate proportions (returns do not have to be strictly constant for this argument to hold). But if the system *is* square, prices are determined and quantities can be varied by small amounts as is the case for constant returns single product systems in the large.

To the best of my knowledge, nobody has ever claimed that this argument for "square systems" can be maintained for all formalizations of demand.

I, for one, have already in my article on "Sraffa and Applied Economics" (Schefold 1985b) drawn attention to the fact that generic non-square solutions may emerge if demand is expressed in terms of neoclassical preferences. And indeed, if we answer the question asked by neoclassical general equilibrium theory, *i. e.* how demand and supply can be equilibrated if there are many factors of production and if individuals have different preferences, it is not at all obvious why square solutions should obtain. To see the problem in its full complexity, imagine a Sraffa-system with land of many different qualities so that different rents arise on different lands which are partly specialized in the production of different commodities. There are agricultural lands, vineyards, mines, hunting grounds etc. There is very little homogeneity among land-owners (this is where the third volume of Marx's 'Kapital' breakes off): wine-growers perhaps have demand functions different from those of mine-owners, and their demand pattern is again different from that of the owners of large agricultural estates on the one hand, owners of small-holdings on the other. Suppose that, nevertheless, we start from a situation in which the Sraffa-system pertaining to this technology is square, at a given rate of profit, and that activity levels are such that the demand arising at the corresponding prices and rent-incomes exactly corresponds to the demand emanating at those prices and incomes, given some kind of price-elastic demand functions. Now suppose a change in the rate of profit. At unchanged levels of demand, prices and rents would again be determined. The change of distribution might entail a change of methods of production but we could be confident again of obtaining a square system. Only, at the new prices and rents, demand would change. For the vector of demand so determined, a changed quantity system would obtain, which in turn implied a new set of prices and rents, and hence new demands. One could go on, iterating this process. Does it converge (Garegnani 1990, 130)? As a matter of fact, we are certain that it would not converge to a generically square solution if the demand functions are derived from neoclassical preferences. Moreover, the sequence of price-changes and changes of rents might affect preferences and the forces determining the rate of profit.

Before trying to improve on such an exercise of *ceteris-paribus*-economics, we should ask whether the neoclassical question is really well posed. Does it not imply that one is detracted from macroeconomic influences on demand, their effect on employment and growth, etc.? Without going deeper into the matter, let us here simply state that the classical approach is for better or for worse shaped to answer other questions. The attempt to insert the discussion of the theory of demand into the theory of growth and employment (Schefold 1990c) leads, I admit, to a very simple formal representation, as has been pointed out by Mainwaring (1990). But the alternative is to make assumptions about a stable functional dependence of demand on prices and incomes so that the interaction with the theory of distribution and employment must lead to special models in view of the great variety of constellations which are possible in different states of accumulation. However primitive it may seem, these are important advantages to the assumption of a given demand vector if it is used wisely in a dynamic context.

When Sraffa referred to requirements for use, he probably meant gross outputs so that the question does not arise of whether the system is stationary, declining, growing, in a steady state or without balanced proportions. In attempting to formalize Sraffa, people have perhaps — and I should primarily blame myself — too quickly availed themselves of the existing tools of mathematical economics. This is why net outputs, not gross outputs, usually have been treated as given, why a uniform rate of growth has been used, why strictly constant returns were assumed and why investment demand was reduced to the banality of taking the rate of growth as given. These simplifications have made it possible to arrive at rigorous proofs but the assumptions should not direct future research nor are they indicative of inherent limitations of the classical approach. Without having the manuscript of Bidard's new book which he announces in his paper, I am at some disadvantage in addressing his arguments. However, I am certainly not in agreement with him when he says: "In general joint production, Sraffa's squareness axiom is hopeless". such a statement can only be based on assumptions about demand which differ from Sraffa's. If requirements for use are given, for instance as gross outputs, the system will generically be square. Bidard may choose different assumptions about demand and about the existence of a relationship which would be stable in the long-run and regulate the reaction of consumers and investors to changes in prices and incomes. If, on his assumptions, Sraffa systems are not generically square, the relative merits of the assumptions should be discussed; logically, the formal argument should be uncontroversial.

3. THE FREE-GOODS RULE

We shall return to the question of constant returns later. Before we discuss special cases of joint production systems, it may be useful to clear up some misunderstandings which have arisen regarding the assumption of free disposal. New theoretical ideas and empirical material on this point has been assembled in a *PhD*-thesis (Brägelmann 1991).

a) Countless substances are freely disposed of like smoke in industrial and agricultural processes.

b) For a limited number of substances, emission limits are fixed or emission certificates issued; disposal is then made costly and its cost is part of the cost of production of the useful commodities. One method for including such processes is discussed in Schefold (1983).

c) An apparent difficulty arises when part of the joint output of a process is disposed of (with or without cost) while another part is sold as a commodity. This seems paradoxical, but it is a frequent occurrence: that part of a product which is turned into a commodity is sold at a price which reflects the expenses for collection, transportation and distribution and, to some extent, if disposal is not free, of the destruction of that part of the product which cannot be used as a commodity. Straw, for instance, is ploughed under in some regions, at negligible costs of disposal since ploughing takes place anyway, whereas it is sold elsewhere at a cost-price reflecting the effort of collection and transportation. When traffic in cities was still based on horses, straw and not wheat was occasionally the main product of agriculture in the vicinity of towns. The disposal of joint products at a cost of collection and transportation, reflecting also disposal costs of residues, is very frequent in the chemical industry. There are then two tendencies: if the dumping of the product as a waste is forbidden, such disposal by giving away at transportation costs tends to get subsidised by the producer who wants to avoid the cost of abatement or removal; in some cases, the producer pays, not the buyer, in order to get rid of a waste. Accordingly, if the product can find new applications, the producers have an interest in fostering the introduction of the corresponding new processes.

d) Some products have been wastes at one time and commodities at another. At one time, people had to pay for being allowed to collect fuel wood in certain forests. Nowadays, after the introduction of cheap fossil fuels, such collection is free and, as long as no logging of timber takes place, actually encouraged, since the removal of dead wood may be ecologically desirable.

It is easy to see that all these possibilities can be accommodated in linear models, including those of the Sraffa-type. And it is obvious that Sraffa did consider the elimination of overproduced commodities: what else is the elimination of an old machine or, extending the concept of commodity, the designation of land which is not fully cultivated as "no-rent land"?

4. SQUARE SYSTEMS

a) The main case to be considered then is the following: g is the rate of growth, r is the rate of profit, $\mathbf{A}$ and $\mathbf{B}$ are the technology matrices from which the methods of production in use will be chosen; $\mathbf{l}$ is the corresponding labour vector and $\mathbf{q}$, $\mathbf{p}$ are the vectors of activity levels and prices, respectively. $\mathbf{d}$ is a vector of demand. Now consider the equations

$$\mathbf{q}(\mathbf{B} - (1+g)\mathbf{A}) \geqq \mathbf{d} \quad (1)$$

$$(\mathbf{B} - (1+r)\mathbf{A})\mathbf{p} \leqq \mathbf{l} \quad (2)$$

$$(r-g)\mathbf{q}\mathbf{A}\mathbf{p} + \mathbf{q}\mathbf{l} = \mathbf{d}\mathbf{p} \quad (3)$$

Lippi and others have shown that, for r and g in a certain range between 0 and a maximum R (it is useful to extend the range sometimes from -1 to R), these equations have solutions in **q** and **p** such that unprofitable activities (2) are not used and overproduced goods receive zero prices (1). The essential condition is that there exists, for some positive rate of growth, some vector **q** which makes it possible to solve equation (1). For this it is a sufficient condition (if all goods are produced and all goods used in some process) that the system generates a surplus of all goods in the stationary state. Further, it can be shown that such a system generates a square solution, with the number of commodities (goods with positive prices) being equal to the number of processes used (methods activated at positive levels). The solution which thus emerges is clearly a system of the Sraffa-type. If g is set equal to -1, we formally obtain Sraffa's case where **d** are requirements for use including investment goods, or gross outputs. From a *formal* point of view, $g = -1$ is special case. *Conceptually*, this is the more general case since it may be discussed without reference to steady growth.

There are two different interpretations of **d**. The easy view is that there is steady growth so that **d** contains only consumption goods, with consumers of both classes demanding — at least on average — consumption goods in the same proportions.

The more difficult case is that where **d** means gross outputs according to the principle of effective demand. Equation (3) then shows that the value of gross output is equal to gross income in this economy; the level of employment is given by **ql** which is also the labour commanded of the real wage. But what about the division of gross output among wage earners and capitalists? Three equations are missing which would show the equalities between the value of investment goods and the saving of capitalists (if workers only consume), between the value of capitalists' consumption goods ("luxuries") and profits for consumptior, and between workers' consumption goods and the wage goods ("necessaries"). If there is a mismatch (for instance because capitalists had arranged for the production of luxuries in an amount corresponding to their own purchasing power and of necessaries in an amount corresponding to the purchasing power of the workers, and if workers now start to demand luxuries while some "necessaries" cannot be sold anymore) the long-period position can possibly not be sustained because the methods of production have to be adapted to the changed proportions of final demand.

Such a possibility does not render the equations (1-3) irrelevant; Sraffa's "self-replacing" state is not necessarily a "self-reproducing" state. I do not think that the requirements for use are, as Bidard writes, "forgotten" by Sraffa, immediately after having been introduced. Rather, one should face the fact that he abstracts from an economic problem which is — like many others — interesting: Whether there is an equilibrium of a certain type (square system, uniform rate of profit etc.) characterized by the equality between income and production not only overall (3) but also at a more disaggregated level. To require it for each individual would mean to introduce individual budget constraints as in neoclassical theory and to ignore the adjustment processes which result if the production and the incomes generated in a self-replacing state do not correspond to the demand arising out of those incomes. If one wants (1-3) to describe a sustainable long-period position and if one does not want to make the simplifying hypothesis that the demand vectors of workers and capitalists are proportional to each other as according to the first interpretation, one simply has to *assume* that incomes and class-specific demands match — an assumption which is not necessary in the case of single product industries with constant returns. Such an assumption is as legitimate as many others which are often only implicit. While it is natural to exclude war or climate change in the discussion of long period positions, it is not so clear why the influence of money holdings on demand are not considered. Is Bidard's emphasis on the inclusion of the nexus between demand and income and his neglect of monetary influences motivated by his economic judgement or by neoclassical tradition?

b) But I am prepared to play according to his rules, if he allows me to treat distribution in a classical manner. Do we get square equilibria if demand and incomes are supposed to correspond? This is again the question of existence.

Garegnani (1990, 137) postulates a "coherence" of the data of a long period position, assuming that they can be arranged in the square form described by Sraffa. Fortunately, there is a, so to speak, "arch-classical" model which precisely meets Bidard's objection and wich justifies belief in "coherence". If we mention workers' consumption, assuming that the real wage consists of "necessaries", these will determine distribution. And if we introduce capitalists' consumption, we expect it to influence effective demand and to represent a primary influence on employment, together with investment. We therefore write:

$$\mathbf{q}(\mathbf{B} - \mathbf{A} - \mathbf{lc}) \geqq \mathbf{s} \tag{4}$$

$$(\mathbf{B} - (1 + r)\mathbf{A})\mathbf{p} \leqq w\mathbf{l} \tag{5}$$

$$r\mathbf{q}(\mathbf{A} + \mathbf{lc})\mathbf{p} = \mathbf{sp} \tag{6}$$

$$\mathbf{cp} = w/(1 + r). \tag{7}$$

Equations (4) mean that activity levels must be such that the goods demanded for net investment and consumption by capitalists are produced or overproduced. The matrix **lc** is the matrix of workers' consumption, with **c** being the row vector of the basket of wage goods demanded per unit of labour. (5) are the usual price equations. Equation (6) shows that the profit out of total capital advanced is sufficient to buy investment goods and consumption goods demanded by capitalists. Equation (7) shows the equality between the discounted ex-post wage rate and the wage bundle consumed per unit of labour. Hence, here we have both a classical or Keynesian determination of distribution and employment, as well as the match between incomes and expenditures of each class. It is easy to see that equations (4-7) have, in general, square solutions.[1]

c) Larger deviations from Sraffa's assumptions about demand do lead to the possibility of non-square solutions. Samuelson (1990a, 279, Note 2) has suggested the use of Engel curves in such a way that the demand equations make the price system determined even if only one method of production, producing all commodities of the economy jointly in one process, is used. I was able to reply that those same Engel curves could lead to square Sraffa systems, with the demand conditions essentially determining activity levels, while prices would be determined as usual. The question of optimality had not been addressed (Schefold 1990c, 316).

Here I have to grant a point to Bidard. He seems to like the assumption of a linear demand function

$$\mathbf{d} = t_w \mathbf{c}_w + t_p \mathbf{c}_p \tag{8}$$

In (8), $\mathbf{c}_w$ is the consumption basket per worker employed and $\mathbf{c}_p$ is the consumption basket per unit of expenditure out of capitalists' profit. The latter is an awkward concept. While it is traditional to regard workers' consumption as proportional (t_w) to employment — this is reflected in equation (6), (7) and (4) —, it is not so clear why capitalists' consumption should be exactly proportional (t_p) to profits (equation 9), with linear Engel curves. Salvadori (1990, 214) had shown that with a demand curve of this type one could obtain a system with one activity less than there are commodities. Against this, I had arqued in my Reply to Salvadori's comment that square solutions could also be constructed (Schefold 1990b, 224). Recently, Bidard has communicated a graphic example to me, to be published in his book, in which he shows that there are constellations such that, among square and non-square solutions of the quantity system, only a non-square solution is cost-minimizing if the demand vector in equation (1) is replaced by the demand vector of equation (8) and if incomes (**ql** is

[1] Equations (4-7) can be reduced to three equations (4), (6), and

$$(\mathbf{B} - (1 + r)(\mathbf{A} + \mathbf{lc}))\mathbf{p} \leqq 0; \qquad (5');$$

conversely, the system (4, 5', 6) is transformed into (4-7) by *defining* w through equation (7) and inserting into (5') to obtain (5). Now it is clear that equations (4, 5', 6) are a special case of equations (1-3), with $\mathbf{A} + \mathbf{lc}$ being the input matrix. The square solution is chosen for a stationary state with the rate of profit at its maximum because labour has no part of the surplus in equations (4, 5', 6). It should be noted that this formal solution of equations (1-3) presupposes that we do not have to deal with those cases where a wage curve of a quadratic Sraffa system breaks off with a finite wage at the maximum rate of profit. I have argued in SCHEFOLD (1978) und SCHEFOLD (1988a), as well as in SCHEFOLD (1988b), that such exceptions may be ruled out, using reasonable assumptions.

the wage in terms of labour commanded while $(r-g)\mathbf{qAp}$ are profits consumed) match expenditures so that the additional conditions are imposed

$$t_w\mathbf{c}_w\mathbf{p} = \mathbf{ql},\ \mathbf{t}_p\mathbf{c}_p\mathbf{p} = (r-g)\mathbf{qAp} \tag{9}$$

d) Using budget constraints, neoclassical models emphasize the interdependence between the level of incomes and the demand for commodities, with the result that the rates of substitution of consumers help to explain relative prices. In the previous "intermediate" model, with equations (1 – 3, 8, 9,), consumption depends on the level of incomes, but the rate of profit remains exogenous. It turns out that we may have solutions where one process less is activited than there are commodities with positive prices. But while such solutions are unsatisfactory because small short-term fluctuations of demand cannot be accommodated without changes of methods of production or of distribution, one may ask in what context a formalization of demand such as that expressed in equation (8) might find a legitimate application in classical analysis. It means turning the classical and Keynesian view on its head to make the level of expenditure out of profits depend primarily on the level of profits themselves. This will be the case if, as in neoclassical uses of linear models, a full-employment constraint is imposed (Samuelson 1990b, 326), determining the level of profits, which then in turn limit capitalists' expenditure. An assumption of this kind is needed to close the system of equations (1-3, 8, 9) with r given. I therefore propose to drop assumption (8). To the extent that there are reserves of labour and other resources, the level of effective demand on the part of entrepreneurs may be constrained only by confidence and by the availability of finance which in turn depends primarily on expected, not on realized profits. In this, what is true for the short run must also hold for long period positions. We assume, on the other hand, that the real wage is given; for simplicity, real wage goods may be assumed to be represented as part of the input matrix.

In order to extend the classical model, we now introduce 'scarce resources' in the form of land. There is less difference between the classical and the neoclassical approach here. It is consistent with the classical tradition to assume that the expenditure of landlords depends on their income. Landlords thus consume a multiple, t, of a fixed basket of commodities $\mathbf{c}$. This leads to the following equations:

$$\mathbf{q}(\mathbf{B}-\mathbf{A}, -\mathbf{Z}) \geqq (\mathbf{s}+t\mathbf{c}, -\mathbf{v}) \tag{10}$$

$$(\mathbf{B}-(1+r)\mathbf{A})\mathbf{p} - \mathbf{Zu} \leqq \mathbf{o} \tag{11}$$

$$r\mathbf{qAp} + \mathbf{vu} = (\mathbf{s}+t\mathbf{c})\mathbf{p} \tag{12}$$

Here we start from a state of self-replacement where $\mathbf{s}$ is the given vector of investment and consumption goods demanded by capitalists.

$\mathbf{Z}$ is a matrix indicating the amount of land of various types in each process. In most cases, one will expect that only one type of land is required to produce a commodity in any given process as in agriculture. *E.g.* wheat is grown on a certain agricultural land and transformed into flour in a mill on some industrial land, while straw is used in yet another location. But processes using several lands simultaneously are formally not ruled out. The total amount of each land available is given by vector **v** which may be assumed to be strictly positive since it does not make sense to list lands which do not exist. **u** is the vector of rents. We now impose the additional conditions:

$$r\mathbf{qAp} = \mathbf{sp} \tag{13}$$

$$\mathbf{vu} = t\mathbf{cp} \tag{14}$$

The intuitive idea is that for any level of demand (with t arbitrarily given) there must be a solution to equations (10-12), where the rate of profit r is determined by the real wage implicit in input matrix **A**. There is then the overall balance between incomes and production (12). If t can be found such that landlords spend what they get (14), it follows from (12) that total profits earned by capitalists are equal to their expenditure (13). This determines **q** and hence employment. There is a labour reserve which can be mobilized e.g. by changing the participation rate, allowing for migration or a subsistence sector.[2]

The proof that the system in general yields square solutions is quite involved; here I only provide a sketch, making reference to Schefold (1988a). I assume that the columns of **Z** do not vanish and that the wage curves pertaining to different q-feasible and/or p-feasible square truncations of the system do not coincide so that they cross only in a finite number of points.

We shall need three further assumptions:

(i) We assume that there are feasible solutions **q** to the inequalities (10) for all t. This means that there is, so to speak, a possibly not very productive but infinitely large plain on which production can be expanded indefinitely after better lands have already been brought under cultivation.[3]

[2] One may assume that the rate of profit is maximized if there are several solutions to (10-12) in order to isolate the dominating technique (Schefold 1988a). In what follows, we shall only consider square solutions, thus excluding flukes of the following kind (which are also ruled out by the linear programme (10′, 11′)):

$$(1 + r)\mathbf{Ap} + \mathbf{z}u = \mathbf{p},\ \mathbf{q}(\mathbf{I} - \mathbf{A}) = \mathbf{s},\ \mathbf{qz} = v,$$

where we have a single product system producing **s**, but by means of a single type of land **z** (with rent u), available to each process, like labour in 'ordinary' single product Sraffa systems. Any of the assumptions mentioned leads to $u = 0$ and thus to a determination of r which is analogous to that of the maximum rate of profit. Otherwise, r can arbitrarily be given to determine prices in terms of the rent-rate $\mathbf{p}/u$. For the conceptual discussion of such a 'monopolistic' or 'absolute' form of rent and its comparison with Sraffa's form, see SCHEFOLD (1989, 242). Mathematically, we have a fluke because the quantity system has $n + 1$ equations to determine $q_1,\dots, q_n$. A variation of **s** would thus lead to an underutilisation of land ($u = o$) or to the introduction of differential rent and $n + 1$ equations.

[3] Strictly speaking, we must have some **q** with $\mathbf{q}(\mathbf{B} - \mathbf{A})\mathbf{s} > t\mathbf{c}$, $\mathbf{qZ} \leqq \mathbf{v}$, in order to ensure the positive rate of profit in the proof below.

(ii) On the other hand, we assume that some rents are positive even if $t = 0$ so that $\mathbf{vu} > 0$. This means that the given demand of capitalists $\mathbf{s}$ cannot be satisfied without using at least one of the better lands fully.

(iii) If t rises, the solutions — if they exist — are assumed to be such that $\mathbf{vu}/\mathbf{cp}$ does not fall. The economy is thus assumed to be "Ricardian": If landlords expand their demand, total rents will increase — whenever they change — faster than the prices of their consumption goods. This seems plausible since an increase of demand will generally tend to raise rents and, in consequence, while the rate of profit falls, prices. But rents may be expected to rise faster than prices because the latter involve also the prices of industrial goods where little land is used.[4]

Proposition I:

If assumption (i) is fulfilled, there are solutions to equations (10-12) which are square, for any given t.

To see this, consider the linear programme

$$\text{Min } \mathbf{xl} \text{ S.T. } \mathbf{x} \geqq 0,\ \mathbf{x}[(\mathbf{B} - (1+r)\mathbf{A}, -\mathbf{Z}) \geqq (\mathbf{s} + t\mathbf{c} - r\mathbf{q}^0\mathbf{A}, -\mathbf{v}) \quad (10')$$

$$\text{Max } [(\mathbf{s} + t\mathbf{c} - r\mathbf{q}^0\mathbf{A})\mathbf{p} - \mathbf{vu}]$$
$$\text{S.T. } \mathbf{p} \geqq \mathbf{o},\ \mathbf{u} \geqq \mathbf{o},\ (\mathbf{B} - (1+r)\mathbf{A})\mathbf{p} - \mathbf{Zu} \leqq \mathbf{l} \quad (11')$$

where $\mathbf{l} > \mathbf{o}$ is some labour vector and where $r > 0$ is chosen so as to fulfill (i). Vector $\mathbf{q}^0$ is in $Q = \{\mathbf{q} \geqq 0,\ \mathbf{ql} \leqq \mathbf{q}^+\mathbf{l}\}\mathbf{q}^+$ solves (10′) for $\mathbf{q}^0 = \mathbf{o}$. There are feasible solutions ($\mathbf{q}$ according to (i) and $\mathbf{p} = \mathbf{o}$, $\mathbf{u} = \mathbf{o}$ respectively). Hence, following Lippi's idea in Schefold (1988a, 121), there is a fixed point $\mathbf{q}^*$ to the mapping $\mathbf{q}^0$ into $X(q^0)$, where $X(\mathbf{q}^0)_c\ Q$ is the set of optimal solutions to (10′), given $\mathbf{q}^0$. The optimal solutions $\mathbf{q}^*$, $\mathbf{p}^*$, $\mathbf{u}^*$ corresponding to $\mathbf{q}^0 = \mathbf{q}^*$ fulfill (10) and (11). If such a solution exists for $r > 0$, it also exists for all r^0, $0 \leqq r^0 \leqq$ r.

In order to prove that the solutions are generically square, one can either introduce a fictitious rate of growth in (10′) and replace $\mathbf{u}$ by $\mathbf{u}/(1+r)$ in (11′) in order to have a formal analogy between the system considered here and that in Schefold (1988a); the "wage curve" then is defined in terms of $\mathbf{d} = (\mathbf{s} + t\mathbf{c}, -\mathbf{v})$ in order to use duality at $r = g$. Or one argues more directly: If there is a p-feasible truncation, consisting, without loss of generality, of the first n processes, with the first n prices being positive, and if the corresponding augmented price vector yelds no loss in process $n+1$, it is clear that only n equations can continue to hold with a small variation of r if the truncation is regular. This argument cannot be applied on the quantity side if the rate of growth cannot be varied so that it becomes

[4] There is a (somewhat contrived) counterexample with $r > g$, fulfilling (ii) but not (iii).

necessary to consider small imaginary generic variations of the positive components of **s** and **v** in order to recognize fluke cases. This presents no problem for **v** since $\mathbf{v} > 0$. Exceptions are possible for means of production as vanishing components of **s** where the corresponding colums of a truncation of $\mathbf{B} - \mathbf{A}$ may also vanish, but this difficulty can be dealt with as in my earlier publications.

We then may raise r in order to find the maximum rate of profit at which the 'wage curve' — here a misnomer because it is $1/((\mathbf{s} + t\mathbf{c})\mathbf{p} - \mathbf{vu})$ — vanishes. I assume here that we may rule out those irregular systems which yield a positive real wage at the maximum rate of profit, as discussed in Schefold (1988a and 1988b). The fundamental theorem of linear programming yelds (12). On account of the 'local non-substitution theorem', the positive components of final demand can be varied in (10) by small amounts without affecting prices or rents except at points where an activity level falls to zero and a method of production has to be changed (affecting prices, rents and — by contrast with points of truncation — in general also the rate of profit). Further, there are critical points where irregularities arise because a truncation of $(\mathbf{s}, -\mathbf{v})$ happens to be an eigen-vector of the corresponding **p**-feasible or **q**-feasible truncation of $(\mathbf{B} - \mathbf{A}, -\mathbf{Z})$.

Proposition II:

If assumptions (i), (ii), (iii) hold, there is $t > 0$ such that a solution to (10-14) exists.

To see this, consider diagram 1:

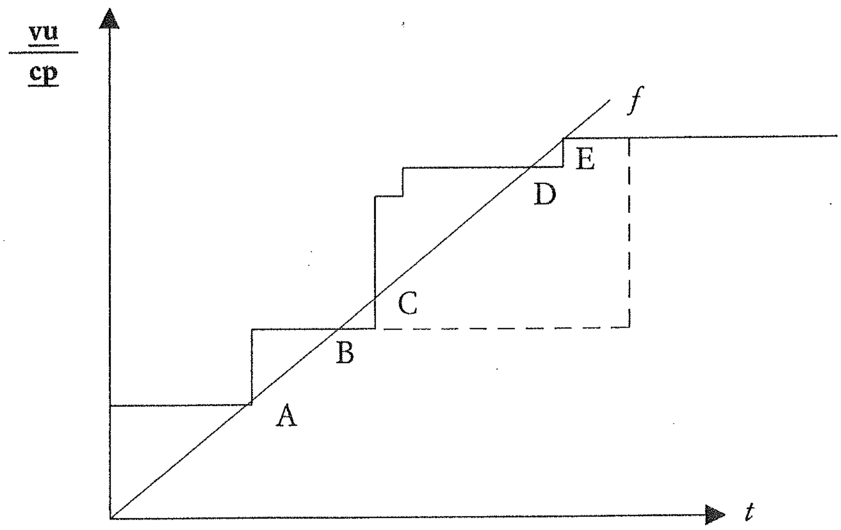

Diagram 1: $\underline{\mathbf{vu}} / \underline{\mathbf{cp}}$ as a function of t (equations 10-12).
f is the 45° – half line.

For each t, there are one or several solutions to equations (10-12). The ratio of total rents to the price of a unit consumption bundle of landlords $\mathbf{vu}/\mathbf{cp}$ rises as a step function (the dotted line in diagram 1 indicates the possibility of multiple solutions), the economy being Ricardian. Because of (ii), the step function is greater than 0 at $t = 0$, and it is bounded because the system of equations admits only a finite number of square truncations. Because of (i), the step function is also defined for large $t > 0$, and is, ultimately, horizontal. Connecting the steps by vertical intervals and choosing one branch (where that is necessary because of multiplicities), we obtain a continuous step mapping which must intersect the 45°-half line. Now there must exist intersections of the type B or D where a horizontal segment of the step mapping intersects the half line. For if the step mapping only touches the half line from above, as in A, it will yet have to cross the half line because of assumption (iii), and similarly, an intersection to the left must have taken place if it touches from below as in E. Finally, an intersection of a vertical segment as in C implies further intersections. Therefore we must have an intersection of the type B or D. At any such point we have $\mathbf{vu}/\mathbf{cp} = t$ and equation (14) is fulfilled. This implies (13) because of (12).

It appears that multiple solutions are here possible such that (14) holds for each t independently of whether the solutions are unique for (10-12) and given t. Generalizations are possible by subdividing the landlords into different classes with different consumption patterns according to the lands they own.

For instance, if there are two classes of landlords, possessing lands $\mathbf{v}^{(1)}$, $\mathbf{v}^{(2)}$; $\mathbf{v}^{(1)} + \mathbf{v}^{(2)} = \mathbf{v}$, and wishing to buy multiples $t^{(1)}$ and $t^{(2)}$ of consumption baskets $\mathbf{c}^{(1)}$ and $\mathbf{c}^{(2)}$, equations (10-13) and

$$\mathbf{v}^{(j)}\mathbf{u}^{(j)} = t^{(j)}\mathbf{c}^{(j)}\mathbf{p};\ j = 1,2; \qquad (14')$$

can be obtained with square solutions. There are solutions to (10-12), with demand in (10) now being given by $\mathbf{s} + t^{(1)}\mathbf{c}^{(1)} + t^{(2)}\mathbf{c}^{(2)}$ which implies a mapping from $(t^{(1)}, t^{(2)})$, into $(y^{(1)}, y^{(2)}) = (\mathbf{v}^{(1)}\mathbf{u}/\mathbf{c}^{(1)}\mathbf{p}, \mathbf{v}^{(2)}\mathbf{u}/\mathbf{c}^{(2)}\mathbf{p})$, the graph of which is a two-dimensional manifold in four-dimensional space (x^1, x^2, y^1, y^2). After connecting the steps, the intersection of this graph with the plane $(x^1 = y^1,\ x^2 = y^2)$ must now yield one or several points of intersection. Critical points are clearly isolated if $\mathbf{c}^{(1)}$ and $\mathbf{c}^{(2)}$ are linearly independent (if not, we are back to Prop. I). The step mapping of diagram 1 can be drawn in function of $t^{(2)}$, given any $t^{(1)}$. The highest point of type 0 which must be obtained can thus be represented as $t^{(2)} = g^{(2)}(t^{(1)})$. This is obviously a positive and bounded step function in the $(t^{(1)}, t^{(2)})$ — plane and it is, we plausibly assume, monotonically rising. Similarly, with the roles of t and t reversed, we obtain $g^{(2)}(t^{(1)})$. Hence, there is generically at least one point of intersection of *steps* of $g^{(1)}$ and $g^{(2)}$ where (14′) holds and the system is locally invariant to small changes of $t^{(1)}$ and $t^{(2)}$.

But another extension, the assumption of price-elastic demand functions, would destroy the squareness-result, as is, perhaps, sufficiently indicated by the following diagram 2:

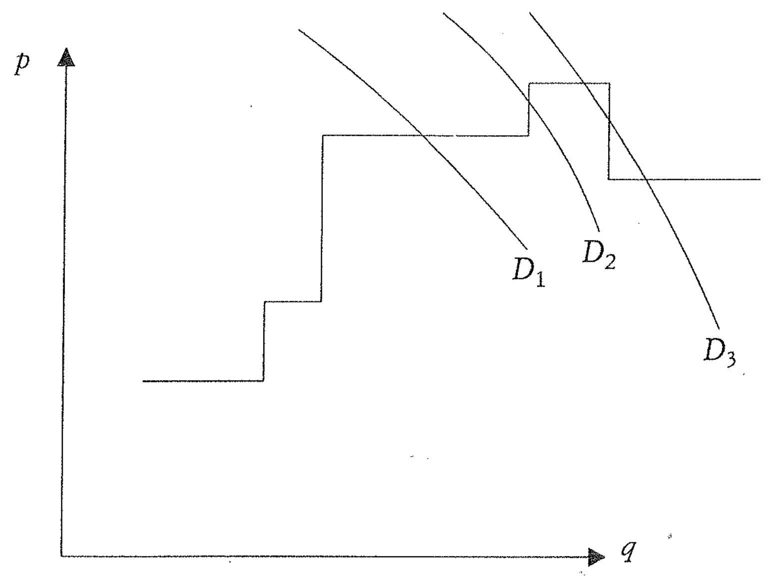

Diagram 2: Price of any commodity in function of the level of output. D_1, D_2, D_3 *demand curves.*

Here, the price has been drawn as a step function of the level of output of a commodity, the production of which is being varied, with the real wage given, as in equations (10-12). The demand functions of landlords necessarily also "jump" whenever their rents change but this effect has here been disregarded. As is shown in Schefold (1989, 244), a square equilibrium would obtain for D_1 but not for D_2. D_3 is indicative of difficulties which might arise because of the possibility of falling sections[5] of the "supply curve".

[5] As already stated, some rents do not necessarily rise monotonically with the rate of profits. Samuelson (1990b, 330) affirms that supply curves must rise. Is this true for our normal cost curves? According to the argument he cites (Samuelson 1946), we have, with $\mathbf{p}^{\mathrm{I}}$ and $\mathbf{p}^{\mathrm{II}}$ being optimal vectors in (11′), with $\mathbf{q}^{\mathrm{I}}$, $\mathbf{q}^{\mathrm{II}}$ optimal fixed point vectors in (10′), corresponding to states of demand t^{I} and t^{II} in (10′), and with $\mathbf{d}^i = (\mathbf{s} + t^i\mathbf{c} - r\mathbf{q}^i\mathbf{A}, -\mathbf{v})$; $i = \mathrm{I}, \mathrm{II}$;

$$\mathbf{d}^{\mathrm{I}}\mathbf{p}^{\mathrm{I}} \geqq \mathbf{d}^{\mathrm{I}}\mathbf{p}^{\mathrm{II}}$$
$$\mathbf{d}^{\mathrm{II}}\mathbf{p}^{\mathrm{II}} \geqq \mathbf{d}^{\mathrm{II}}\mathbf{p}^{\mathrm{I}}.$$

Adding these inequalities yields $(\mathbf{d}^{\mathrm{I}} - \mathbf{d}^{\mathrm{II}})(\mathbf{p}^{\mathrm{I}} - \mathbf{p}^{\mathrm{II}}) \geqq 0$ so that an increase of only one component of $\mathbf{d}$ between states I and II implies in fact that the corresponding component of $\mathbf{p}$ must also rise. The situation which Samuelson has in mind clearly is that where, in terms of our model, $r = 0$ and $\mathbf{c}$ is a unit vector. But the conclusion is not obvious and the normal cost curve is not always monotonous for $r > 0$ as can be shown, extending the familiar diagrammatic technique, for a corn economy with intensive rent on one land and with three methods. In the example, one cost-minimizing system of order two exists for low levels of output, and two exist for higher ones. The transition to the dominating technique with the increase of output involves, paradoxically, a fall of price and rent in terms of labour commanded.

I prefer to call it "normal cost curve" in this context since the effects of changes of net or gross outputs on all other outputs and prices are simultaneously taken into account while distribution — real wage or rate of profit — is kept fixed. Partial equilibrium supply curves in neoclassical theory do not reflect this interdependence. The diagram illustrates that we should probably obtain equilibria under suitable conditions if we assumed elastic demand curves, but they would not necessarily be stable, nor square or unique. I discussed the possibility of an equilibrium of the type obtained with D_2 first in 1972[6] and I proposed to interpret the constellation as the determination of a market price. The advantage of the approach chosen here is that it avoids demand functions and leads to more definite results regarding the structure of production. Using the squareness of the system and hypotheses about the structure of land use, *e.g.* the assumption that each process uses at most one type of land (to which may be added the assumption of single production or of a grouping of joint products), one may derive the familiar types of rent (extensive, intensive), the patterns of specialization etc.

As an example, consider the type of rent which has been called external intensive rent: One land (*e.g.* a forest) is used fully to produce a good (fuel). Rent and price of the product can be determined although there is no other fuel-producing land as in extensive differential rent and although fuel is not produced by two methods as with intensive rent because fuel is used, not produced, in two processes. It is burned to produce heat, either rather inefficiently by means of a hearth or, more efficiently, by means of a stove, so that, with other input prices being given, we have three equations to determine rent, the price of fuel and the value of heating. This example is an instance of the determination of a price in a Sraffa system on the "demand side" by introducing domestic processes of production. The system of industrial processes here appears not to be square and to leave an indeterminacy regarding the rent of the forest and the price of wood which call for an additional determination on the demand side. One might think that neoclassical demand curves are needed, but a classical determination is possible through the addition of the two domestic processes. To put it another way: We are here, so to speak, on a vertical part of the normal cost curve shown in diagram 2 since the forest is used fully and output cannot be expanded. Yet we may increase the production of heat continuously, with rent and the price of fuel remaining constant, if the number of stoves is increased and the number of hearths reduced so that the amount of fuel burnt stays constant. This possibility of a variation of output within a certain range at constant prices and a fixed rate of profit is made possible by, and simultaneously implies, "squareness".

One may be tempted to regard the introduction of neoclassical demand

[6] C. C. von Weizsäckeer suggested the possibility.

functions as an additional feature of the model which would make it more general. And the next step would then be to attempt to replace the step functions of the normal cost curves by continuous, smooth and possibly monotonic supply curves. However, a change in the uniform rate of profit (the determination of which is a problem in neoclassical theory to which we shall return) may upset the ordering of the lands as to their "fertility", or the cost of production on them, so that the supply curves would exist only for a partial analysis and would be not suitable for the analysis of the interdependence of industries. Finally, if we abandon "squareness", even small changes in the composition of output necessarily affect distribution and normal prices so that it is difficult to see how small short term fluctuations of demand could be accomodated and how room for a Keynesian multiplier analysis could be preserved, with, for instance, an expansion of demand which would leave prices and distribution, within limits, essentially constant.

Nobody can be blamed for a belief that there is some persistence in consumer preferences — possibly even comparable to the persistence of technological conditions —, and that this should be represented in our models. As we have seen, the approach pursued here must to a large extent abstract from an ex-ante consideration of price reactions of consumers — although not completely, as I have attempted to show else-where. It recommends itself not only because of the various critiques of neoclassical theory, but also because there are important advantages deriving from the abstractions made in classical theory: it is rich in the consideration of heterogeneous influences on distribution, accumulation and effective demand which are so difficult to get into focus once it is regarded as axiomatic that theoretical analysis must start from a general equilibrium model.

Having thus presented some formal analyses, some intuition and some theoretical considerations in defence of the square joint production systems which Bidard attacks, I now turn to his perception of the relationship between classical and neoclassical theory.

6. CAPITAL THEORY, INTERTEMPORAL MODELS AND LONG-PERIOD EQUILIBRIUM

Bidard writes: "The neoclassical version of capital theory is no longer in the state it was in the twenties" — true, but intertemporal equilibrium started with Lindahl and Hayek, precisely in the twenties (Milgate 1982, 129-136), and I have argued that objections to the intertemporal theory are already contained in Sraffa's critique of the latter. At any rate, the difficulty of reconciling intertemporal theory and the notion of long-period equilibrium is evident in Burmeister's (1980) book where it is admitted that "regularity" (*i.e.* the absence of capital reversals and reswitching) is required to achieve convergence to long-period equilibrium. The contributions by Malinvaud, Arrow, Debreu were highly ingenious. Yet the critique of capital theory applies, in my view, in the following manner to "the modern construction".

The intertemporal equilibrium of the Debreu-type, with given endowments and a finite time horizon, allows two related interpretations: the equilibrium may express a coordination by agents of their future plans at discounted prices in one moment of time. Or we may imagine that these plans are actually carried out and that, as time progresses, endowments are gradually used and goods produced according to the demands foreseen at the beginning of the first period until stocks are depleted in the end. We adopt this latter interpretation. The arbitrary nature of the length of the time horizon at which the economic world thus comes to an end (unless — and this alternative is no conceptual improvement — arbitrary stocks are to be produced for the final period to allow future production beyond the time horizon) suggests that it would be best to shift the time horizon as far as possible — mathematically speaking, to infinity. To begin with, let us assume that the time horizon is "far away". What, then, happens in the process?

Assume that population is constant, that preferences do not change and that natural endowments are also immutable. The amounts and the distribution of initial capital goods are, by contrast, arbitrary at the beginning. For instance, the economy may have inherited from the past a large stock of grain. It is plausible that, with perfect foresight, various corrective measures will lead to a reduced grain supply over time to match actual needs from period to period. There are two mechanisms to achieve this in the neoclassical model:[3] If initial grain prices are low, this will induce consumers to demand more grain; they substitute it for rice, say. If grain is thus cheap initially, it may similarly be used in production; in the extreme, it is used as fodder. The model also allows that the possibilities for substitution are limited. If there are, for this reason, excess endowments in the beginning, they will receive a zero price; the excess is disposed of freely. The holding of speculative stocks, by contrast, is not part of the neoclassical story. It comes in as a Keynesian modification.

It is clear that, as time proceeds, in the absence of disturbing events, like changes of factor supplies or of the available technology, supply will adapt to the permanent needs. Since relative quantities produced will thus tend to become constant, relative prices will also converge over time. As a matter of fact, it has been proved in a series of papers published in the 1980ies that general equilibrium models of the Debreu type have turn-pike properties: as the time horizon tends to infinity, relative quantities and prices converge as a stationary position is reached. Elaborating on the work started by Bewley (1982) and others, Epstein (1987) has shown that the "terminal" stationary state is, under certain conditions regarding preferences etc., independent of the initial endowments of capital goods.

The process of the transition is best described using the concept of own rates of interest. If, in our example, the initial excess supply leads to initially low grain prices, the own rate of interest (the ratio of the value of the grain price of today to the grain price of tomorrow minus one) will be negative because grain prices will, starting from low levels, be rising, not falling, as discounted prices normally do in a steady state. Other own rates of interest, of commodities which are initially scarce, will be high and falling. The turn-pike result states that these own rates of interest will in the long run gradually converge towards a common value which is "the" rate of interest or rate of discount of the economy as the whole, and to this the consumers adapt. Epstein's (1987) formalisation of recursive preferences is only one of several remarkable aspects of his proof of the turn-pike property: The rates of time preference of consumers are not clumsily given from outside as in many textbook versions, but are endogenously determined as resulting from intertemporal evaluations of alternative future consumption paths and, under suitable conditions, they all converge towards "the" rate of interest.

The initial arbitrariness of the endowments and the consequent initial inequality between the different own rates of interests does not imply that something might be gained by arbitrage within the intertemporal equilibrium. As in the well known example of international exchange between currencies, different rates of interest for Lire, say, and Pounds, may be compatible with a given exchange rate of Lire and Pounds today, if the higher rate of interest on Lire and the lower rate of interest on Pounds is compensated by a lower exchange rate of Lire for Pounds in the future. Thus, in our example, the rate of return of all activities will be uniform and negative if measured in grain, and it will be uniform and positive if measured in rice. But these different rates of return will all converge towards the same in the long run, under the conditions stated.

This explains why Joan Robinson declared that she was confused when she asked what the most general modern theory of value and distribution was and received the answer that it was intertemporal theory. For the intertemporal model does not represent a long run equilibrium position in the classical sense with a uniform rate of profit; in fact, a peculiar type of such a position emerges only as the terminal state, as we now know, according to the turn-pike propositions.

Initially, because of the arbitrary nature of the endowments of capital goods, we have a kind of disequilibrium — or temporary equilibrium — insofar as the amounts supplied have yet to be adapted to the conditions of the stationary state which emerges only in the long run. From the Keynesian point of view, it could even be asked why the expectations of the owners of stocks available in the beginning did not influence the investment behaviour of producers. If the initial endowments of grain are high, not because of an accidental bumper harvest, but because producers had expected a demand for them, the deceived expectations should play a role in the model; but they don't (Schefold 1985a). In this view, we have a disequilibrium with insufficient specifications, and there is not that much difference between temporary (*e.g.* Hahn) and intertemporal equilibria. Because of the assumption of perfect foresight, however, a gradual adaptation takes place in the intertemporal case, provided there is room for substitution in production and consumption to react to changing relative prices.

It is clear that the long period equilibrium which thus emerges only as time goes on is peculiar in several respects: it is a stationary state (or, in some cases, a state of steady growth which does not allow for gradual shifts of demand according to different Engel-elasticities with rising incomes). More importantly, it is a state of full employment of capital and labour, and the associated distribution is regulated by supply and demand. The classical vision of long period positions allows for gradual changes in the composition of output, and full employment of labour is not assumed. In consequence, the theory of distribution is byforce different.

The intertemporal equilibrium, therefore, is to be interpreted, from a classical and Keynesian perspective, as a special form of adaptation of the economy in disequilibrium conditions to a final stationary equilibrium at full employment, and this adaptation takes a very special form because the change of relative prices makes it possible to maintain full employment throughout the process of adaptation; the fundamental reasons being that the future is foreseen, that prices are flexible and that substitution works in production and consumption. In the transition, there is no single rate of interest but many; of the diverse own rates of interst, none can be identified as "the" rate of interest pertaining to money or as "the" natural rate of interest which would equalize saving and investment.

It is often argued that the hypothetical character of the assumptions is to be justified not on grounds of realism but because the model makes it possible to show the working of the invisible hand with its welfare implications in the most elegant and general way possible. I cannot deny the aesthetic attraction of the formalism, but if the claims raised about the welfare aspects of a model and the virtues of the invisible hand are to have any practical implication, the model must be shown to be an — albeit abstract — representation of the relevant aspects of the real world and not one that leads to essentially different conclusions as soon as the abstraction is modified to allow for some degree of realism.

The Keynesian critique concerns the treatment of the assumption of investment behaviour based on perfect foresight and the properties of money in an uncertain world, in particular the lack of a determination of the money rate of interest. I cannot deal with such themes here.

But consider the intertemporal model from the point of view of capital theory. Recall the well-known one sector model with an infinite horizon, *i.e.*, with a production function and with one consumer who maximizes his welfare over an infinite number of periods by discounting his utility.

This model, apparently more important today than ever as a starting point for macroeconmics (Blanchard/Fisher 1989), has a terminal state which is characterized by the modified golden rule: The marginal productivity of capital tends towards the rate of growth of the labour force, augmented by the rate of discount.

The path yielding this approximation from arbitrary initial conditions is defined by two differential equations with the following intuitive content: starting from an initial position in which the capital-labour ratio is low, the marginal productivity of capital is gradually lowered and capital per head is accumulated as long as the rate of return exceeds the steady state value; this means that consumption per head grows. On the other hand, there is investment per head as long as output per head exceeds current consumption per head. It is the traditional story which reappears here in modern disguise: as the rate of interest falls and capital becomes cheaper and is accumulated, consumption per head rises.

According to Samuelson's 'parable' in Burmeister (1980), this inverse relationship between consumption per head and the rate of interest can also be explained intuitively by assuming that an economy with given population and a small capital stock repeatedly saves, foregoing consumption, in order to accumulate capital so that steady states with more consumption and a lower rate of interest are reached in successive steps. The existence of the well-behaved production function thus implies that consumption is a monotonically falling function of the rate of interest.

Obviously, the convergence towards the terminal state is disturbed or made impossible if there is no well-behaved production function: with reswitching and capital reversals, it becomes possible that the fall in rate of interest leads to a choice of technique which causes capital per head and consumption per head to fall rather than to rise. The path towards the terminal states will then be deflected. In Burmeister's (1980) and Epstein's (1987) multisectoral models, the 'perverse' effects of capital theory are excluded and "regularity" is assumed by *directly* postulating the existence of an inverse monotonic relationship between consumption per head and the rate of return. In the one sector model, this postulate is not an axiom but follows from the existence of a well-behaved production function, coupled with the hypothesis that the rate of interest is equal to the marginal product of capital, while consumption per head is, in the stationary state, equal to output per head. Epstein, in his multisectoral model, defines regularity by requiring that, as the rate of return is lowered, the marginal product of each capital good falls, more of each capital good is available and consumption per head rises in the stationary state. "This assumption is satisfied in the single sector model and in some multisector models." (Epstein 1987, 341). But it is clear that "regularity" comprises several properties: the identification of the rate of return with marginal productivity on the one hand, an assumption about technology on the other. There is no direct focus on the demand function for capital which really causes all the trouble.

It would thus appear that the assumption of a "neoclassical technology", i.e. one which excludes reswitching and perverse Wicksell effects, is necessary not for the existence of an intertemporal equilibrium but for the possibility of interpreting it as the explanation of distribution in a long period equilibrium by affording the possibility of a transition towards it. I do not assert that this is the most important aspect of the classical critique of neoclassical theory but it seems to be the most topical, and it fits Bidard's objection precisely: if we ask neoclassical theorists about their theory of capital and distribution in the long period and they point to the achievements of intertemporal theory, essentially the same objections arise as with the other attempts to found a theory of capital on supply and demand: the same assumption has to be made in order to guarantee the existence of the equilibrium as, to use Samuelson's expression, a permanent state, *i.e.* as the state towards which the economy tends, the intertemporal path being only 'transient'. The analogy with the first debate on capital theory is close: It was said that if there is reswitching, the production function underlying the steady state towards which the economy converges in Solow's (1956) model does not exist. Now, we have: "A unique steady state exists if the technology is suitably restricted. Under some additional restrictions, it is shown that all bounded and interior efficient allocations converge to the steady state..." (Epstein 1987, 329). The restriction excludes reswitching etc.

7. CONCLUSIONS

The essential point of the criticism concerns the factor demand curves. The discovery that factor demand curves may be positively sloped in the relevant range, not negatively as is necessary for stability, have not impressed neoclassical theorists that much because, they say, sufficiently general proofs for stability are not available anyway, not even in pure exchange economies. But, against this argument of dispair, it may be argued that the instability regarding production is of greater economic relevance since it is linked to macroeconomic concerns. We have an important example from Keynesian theory which illustrates the effects of a positively sloped factor demand curve: if the level of the demand for investment and government expenditure is given, the demand for labour is likely to rise, not fall, with an increase in the real wage rate in a closed economy, since the increase of wages creates demand for the products of labour.

Convincing conditions of sufficient generality which ensure a well-behaved technology have not been proposed. We therefore should not seek for those special assumptions under which the neoclassical theory might work but for a different theory of distribution and employment altogether. Keynesian modifications of neoclassical full employment theory are still being discussed by the mainstream economists. I am reminded here of the discussion of the Ptolemaic world system: if planets do not move in circles although circles are thought to describe their behaviour, epi-cycles are invented. Why do we not turn to the ellipses straightaway, as Kepler did, forgetting about the circles and the harmonies of spheres, which would mean in our context: why do we not turn to a theory of value which does not presuppose full employment as the natural state?

The denial of the generality of the neoclassical theory of distribution and employment in the long run does not, of course, imply that full employment might not be reached through other forces such as a strong dynamism of investment and adaptation on the supplyside at a given real wage such as changes in the participation rate. Inflationary pressures set an upper limit. The lower limit is more difficult to define.

If we want to use the classical method to represent economies in various states of accumulation, we should free ourselves also of the notion that, normally, only steady states are compatible with a uniform rate of profit. In fact, there cannot be steady growth in the presence of land or of exhaustible resources or if demand for some goods expands more rapidly than for others because of changes of tastes or of methods of production. Yet, it is convenient in all these cases to assume that the sectoral rearrangement of the economy leads to a gradual adaptation of activity levels where competitive forces are sufficiently strong to cause capital to migrate to the faster growing sectors and to abandon those which are growing more slowly or contracting so that the tendency for the equalization of the rate of profit is present permanently, as in the simpler case of steady growth. In a competitive stationary economy, the causes for inequalities in the rates of profit are accidental while sectoral shifts imply a systematic reason for actual rates of profit to differ from normal ones. Nevertheless, the tendency towards the uniform rate of profit may be present in all cases and provide sufficient reason to *define* prices of production on the basis of a uniform rate and to use them in the analysis. I am not arguing against models which use a more complex dynamics of the adaptation of prices and quantities but the advantage of greater "realism" and complexity must be balanced against the loss of perspicacity which might be associated with "classical dynamics" where, I fear, 'anything goes' unless the theory of prices of production is retained as a firm framework of reference.

Johann Wolfgang Goethe-Universität, Frankfurt am Main.

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