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Part III

Market Prices and Natural Prices: A Model with a Value Effectual Demand

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1. INTRODUCTION

Thirty years ago, Piero Sraffa published his 'Production of Commodities by Means of Commodities', a slender book of about 120 pages. It led to a revival of the classical approach to the theory of value and distribution and a renewed interest in the economists of the late eighteenth and early nineteenth century, most prominently among them Adam Smith and David Ricardo. In the first years after the publication of the book the debate concentrated on the criticism of the 'neoclassical' economic theory, as already indicated in the subtitle of Sraffa's book 'A Prelude to the Critique of Economic Theory'. It centered on questions of production, capital and distribution theory. Only in the Seventies an attempt had been made to develop a coherent positive theory on this basis.

The relation between natural prices and market prices is essential for such a positive theory, but is still a rather controversial and unsettled topic in the discussione among the 'neo-Ricardians' (cf. *e.g.* the models of Nikaido, 1983, 1985, Duménil/Lévy, 1987, 1988, 1989, Hosoda, 1985, Boggio, 1985 and Flaschel/Semmler, 1987 and the discussion of Steedman, 1984, Arena/Torre, 1986). According to a widespread opinion that can be traced back to Smith, market prices are seen to gravitate to or around natural prices. The gravitation is usually considered as being engendered by the classical process of competition. If the supply in one sector is too small relative to demand, the respective market price and the respective sectoral rate of profit are supposed to rise. The positive change in the rate of profit attracts capital towards the sector under consideration. The supply is increased and the original 'disequilibrium' reduced.

The central issue of the following paper is the seemingly simple question whether market price formation according to market disequilibria and capital reallocation, guided only by information about the sectoral market rates of profit, can be sufficient to substantiate the role of natural prices as centers of gravitation for market prices. This is not as obvious as it might seem. It is the classical conception of production which introduces one potentially destabilizing element. This approach typically uses produced inputs to substantiate the notion of capital. Sectors are technologically interdependent and this causes specific problems for the capital reallocation process. A shift of productive capacity not only changes the supply in the various sectors, but at the same time the demand (for input use) in the same sectors. It is on the problems which arise from this technological interdependency between sectors that the emphasis of the present work rests.

The following paper provides a formal model to substantiate this allusive picture of gravitation. It combines the central stabilizing and the central destabilizing component as delineated above and it is based on the idea that the different components of demand are subject to specific constraints. It is shown that the 'natural solution' conceptualized by Sraffa prices and a steady growth path is a fixed point solution for the specified model dynamics. An (economically meaningful) range for the coefficient of adaptation exists which renders the fixed point locally asymptotically stable. At the limiting value for this coefficient of adaptation the time path undergoes a Hopf or eventually a flip bifurcation.

2. THE MODEL SPECIFICATION

2.1. *Introduction*

The natural solution is given by Sraffa prices and a steady state growth path. The (equilibrium) growth rate is given by the long-term expectations of producers which are assumed to be equal for the different sectors (and which are eventually influenced by the interest rate of financial capital).

The model for the disequilibrium dynamics is specified in discrete time with the production period structuring time. At the beginning of each production period the market processes evolve. The model depicts these market processes, which determine market prices and market quantities as a result of the confrontation between the quantity supplied to the market and the quantity demanded for input purposes and for final consumption. Without considering inventories the quantity supplied to the market is simply the quantity produced in the previous period. The behavior of the two components of demand derives from an essential asymmetry in the perceived constraints, especially in the possibility of short term financing.[1]

The demand for input purposes is based on short-term production plans. Short-term expectations modify the longterm expectations and are formed according to differences in the realized sectoral market rates of profit. Therefore, production (and accumulation) plans in the short-term do not

[1] This emphasis on the access to short-term finance corresponds to the concepts put forward by BENETTI/CARTELIER, 1980.

coincide with the long term growth path. The planned production implies an input demand which reflects the shift of productive capacity between the two sectors (according to the sectoral rates of profit). In the model, producers are subject to soft budget constraints (because implicitly assuming that they have easy access to short term finance), but to (subectively) rigid constraints with regard to their expectations on the sectoral quantity development. Therefore, input demand is considered to be always satisfied in quantity terms, though possibly rationed in value terms.

The other component of demand is that for final consumption. A strict classical savings behavior is assumed to prevail.

Profit income is entirely saved. Changes in this supply of financial capital do not directly influence the production sector, but only indirectly through the long term expectations. (This relation, however, is not analyzed in the following paper.)

Workers' consumption is not included in the technology matrix. At the end of the production period, money wages are paid, which are entirely and immediately spent in the commodity market. Workers are assumed to be subject to a rigid budget constraint, because they do not have access to short term finance. For simplicity, it will be assumed that the composition of consumption is exogenously given and constant over time. The level of consumption, however, will vary with the level of (actual) employment.

The allocation of the quantity brought to the market directly follows from the assumption that input demand is satisfied in quantity terms. This mechanism controls the quantity path of the (market) quantities. Market prices ultimately depend on the comparison between the quantity supplied to the market minus the input demand and the value demand for final consumption.

Market prices, in their turn, imply specific values for the market rates of profit, which guide the next decision on input demand and interconnect the market processes taking place in successive points of time.

The paper focuses on the question whether such a market price dynamics describes a 'process of gravitation'. I try to use the most simple model specification which will still allow an answer to this question. Therefore only two production sectors are specified. Both commodities are assumed to serve input and consumption purposes. They are produced by using the same two commodities, though in different proportions, and labor as input. No fixed capital, no joint production, and no changes of technique are allowed for. Producers do not differ within a particular sector; instead, we assume one representative producer for each sector. Labor market interactions are not taken into consideration; it is assumed that they do not directly influence the reallocation process between the sectors.[2]

[2] The implicit assumption is that for 'small' labor market variations the labor supply is elastic (also because of the existence of unemployment or overtime work) and that 'fundamental' labor market disequilibria would induce technical changes (which are beyond the scope of the present analysis), cf. also SCHEFOLD, 1979, 187.

Using the money wage rate as numéraire, market prices are expressed as labor commanded prices.

The proposed model shows some distinctive features in comparison to models found in the literature. The determination of the level of market prices by the principle of effectual demand interpreted in value terms was first suggested by Benetti (1979, 1981) and later implicitly taken up by Nikaido (1983, 1985, cf. for a discussion, Kubin, 1989, 1991). In contrast to the former, the present model specifies a Ricardian variant of the process with an interdependent production technology and production prices as the natural price solution. In contrast to the latter, it considers input demand as satisfied in quantity terms. In contrast to both interpretations the present one does not assume the value of the effectual demand to be given exogenously, but relates it explicitly to the (money) wage payment.

2.2. *The formal framework*

2.2.1. The natural solution

We analyze a two sector model with a given technology matrix A (with the elements a_{ij}), exhibiting constant returns to scale, being indecomposable and productive. The wage is not directly included in the technology matrix, but it is allowed for separately. Natural prices, indicated by PN_i, therefore have to cover the replacement cost of used inputs, a profit income distributed in proportion to the value of used capital at a uniform rate rn, and wage income distributed post factum and in proportion to the labor input l_i (per unit of output) at a uniform rate w. These considerations make it possible to specify the following price equations:

$$\begin{aligned} PN_1 &= (a_{11} \cdot PN_1 + a_{21} \cdot PN_2) \cdot (1 + rn) + w \cdot l_1 \\ PN_2 &= (a_{12} \cdot PN_1 + a_{22} \cdot PN_2) \cdot (1 + rn) + w \cdot l_2 \end{aligned} \tag{1}$$

Choosing the wage rate as numéraire[3]

$$w = 1 \tag{2}$$

closes one degree of freedom. Prices are then expressed as labor commanded prices, which depend parametrically on the natural rate of profit. The natural rate of profit, in its turn, has to be determined exogeneously and can vary between zero and the maximum rate as given by the maximum eigenvalue lm of the technology matrix

$$0 < rn < \frac{l - lm}{lm} := rn^{\max} \tag{3}$$

[3] Therefore, questions concerning differences between the natural and the market wage rate cannot be addressed.

Solving the equation system for the prices results in

$$(4) \quad PN_1 = \frac{[1 - a_{22} \cdot (1 + rn)] \cdot l_1 + l_2 \cdot a_{21} \cdot (1 + rn)}{[1 - a_{11} \cdot (1 + rn)] \cdot [1 - a_{22} \cdot (1 + rn)] - a_{12} \cdot a_{21} \cdot (1 + rn)^2}$$

$$(5) \quad PN_2 = \frac{PN_1 \cdot [1 - a_{11} \cdot (1 + rn)] - l_1}{a_{21} \cdot (1 + rn)}$$

The productivity of the technology matrix is equivalent to the following conditions (cf. Takayama, 1974, 392, theorem 4.D.2. IV' and VI')

$$(6) \; [1 - a_{11} \cdot (1 + rn)] \cdot [1 - a_{22} \cdot (1 + rn)] - a_{12} \cdot (1 + rn) \cdot a_{21} \cdot (1 + rn)] > 0$$

as well as

$$(7) \quad 1 - a_{11} \cdot (1 + rn) > 0 \qquad 1 - a_{22} \cdot (1 + rn) > 0$$

These conditions guarantee that for a natural rate of profit smaller than its maximum value in both price equations (equ. (4) and (5)) the numerator as well as the denominator is positive.

The gross output ratio attains its natural value (xn), if it is sufficient to cover the replacement of used inputs, to extend the inputs by the expected growth rate g^e (*i.e.* to cover the net investment), and if the value of the remainder, *i.e.* the supply to the market for consumption goods, equals the value of the consumption demand. The latter equals the wage income, under the (classical) assumption that wages are entirely spent (with c indicating the part used for commodity 1), whereas profit income is entirely saved.

For commodity 1, we obtain

$$(8) \quad PN_1 \cdot [XN_1 - a_{11} \cdot XN_1 \cdot (1 + g^e) - a_{12} \cdot (1 + g^e) \cdot XN_2] = \\ = c \cdot (l_1 \cdot XN_1 + l_2 \cdot XN_2)$$

and solving for xn

$$(9) \quad xn = \frac{PN_1 \cdot a_{12} \cdot (1 + g^e) + c \cdot l_2}{PN_1 - PN_1 \cdot a_{11} \cdot (1 + g^e) - c \cdot l_1}$$

(A similar relation between xn and PN_2 could be derived starting with the equilibrium for commodity 2.)

The natural gross output ratio, therefore, depends on the technology (a_{ij} and l_i), on the composition of consumption demand (c), on the planned growth rate (g^e), and on the natural rate of profit (as contained in PN_i).

The natural position is further defined by an equilibrium between savings (*i.e.* the entire profit income) and (net) investment (i.e. in a model without fixed capital the expansion of the circulating capital as measured by g^e)).

$$(10) \quad rn \cdot (PN_1 \cdot a_{11} + PN_2 \cdot a_{21} + PN_1 \cdot a_{12} + PN_2 \cdot a_{22}) = \\ = g^e \cdot (PN_1 \cdot a_{11} + PN_2 \cdot a_{21} + PN_1 \cdot a_{12} + PN_2 \cdot a_{22})$$

$$(11) \quad rn = g^e$$

Under the assumptions that wage income is entirely spent for consumption and the profit income is entirely saved, the natural rate of profit equals the growth rate (in the natural position). Only one of these variables can be determined exogenously. In our interpretation it is the planned or expected growth rate which is determined by long term expectations and which is therefore exogenous for the model under consideration. In the equations for the natural price of commodity 1 (cf. equ. (4)) and for the productivity (cf. equ. (6) and (7)), rn can be replaced by g^e.

The productivity conditions are now open to a direct intuitive interpretation: The system remains viable if the intended growth rate does not exceed the available physical surplus. This is assumed to hold throughout the model analysis.

2.2.2. The disequilibrium dynamics

We start the formal exposition by specifying the quantity dynamics. The formation of expectations concerning future production is essential for this part of the model. The expected long term growth rate (g^e) which is, by assumption, common to both sectors and exogenously given, is modified according to the perceived sectoral market rates of profit $r_i(t)$.

With $x(t)$ indicating the ratio of the sectoral gross outputs we propose the following expressions for the planned sectoral growth rates $g_i(t)$:

(12) $$g_1(t) = g^e + a \cdot b_1(x(t)) \cdot f(r_1(t), r_2(t))$$

(13) $$g_2(t) = g^e - a \cdot b_2(x(t)) \cdot f(r_1(t), r_2(t)).$$

'a' is the (positive) coefficient of adaptation and f the function of interpreting sectoral market rates of profit subject to the following restrictions

(14) $$f(r_1 = r_2) = 0, \qquad \frac{\partial f(.)}{\partial r_1(t)} > 0, \qquad \frac{\partial f(.)}{\partial r_2(t)} < 0.$$

$b_i(.)$ measures the influence of the relative sector size, which modifies the planning in pure growth rates in the direction of a planning in terms of production levels (cf. for an interpretation Kubin, 1991). It exhibits the following sectoral properties:

(15) $$b_i(.) > 0 \qquad b_i(1) = 1$$

(16) $$\frac{\partial b_1(.)}{\partial x(t)} < 0, \qquad \frac{\partial b_2(.)}{\partial x(t)} > 0.$$

Market prices are determined by the principle of effectual demand confronting a supply in quantity terms and a demand in value terms:

The sectoral supply $S_i(t)$ equals the respective gross output in the previous period $X_i(t-1)$ after having deducted the inputs necessary for the gross output planned for period t.[4] The gross output planned for period t is the gross output in period $(t-1)$ adjusted by the growth rate expected at the end of period $(t-1)$, $g_1(t-1)$ and $g_2(t-1)$.

(17) $$S_1(t) = X_1(t-1) - a_{11} \cdot X_1(t-1) \cdot [1 + g_1(t-1)] - a_{12} \cdot X_2(t-1) \cdot [1 + g_2(t-1)]$$

(18) $$S_2(t) = X_2(t-1) - a_{21} \cdot X_1(t-1) \cdot [1 + g_1(t-1)] - a_{22} \cdot X_2(t-1) \cdot [1 + g_2(t-1)]$$

The demand in value terms refers to the money wages (paid at the rate w at the end of the last period) which are entirely used for consumption. With c and $(1-c)$ denoting the (exogenously given) sectoral splitting, and with l_i sectoral labor coefficients, the sectoral market prices $P_i(t)$ result from the following equivalence of exchange:

(19) $$P_1(t) \cdot S_1(t) = c \cdot w \cdot [l_1 \cdot X_1(t-1) + l_2 \cdot X_2(t-1)]$$

(20) $$P_2(t) \cdot S_2(t) = (1-c) \cdot w \cdot [l_1 \cdot X_1(t-1) + l_2 \cdot X_2(t-1)]$$

Setting

(21) $$w = 1 \qquad \text{(numéraire)},$$

inserting for $S_i(t)$ and dividing by $X_2(t-1)$ ultimately leads to

(22) $$P_1(t) = \frac{c \cdot [l_1 \cdot x(t-1) + l_2]}{x(t-1) - a_{11} \cdot x(t-1) \cdot [1 + g_1(t-1)] - a_{12} \cdot [1 + g_2(t-1)]}$$

(23) $$P_2(t) = \frac{(1-c) \cdot [l_1 \cdot x(t-1) + l_2]}{1 - a_{21} \cdot [1 + g_1(t-1)] \cdot x(t-1) - a_{22} \cdot [1 + g_2(t-1)]}$$

Valuing capital, *i.e.* input goods, at their replacement prices, the following equations define the sectoral market rates of profit $r_i(t)$:

(24) $$P_1(t) = [a_{11} \cdot P_1(t) + a_{21} \cdot P_2(t)] \cdot [1 + r_1(t)] + l_1$$

(25) $$P_2(t) = [a_{12} \cdot P_1(t) + a_{22} \cdot P_2(t)] \cdot [1 + r_2(t)] + l_2$$

(26) $$r_1(t) = \frac{P_1(t) - l_1}{a_{11} \cdot P_1(t) + a_{21} \cdot P_2(t)} - 1 \qquad \text{with}$$

(27) $$\frac{\partial r_1(t)}{\partial P_1(t)} = \frac{a_{21} \cdot P_2(t) + a_{11} \cdot l_1}{[a_{11} \cdot P_1(t) + a_{21} \cdot P_2(t)]^2} > 0$$

(28) $$\frac{\partial r_1(t)}{\partial P_2(t)} = \frac{[l_1 - P_1(t)] \cdot a_{21}}{[a_{11} \cdot P_1(t) + a_{21} \cdot P_2(t)]^2} < 0 \qquad \text{for } P_1(t) > l_1$$ [5]

[4] We are allowed to deduct this input demand because of the assumption that the input demand is satisfied in quantity terms.

[5] This condition is satisfied, if the net output in each sector is at least sufficient to cover the own labor costs.

$$(29) \qquad r_2(t) = \frac{P_2(t) - l_2}{a_{12} \cdot P_1(t) + a_{22} \cdot P_2(t)} - 1 \qquad \text{with}$$

$$(30) \qquad \frac{\partial r_2(t)}{\partial P_1(t)} = \frac{[l_2 - P_2(t)] \cdot a_{12}}{[a_{12} \cdot P_1(t) + a_{22} \cdot P_2(t)]^2} < 0 \qquad \text{for } P_2(t) > l_2$$ [6]

$$(31) \qquad \frac{\partial r_2(t)}{\partial P_2(t)} = \frac{a_{12} \cdot P_1(t) + a_{22} \cdot l_2}{[a_{12} \cdot P_1(t) + a_{22} \cdot P_2(t)]^2} > 0$$

After these market processes, *i.e.* the determination of the market prices according to the principle of effectual demand, the production period follows. Assuming that the planned input demand is always satisfied in quantity terms, the expected growth rates of the sectoral gross output is realized. Therefore the following expression describes the quantity development of the system:

$$(32) \qquad x(t) = x(t-1) \cdot \frac{1 + g_1(t-1)}{1 + g_2(t-1)}$$

Denoting with $g(t)$ what is common to both sectoral expectations:

$$(33) \qquad g(t) = a \cdot f(r_1(t), r_2(t)),$$

results in the following expressions for the sectoral growth rates

$$(34) \qquad g_1(t) = b_1(x(t)) \cdot g(t) + g^e$$

$$(35) \qquad g_2(t) = -\, b_2(x(t)) \cdot g(t) + g^e.$$

Equ. (32) after having inserted equ. (34) and (35), and equ. (33) after hving inserted equ. (26), (29), equ. (22), (23), and equ. (34) and (35) form a two dimensional non linear system of difference equations with the variables $x(t)$ and $g(t)$.

2.3. *The natural solution as a fixed point*

It is easily checked that the conditions for the natural solution define a fixed point of the specified dynamic system.

$$(36) \qquad r_1(t) = r_2(t) = g^e$$

$$(37) \qquad g_1(t) = g_2(t) = g^e$$

$$(38) \qquad x(t) = xn$$

$$(39) \qquad P_i(t) = PN_i$$

[6] This condition is satisfied, if the net output in each sector is at least sufficient to cover the own labor costs.

2.4. *The stability properties of the fixed point*

2.4.1. Linearization

In order to analyze the stability properties of this fixed point solution we linearize the system by forming the Jacobian matrix at the fixed point. For the elements of the first line we insert equ. (34) and (35) into equ. (32), form explicitly the respective derivatives and use the natural position for simplification. We ultimately get

(40) $$\frac{\partial x(t)}{\partial x(t-1)|_{nat}} = 1$$

(41) $$\frac{\partial x(t)}{\partial g(t-1)|_{nat}} = \frac{b_1(.) + b_2(.)}{1 + g^e} \cdot xn$$

For the elements of the second line of the Jacobian matrix the derivatives of

(42) $$g(t) = a \cdot f(r_1(P_1(x(t-1), g(t-1)), P_2(x(t-1), g(t-1))), \\ r_2(P_1(x(t-1), g(t-1)), P_2(x(t-1), g(t-1))))$$

are to be determinated and evaluated at the fixed point. The first one can be split into the following product:

(43) $$\frac{\partial g(t)}{\partial x(t-1)|_{nat}} = a \cdot \frac{\partial f(t)}{\partial x(t-1)} =$$

$$a \cdot \left\{ \frac{\partial f(t)}{\partial r_1(t)} \cdot \left[\frac{\partial r_1(t)}{\partial P_1(t)} \cdot \frac{\partial P_1(t)}{\partial x(t-1)} + \frac{\partial r_1(t)}{\partial P_2(t)} \cdot \frac{\partial P_2(t)}{\partial x(t-1)} \right] + \right.$$

$$\left. + \frac{\partial f(t)}{\partial r_2(t)} \cdot \left[\frac{\partial r_2(t)}{\partial P_1(t)} \cdot \frac{\partial P_1(t)}{\partial x(t-1)} + \frac{\partial r_2(t)}{\partial P_2(t)} \cdot \frac{\partial P_2(t)}{\partial x(t-1)} \right] \right\}$$

For the sign of this derivative we recall equ. (14), (27), (28), (29) and (30). Forming the derivatives of $P_i(t)$ explicitly, it further can be shown that

(44) $$\frac{\partial P_1(t)}{\partial x(t-1)|_{nat}} =$$

$$= \frac{\{-l_1 \cdot a_{12} \cdot (1 + g^e) - l_2 \cdot [1 - a_{11} \cdot (1 + g^e)]\} \cdot (PN_1)^2}{c \cdot [l_1 \cdot xn + l_2]^2} < 0$$

(45) $$\frac{\partial P_2(t)}{\partial x(t-1)|_{nat}} =$$

$$= \frac{\{l_1 \cdot [1 - a_{22} \cdot (1 + g^e)] + l_2 \cdot a_{21} \cdot (1 + g^e)\} \cdot (PN_2)^2}{(1 - c) \cdot [l_1 \cdot xn + l_2]^2} > 0$$

Therefore

$$(46) \qquad \frac{\partial g(t)}{\partial x(t-1)}\bigg|_{nat} = a \cdot \frac{\partial f(t)}{\partial x(t-1)} < 0.$$

The last partial derivative can be split in the following product

$$(47) \qquad \frac{\partial g(t)}{\partial g(t-1)}\bigg|_{nat} = a \cdot \frac{\partial f(t)}{\partial g(t-1)} =$$

$$a \cdot \left\{ \frac{\partial f(t)}{\partial r_1(t)} \cdot \left[\frac{\partial r_1(t)}{\partial P_1(t)} \cdot \frac{\partial P_1(t)}{\partial g(t-1)} + \frac{\partial r_1(t)}{\partial P_2(t)} \cdot \frac{\partial P_2(t)}{\partial g(t-1)} \right] + \right.$$

$$\left. + \frac{\partial f(t)}{\partial r_2(t)} \cdot \left[\frac{\partial r_2(t)}{\partial P_1(t)} \cdot \frac{\partial P_1(t)}{\partial g(t-1)} + \frac{\partial r_2(t)}{\partial P_2(t)} \cdot \frac{\partial P_2(t)}{\partial g(t-1)} \right] \right\}$$

with

$$(48) \qquad \frac{\partial P_1(t)}{\partial g(t-1)} = \frac{-(-a_{11} \cdot b_1 \cdot xn + a_{12} \cdot b_2) \cdot (PN_1)^2}{c \cdot (l_1 \cdot xn + l_2)}$$

$$(49) \qquad \frac{\partial P_2(t)}{\partial g(t-1)} = \frac{-(-a_{21} \cdot b_1 \cdot xn + a_{22} \cdot b_2) \cdot (PN_2)^2}{(1-c) \cdot (l_1 \cdot xn + l_2)}$$

No estimation of the sign is possible, because the sign of the derivatives of the prices remains ambiguous.

Equ. (40), (41), (43) and (47) give the expressions for the elements of the Jacobian matrix (evaluated at the fixed point). Its trace and its determinant can be written as follows:

$$(50) \qquad \text{trace}\,(J) = 1 + a \cdot \frac{\partial f(t)}{\partial g(t-1)}$$

$$(51) \qquad \det(J) = a \cdot \frac{\partial f(t)}{\partial g(t-1)} - xn \cdot \frac{b_1 + b_2}{1 + g^e} \cdot a \cdot \frac{\partial f(t)}{\partial x(t-1)}$$

2.4.2. Some properties in detail

2.4.2.1. Overview

We use the graphical representation in the determinant/trace space in order to analyze the stability properties of the fixed point (cf. *e.g.* Sargent, 1987). The coefficient of adaptation is the parameter, which defines a family of equation systems (with an equal fixed point).

1. If the coefficient of adaptation equals zero, the determinant of the Jacobian matrix is zero as well and its trace equals 1. Regardless of all other parameters (such as technology, structure of final demand, and the reaction function) the determinant/trace combination appertaining to every possible family of equation systems will be found at point T (cf. fig. 1).

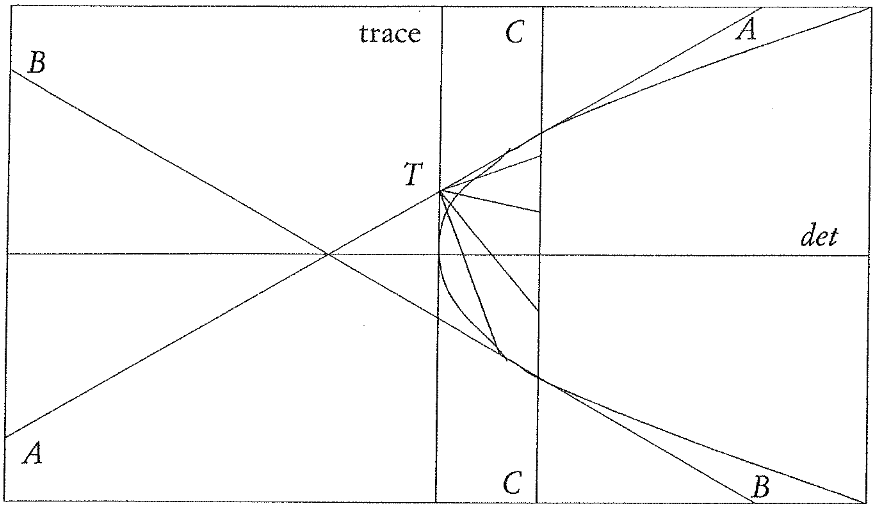

Figure 1

2. If the coefficient of adaptation increases, the determinant/trace position leaves this point following a straight line. The direction of this movement generally depends on the numeric specification of the other parameters. Nevertheless, some cases may be excluded.

3. The determinant/trace position moves for all possible specifications of the equation system to the right hand side (*i.e.* $\delta det(J)/\partial a$ is always positive), and never to the left hand side.

4. For (economically meaningful) positive values of the coefficient of adaptation, it never enters the unstable region above line *AA*. Therefore, no fold bifurcation is to be expected to occur for positive values of the coefficient of adaptation (cf. Whitley, 1983).

5. Therefore, only two possibilities remain:
The determinant/trace position may eventually cross the line *CC* and a Hopf bifurcation occurs. We subsequently denote the appertaining limiting value for the coefficient of adaptation with $a1$.
The determinant/trace position may also cross the line *BB*, where a flip bifurcation is to be expected to occur (cf. Whitley, 1983). $a2$ indicates the respective limiting value for the coefficient of adaptation.

6. Which of the two cases occurs, depends on the numerical specification of the other parameters of the equation system.[7]

Now we turn to the derivations in detail.

[7] For a simplified specification conditions for the occurrence of the two cases can explicitly be derived (cf. Kubin, 1991).

2.4.2.2. Property 1

The first property that the path of the determinant/trace combination starts at the point $T(0/1)$ follows directly from inserting $a = 0$ into equ. (50) and equ. (51).

2.4.2.3. Property 2

The second property, that the determinant/trace path is a straight line is implied by

$$\frac{\partial (trace\ (J))}{\partial a} = \frac{\partial f(t)}{\partial g(t-1)} = \text{const.} \tag{52}$$

$$\frac{\partial (det\ (J))}{\partial a} = \frac{\partial f(t)}{\partial g(t-1)} - xn \cdot \frac{b_1 + b_2}{1 + g^e} \cdot \frac{\partial f(t)}{\partial x(t-1)} \tag{53}$$

both expressions being constant (*i.e.* not dependent on the coefficient of adaptation) for a given system of equations.

2.4.2.4. Property 3

The third property states that

$$\frac{\partial (det\ (J))}{\partial a} > 0 \tag{54}$$

$$\frac{\partial f(t)}{\partial g(t-1)} - xn \cdot \frac{b_1 + b_2}{1 + g^e} \cdot \frac{\partial f(t)}{\partial x(t-1)} > 0 \tag{55}$$

or inserting from equ. (47) and (43)

$$\begin{aligned} &\frac{\partial f(t)}{\partial r_1(t)} \cdot \left\{ \left[\frac{\partial r_1(t)}{\partial P_1(t)} \cdot \frac{\partial P_1(t)}{\partial g(t-1)} + \frac{\partial r_1(t)}{\partial P_2(t)} \cdot \frac{\partial P_2(t)}{\partial g(t-1)} \right] - \right. \\ &\left. - xn \cdot \frac{b_1 + b_2}{1 + g^e} \cdot \left[\frac{\partial r_1(t)}{\partial P_1(t)} \cdot \frac{\partial P_1(t)}{\partial x(t-1)} + \frac{\partial r_1(t)}{\partial P_2(t)} \cdot \frac{\partial P_2(t)}{\partial x(t-1)} \right] \right\} > \\ &> - \frac{\partial f(t)}{\partial r_2(t)} \cdot \left\{ \left[\frac{\partial r_2(t)}{\partial P_1(t)} \cdot \frac{\partial P_1(t)}{\partial g(t-1)} + \frac{\partial r_2(t)}{\partial P_2(t)} \cdot \frac{\partial P_2(t)}{\partial g(t-1)} \right] - \right. \\ &\left. - xn \cdot \frac{b_1 + b_2}{1 + g^e} \cdot \left[\frac{\partial r_2(t)}{\partial P_1(t)} \cdot \frac{\partial P_1(t)}{\partial x(t-1)} + \frac{\partial r_2(t)}{\partial P_2(t)} \cdot \frac{\partial P_2(t)}{\partial x(t-1)} \right] \right\} \end{aligned} \tag{56}$$

Paying regard to equ. (14) this propery holds because the expression in {} brackets on the left hand side of equ. (56) can be shown to be positive, and the one on the right hand side to be negative.

In order to show the left hand side to be positive, we insert from equ. (27), (48), (28), (49), (44), (45), and collect the terms with b_1 and b_2, respectively, on opposite sides of the inequality sign (for details cf. Kubin, 1991).

In order to show the right hand side to be negative, we insert from equ. (30), (48), (31), (49), (44), (45) and collect again the terms with b_1 and b_2, respectively, on opposite sides of the inequality sign (for details cf. Kubin, 1991).

2.4.2.5. Property 4

The fourth property states that the determinant/trace path of the system never enters the unstable region above line *AA*. To show this property it has to be checked whether the slope of the path is smaller than 1 (*i.e.* the slope of line *AA*).

(57)
$$\frac{\dfrac{\partial (trace\,(J))}{\partial a}}{\dfrac{\partial (det\,(J))}{\partial a}} < 1$$

The denominator is positive (cf. property 3). We insert from equ. (52) and (53) and ultimately get

(58)
$$0 < -xn \cdot \frac{b_1 + b_2}{1 + g^e} \cdot \frac{\partial f(t)}{\partial x\,(t-1)}$$

which holds because of equ. (46).

2.4.2.6. Property 5

The fifth property states that the determinant/trace path may eventually cross either (property 5a) the line *CC* with a Hopf bifurcation occurring (at some limiting value a1 of the coefficient of adaptation) or (property 5b) the line *BB* with a flip bifurcation occurring (at some limiting value a_2).

The determinant/trace path crosses the line *CC* as long as

(59)
$$l > \frac{\dfrac{\partial (trace\,(J))}{\partial a}}{\dfrac{\partial (det\,(J))}{\partial a}} > -3$$

The left hand inequality has already been shown (cf. property 4). Recalling that the denominator is positive (cf. property 3) and inserting from equ. (52) and (53), the right one ultimately reads

(60)
$$\frac{\partial f(t)}{\partial g\,(t-1)} > \frac{3}{4} \cdot xn \cdot \frac{b_1 + b_2}{1 + g^e} \cdot \frac{\partial f(t)}{\partial x\,(t-1)}$$

If the determinant/trace path crosses the line *CC*, both eigenvalues of the Jacobian matrix are complex conjugates and their modulus (*i.e.* the determinant of the Jacobian matrix) equals 1. This condition makes it possible to fix the following limiting value $a1$ for the coefficient of adaptation (rendering the determinant equal to unity, cf. equ. (51)):

$$(61) \qquad a1 = \frac{1}{\frac{\partial f(t)}{\partial g(t-1)} - xn \cdot \frac{b_1 + b_2}{1 + g^e} \cdot \frac{\partial f(t)}{\partial x(t-1)}}$$

That this limiting value is economically meaningful, *i.e.* positive, is implied by equ. (60) (recalling equ. (46)).

It can further be shown that at $a1$ a Hopf bifurcation occurs. The following conditions should hold:

1. The eigenvalues of the Jacobian matrix valued at the fixed point $l_{1,2}$ should be complex conjugates.

This property is one of the conditions for deriving $a1$.

2. Their absolute value should equal 1.

This property, too, is a condition for deriving the limiting value.

3. $$l^i(a_1) \neq 1 \qquad \text{for } i = 1,\ldots,4$$

This property would be cumbersome to show and is, thus, assumed to hold.

4. $$\frac{\partial}{\partial a}(|l(a)|)\bigg|_{a=a1} = \frac{\partial}{\partial a}(det(J)) > 0$$

which holds for every specification of the system (as shown above, cf. property 3).

Property 5*b* states that the determinant/trace path eventually crosses the line *BB* with a flip bifurcation occurring (at some limiting value $a2$).

The determinant/trace path crosses the line *BB* as long as

$$(62) \qquad \frac{\frac{\partial(trace(J))}{\partial a}}{\frac{\partial(det(J))}{\partial a}} < -3 \qquad \text{and} \qquad \frac{\partial(det(J))}{\partial a} > 0$$

While the right inequality always holds, the left one reads (after having inserted accordingly, cf. above equ. (59) — (60))

$$(63) \qquad \frac{\partial f(t)}{\partial g(t-1)} < \frac{3}{4} \cdot xn \cdot \frac{b_1 + b_2}{1 + g^e} \cdot \frac{\partial f(t)}{\partial x(t-1)}$$

If the determinant/trace path crosses the line *BB* the eigenvalues of the Jacobian matrix (eveluated at the fixed point) are real numbers and one equals -1. The following condition allows one to derive a second limiting value $a2$ for the coefficient of adaptation (cf. Goldberg, 1958):

(64) $$1 + trace(J) + det(J) = 0$$

Inserting from equ. (50) and (51) leads to

(65) $$a_2 = \frac{-2}{2 \cdot \dfrac{\partial f(t)}{\partial g(t-1)} - xn \cdot \dfrac{b_1 + b_2}{1 + g^e} \cdot \dfrac{\partial f(t)}{\partial x(t-1)}}$$

That this equation determines an economically meaningful, *i.e.* positive limiting value *a2* for the coefficient of adaptation, is again implied by equ. (63) and (46).

3. SUMMARY

The central concern of the present paper is the relation between market prices and natural prices in the classical approach to economic theory. The natural position is often conceptualized as being independent[8] of the particular, unsystematic market processes. According to this view (cf. *e.g.* Garegnani, 1976, Milgate, 1982) the analysis of market prices is of secondary importance in comparison to the analysis of the natural position. The motion of market prices is held to be regulated by the natural position. Market prices gravitate to or around the natural position. The specific form of this dynamics may be rather unsystematic; however, it is of subordinate interest because it is judged as not feeding back to the natural position.

The formal analysis of the model showed the following results. First, the natural solution is a fixed point of the specified market dynamics. Second, a range for the coefficient of adaptation exists which renders this fixed point locally asymptotically stable. It is crucial that this property does not depend on the technology matrix (in contrast to Nikaido's results, 1983, 1985; cf. for a discussion, Kubin, 1989). Third, at some limiting value of this coefficient of adaptation the time path of the system undergoes a (Hopf or flip) bifurcation, where a motion of the system without predetermined period is to be expected. For a somewhat simplified model further results could be derived (cf. Kubin, 1991): Simple conditions could be specified for separating the two bifurcation types. Computer simulations revealed different unsystematic (yet bounded) dynamic patterns which can occur for adaptation coefficients greater than the limiting value. Adding a lagged term to the reaction function increases the stable range for the coefficient of adaptation. All these results support the gravitation hypothesis.

In order to conclude, I would like to point out that the unsystematic

[8] This does not amount to considering the natural position as constant or invariable; it only maintains that the laws governing the natural position differ from those controlling the (unsystematic) market processes (cf. Deleplace, 1984).

(yet bounded) motion of the system in a neighborhood of the fixed point solution is not only a curious or, at best, an aesthetically appealing feature of the suggested model (cf. Farmer/Kubin, 1990). First, to begin with the most obvious aspect, this type of motion enlarges the range of the coefficient of adaptation for which the hypothesis of gravitation is not violated. For the gravitation idea it is not necessary that the time path converges to the natural solution; it suffices if the time path remains close to the natural position. Second, it is exactly this type of motion which is highly compatible with the underlying idea that the natural position and its determinants are independent of the market processes. This concept appears only to be intuitively reasonable if the market processes are unsystematic. Any system in their motion could eventually be learned by economic agents. Their reactions, in turn, would modify the determinants of the natural position (as there are technology, effectual demand and income distribution). An unsystematic motion of the market variables inhibits these reactions. Third, the (positive) information which an *unsystematic* type of motion can convey is that the system is in a critical phase. Economic agents lose confidence in their decision basis and it could be supposed that they react more cautiously. An unsystematic type of motion, therefore, allows the economic agents to modify their behavior. This is not the case for a strictly diverging motion. In a broader context where also the functional relationships are allowed to vary, an unsystematic (but still bounded) type of motion can be considered as increasing the overall stability of the system (cf. Day, 1981, Vercelli, 1982).

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