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Gravitation Theory: Two Illustrative Models

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During the last few years, an increasing interest has arisen in rediscovering the old classical problem of the gravitation of market prices around natural values. Such a revival is not merely the result of an increasing attention paid to the classical period by historians of economic thought. There are indeed strong theoretical reasons for returning to the theory of gravitation.

On one side, the revival was due to the so-called "neo-Ricardian" economists. Some of these indeed defended the necessity of using what P. Garegnani (1976) called the method of long-run positions comparison, to rebuild a modern classical economic theory. Now, the "surplus approach" version of this method appeared to be founded on the Smith-Ricardo-Marx theory of the equalization of sectorial rates of profit. It is therefore natural that, in defending this version, Garegnani (1976) was lead to stress the importance of the arguments developed by Smith in the chapter of the Wealth of Nations dedicated to the "natural and market price of commodities" (A. Smith (1977), book 1, chapter VII), Rightly or wrongly, the validity of these arguments then appeared to be crucial in the proving of the logical consistency of the modern reformulation of the surplus approach.

On the other side, in the mid-seventies, some economists supporting the idea of renovating the classical approach of prices saw in the revival of the theory of gravitation a means to study the problem of the stability of these prices. There was indeed a priori no reason for a modern classical theory of prices to ignore the problems that the neo-classical approach was itself facing. In this perspective, M. Egidi's (1975) pioneer article contributed to insert the problems of proving the stability of production prices and studying the process of their formation on the research agenda of the modern renovation of the classical theory of prices.

Since Egidi's and Garegnani's contributions, the literature dedicated to gravitation theory has grown and revealed the complexity of the problem.

This complexity entails at least three main questions which have to be faced.

The first question regards the dichotomy of "short run" and "long run" theories. Is it possible to assume that "the adjustment of individual outputs involved in the gravitation towards natural prices could be conceived as an adaptation to quantities already fixed, *i.e.* determined in another part of the theory" (P. Garegnani (1983) p. 132). Such a dichotomy first of all implies that short run supply adaptations are limited to variations in the degrees of utilization of productive capacities and do not involve changes of the productive capacities themselves. Now, how can we be sure that the gravitation adjustment process of quantities will not entail such changes, especially if returns to scale are not constant? The dichotomy of short and long run processes seems also to imply that "the factors affecting the forces which determine long-period normal values change slowly and not by frequent 'jumps' or 'jerks'" (M. Milgate (1982) p. 30). But this assumption is too stringent: a period of numerous and repeated technical innovations or the occurrence of an economic crisis clearly contradict a slow change of normal magnitudes. There is, lastly, no reason to limit the dynamics to path-independent ones. L. Pasinetti's conception is more convincing, according to which "the relevant point is that the very nature of the process of long-run growth requires a structural dynamics which leads to difficulties in the short run", "the one implies the other" and, therefore, "the whole process has to be accepted and tackled in its entirety" (L. Pasinetti (1981) p. 243).

The second question is related to microeconomic behaviors within the gravitation process. Numerous gravitation models indeed limit their dynamics to a cross-dual mechanical determination of prices and quantities, the microfoundations of which are scarcely made explicit. Now, microeconomic behaviors may differ according to the strength and the type of economic uncertainty, to the nature of the adjustment or to the ends of the agents. These possible differences have to be investigated and they may play an important role in the issue of the gravitation problem. An interesting illustration of this second question may also be found in Garegnani (1983), which simultaneously accepts the idea of "incorrect expectations" p. 133) creating deviations and the assumption that the "normal" rate of profit "is also the rate of profit which that present experience will lead entrepreneurs in general to expect in the future from their current investment" (p. 27).

A final question comes from the nature of markets and market adjustments. We all know that in Smith, prices were considered to be clearing-market prices (R. Arena (1978)). This does not mean however that Smithian market prices are equilibrium prices or Walrasian prices. Even in Smith, market-clearing equilibria can be associated with market prices which differ from natural ones. Expected and realized profitabilities can indeed differ. Moreover, Smithian analysis does not sum up the richness of the classical theory of markets. Now, in modern gravitation models, except for literary fidelity to Smith, we are not obliged to accept a market-clearing assumption. More generally, classical analysis does not seem to welcome a unique canonical model of the market, analogous to the Walrasian one. A typology of markets may be introduced and made precise, according to specific institutional context.

Both the following models will throw a light on the preceding questions. The role of reproduction will involve the necessity of reconsidering the independency of short and long run changes. Microeconomic behaviours will be taken into account and the analysis will show their diversity. We will observe, finally, that very different forms of classical market adjustment processes may be described in a classical framework.

1. A MARKET-CLEARING MODEL OF GRAVITATION

This first model is roughly analogous to the one already presented in Arena, Froeschle and Torre (1988). Its source of inspiration is Adam Smith's Wealth of Nations, even if we do not claim to present "what Smith really meant": within each annual cycle of production and exchange, market prices are indeed assumed to clear the market and to adjust quantity discrepancies between supply and demand.

As in classical theory, two groups of agents are made distinct: firms and workers. Wages are included in advanced capital and their natural level is related to the value of a wage-good basket, the structure of which is given exogenously. Therefore, firms appear to be the sole active economic agents on the markets. They demand wage-goods and capital-goods and produce these commodities in single production processes. All the productive techniques of a particular industry are assumed to be identical. Complementarity of inputs is supposed but returns to scale can be variable. In formal terms, production coefficients $a_{ij}^{(t)}$ are polynomial functions of the elements of a diagonal matrix $Q^{(t)}$:

$$A^{(t)} = A_0 + Q^{(t)} A_1 \qquad (1)$$

where A_0, A_1 are given square matrices the values of elements of which make it possible to define the nature of returns to scale, according to the sign of A_1 and eA_1 ($e = 1,...1$), and $Q^{(t)}$ represents the current diagonal matrix of actual sectorial levels of activity.

Productive decisions

The period begins with firms productive decisions. In order to determine the quantities of commodities produced and brought to the market, firms take into account market performances of the previous period. These performances are given by past realized rates of profit. They provide the data according to wich firms form their current rates of profit expectations. Thus:

$$\Delta^{e(t)} = B\Delta^{m(t-1)} + (I - B)\Delta^{m(t-1)}$$

where $\Delta^{m(t-1)}$ is a diagonal matrix of market realized rates of profit within period i, $\Delta^{e(t)}$ is the diagonal matrix of industrial expected rates of profit within the current period, an B is a diagonal matrix the element of which are such that $0 \leq b_{ii} \leq 1$.

Firms profit expectations are thus extrapolative. In the choice of their levels of activity, firms take into account their situations within the general hierarchy of industrial rates of profit. They bear in mind two rates: their industrial expected rate of profit and a social index of profitability which is the "normal" rate of profit at a point of time.

This "normal" rate of profit at a point of time coincides with the expected average rate of profit in period t:

$$r^{e(t)} = b\, r^{(-1)} + (1 - b)\, r^{(t-2)}$$

where $r^{(t)}$ is the actual average rate of profit for period t, and b a constant such that $0 \leq b \leq 1$.

The determination of $r^{e(t)}$ is therefore extrapolative, as the industrial expected rates of profit.

The actual average rate of profit for period t is equal to:

$$\bar{r}^{(t)} = \frac{q^{(t)}\,[p^{m(t)} - A^{(t)}\,p^{m(t-1)}]}{q^{(t)}\,A^{(t)}\,p^{m(t-1)}}$$

where $q^{(t)}$ is the row-vector of actual current levels of activity, and $p^{m(t)}$ is the column-vector of market prices for period t. Means of production used in period (t) are indeed evaluated at their purchase prices, *i.e.*, at market prices of period ($t - 1$).

Firms' desired current levels of activity are, finally, given by the row-vector $q_d^{(t)}$:

$$q_d^{(t)} = q^{(t-1)}[I + S(\Delta^{e(t)} - \bar{r}^{e(t)}I)]$$

where S is a semi-positive current matrix the elements of which express the sensivity of producers to the perception they have about their situations within the intersectoral hierarchy of profit rates.

In other words, the supply change motive appears to be here the difference between the expected industrial rate of profit and the normal one. Firms increase or decrease their levels of activity according to the positivity or the negativity of this difference.

Formation of productive advances

Using expression (2), it Is easy to determine matrix $A_d^{(t)}$ of current desired efficient production coefficients:

$$A_d^{(t)} = A_0 + Q_d^{(d)} A_1 \qquad (2)$$

Desired and actual production coefficients or levels of activity are made distinct because the economic system is assumed to be constrained by a physical viability rule: firms have to find current means of production among the gross product of previous period:

$$q^{(t)} A^{(t)} \leq q^{(t-1)}$$

If this constraint is not compatible with desired productive decisions, producers are assumed to repeat the actual decisions of the previous period. If these also appear to be unfeasible, the gravitation process is then considered to be itself non-feasible: productive dynamic process, which do not respect twice the physical viability rule are thus systematically discarded. We will call these processes "non feasible paths".

Formation of supply prices

Activity levels $q^{(t)}$ are actually developed and offered to market. Supply prices are proposed by producers, applying expected industrial rates of profit to actual cost of production. They are equal to:

$$p^{e(t)} = (I + \Delta^{e(t)}) A^t p^{m(t-1)}$$

where $p^{e(t)}$ is the vector of supply prices.

Determination of demands

Demand behaviors are the result of two types of influence. On the one hand, consumers take into account their past experiences. They adopt adaptive behaviors, remembering the difference they noticed between their demands and the supplies of the previous period, at the supply prices, *i.e.*, before market adjustments. On the other hand, consumers are also sensitive to substitution effects which are implied by general interdependence of prices. In this prospect, relative demands are given by:

$$q'^{m(t)} = q^{m(t-1)} + [q^{(t-1)} - q^{m(t-1)}] T + d^t \qquad (3)$$

where $q^{m(t-1)}$ are demands of the previous period, T is a semi-positive diagonal matrix of constant elements and d^t expresses the complex influence of substitution effects resulting from general price interdependence.[1]

We also know that the aggregate demand and supply are equal, so that:

$$q^{(t)} p^{e(t)} = q^{m(t)} p^{e(t)} \qquad (4)$$

Combining (3) and (4), we can take into account income effects and therefore determine current demands at supply prices:

$$q^{m(t)} = q'^{m(t)} (q^{(t)} p^{e(t)} / q'^{m(t)} p^{e(t)})$$

[1] For a precise specification of d^t, see Arena, Froeschle and Torre (1988), p. 1105.

Determination of actual transactions and market prices

In agrement with a Smith's conception, market prices are assumed to be perfectly flexible within the period and, therefore, to be able to clear the markets.

The market-clearing rule is as simple as possible: the market price level is the one which allows the sale of the whole supplied quantity of commodities. This price s thus equal to:

$$p^{m(t)}_i = \frac{q^{m(t)}_i \, p^{e(t)}_i}{q^{(t)}_i} \, , \, V$$

where $p^{m(t)}_i$, $q^{m(t)}_i$, $p^{e(t)}_i$, $q^{(t)}_i$ represent the *i*th respective components of vectors $p^{m(t)}$, $q^{m(t)}$, $p^{e(t)}$ and $q^{(t)}$.

Determination of market rates of profit

Production costs are known after the development of production processes. The knowledge of actual market prices thus allows firms to reckon their market rates of profit, *i.e.*, those actually obtained in the different industries, which generally differ. The diagonal matrix of market profit rates is indeed equal to $\Delta^{m(t)}$, and given by:

$$p^{m(t)} = (I + \Delta^{m(t)}) \, A^t p^{m(t-1)}$$

This market-clearing model attempts to give at least partial answers to our introductory questions.

It first takes into account the problems generated by productive capacity limits when supply charges occur and when returns to scale are non-constant. They are summed up by the physical viability rule. The numerical experiments treatment of the model allows us to emphasize its effects. The presence of this rule indeed implies the appearance of a very high number of "non viable paths" superior, in all cases, to 50% of experiments development with the help of the Monte-Carlo method.[2] Its absence is, on the contrary, a strong guarantee of convergences to prices of production, at least when the economy is compounded of a small number of industries. According to intuition, increasing returns exert a negative effect but only when the viability rule is discarded: when it is taken into account, they may have, on the contrary, a positive effect, because they reduce the magnitude of production coefficients and, therefore, the weight of the physical viability rule.

However, these remarks answer only to a part of the problems related to the interference between long run and short run theories.[3]

[2] For a detailed exposition of numerical experiments and of their results, see ARENA, FROESCHLE and TORRE (1988) pp. 1107-1115.

[3] For some results in this prospect, see ARENA and FROESCHLE (1986).

The model also tries to face the problem of the introduction of microeconomic behaviors. Adaptive expectations are assumed. Periodical revisions of levels of activity are considered. Finally, the expected average rate of profit (interpreted as the "normal" one at a point of time) is used as a social index of profitablility.

We might also notice that market-clearing adjustments are introduced to give a Smithian flavour to the model.

This first model confirms essentially I. Steedman's (1984) conclusion according to which "the moral is simply that there is a genuine question at stake there, whose answer is not self-evident" (p. 137).

On one hand, the number of convergences indeed depends crucially upon the nature of firms' behaviors: as expected, the strength of the reaction of agents reduces the probability of convergences and, if it is too brutal, it can hinder the gravitation process.[4]

On the other hand, the results also confirm those of Steedman (1984), according to which, as soon as the number of industries is superior to two, "deviations of market prices from natural prices need not be positively correlated with the deviations of industry profit rates from the normal rate, in the presence of produced means of production" (p. 127). When the number of industries increases, the frequence of convergences does indeed fall drastically (see Froeschle (1985). This result reinforces the necessity for deeper investigation, even in market-clearing models. However, it is also interesting to offer the alternative assumption to the reader, introducing inventories and possibilities of inventory changes.

2. AN INVENTORY MODEL OF GRAVITATION

This second model of gravitation is very different from the previons one. Its source of inspiration is no longer the Wealth of Nations, but Cantillon's *Essai sur la Nature du Commerce en Général* and MacCulloch's *Principles of Political Economy*.

Two main features make it distinct from the market-clearing model.

According to the classical tradition, agents are still assumed to be gathered in two groups: firms and workers. Moreover, wages are still included in advanced capital. But firms are no longer the sole active economic agents on the markets. A third group appears, which corresponds to what R. Cantillon called "entrepreneurs de leur propre travail" or "marchands" (R. Cantillon (1952) — first part Chapter XIII) and J. R. MacCulloch designated as "merchants", "speculators" or "dealers" (J. R. MacCulloch (1965) — Part II — Chapter III). These "merchants" are agents who buy commodities in order to sell them at a price which is expected to be superior to the purchase price. Therefore, as J. R. MacCulloch writes, "speculation

[4] See R. Arena, Froeschle and Torre (1988).

is really only another name for foresight" (J. R. MacCulloch (1965) p. 259). To take R. Cantillon's example, "le drapier est un entrepreneur qui achète des draps et des étoffes du manufacturier à un prix certain, pour les revendre à un prix incertain, parce qu'il ne peut pas prévoir la quantité de la consommation" (R. Cantillon (1965) p. 30).

The introduction of "merchants" in to the models implies the possible appearance of inventories and inventory changes. Therefore, gravitation processes can no longer be described as market-clearing ones. As in Cantillon (chapter XIV of the first part of the Essai, the flexibility of market prices can be imperfect or insufficient, allowing the appearance of deliberate or involuntary inventories which can survive "d'année en année" (p. 35), *i.e.* last more than an annual cycle of production of exchange. The same possibility is considered by J. MacCulloch, when this author writes that "it must not be suposed that this adjustment of the supplies of produce, according to the variations in the effectual demand, is always speedily or easily brought about" (J. R. MacCulloch (1965) p. 254).

These two features help us to present the major characteristics of our second model. Its specificity in the classical context results from the inventory demand it exhibits, which was previously introduced by G. Laroque in recent papers.[5] We shall adapt the dynamics presented in these works to a more complex economy in which the classical forces generated by the differenciation of profits interact with the speculative behavior of merchants.

Agents' expectations and beahviors

If we suppose that wage-goods are included in the means of production, the market is only visited by merchants and firm-owners. At time t, firm-owners held money balances $M^{(t)}$resulting from previous periods and earn current profits $\pi^{(t)}$. They demand goods produced by the economy: a variable quantity of the luxury-good $\underline{q}_1^{m(t)}$ which is a function of their income and of their wealth, and also a given quantity of the basic-good $\underline{q}_2^m$.[6]

$$\underline{q}_1^{m(t)} = \frac{\alpha\pi(t)}{p_1^{(t)}} + \frac{\beta M(t)}{p_1^{(t)}}$$
$$\underline{q}_2^{m(t)} = \underline{q}_2^{m} \qquad ,\forall t$$

where $p_1^{(t)}$ is the price of luxury goods at time t

α and β given constants such that

$0 \leq \alpha \leq 1$

$0 \leq \beta \leq 1$

[5] G. Laroque (1989 a and b).

[6] The model is a two sector model, for the sake of simplicity, *i.e.*, in order to reach a general analytical solution. However, it is always possible to study a model with more industries, using numerical experiments.

Merchants or speculators buy luxury goods in period t to sell them in period $(t+1)$. The inventory demand $S^{m(t)}$ depends on their expectations in the following way:

$$S_1^{m(t)} \begin{cases} = 0 \text{ when } p_1^{e(t+1)} < p_1^{(t)} \\ \text{or} \begin{cases} & p_1^{e(t+1)} = p_1^{(t)} \\ \text{and} & q_1^{S(t)} < q_1^{m(t)} \end{cases} \\ \in [0, \bar{S}_1] \text{ when } p_1^{e(t+1)} = p_1^{(t)} \\ \qquad \text{and } q_1^{S(t)} = q_1^{m(t)} \\ = \bar{S}_1 \text{ when } p_1^{e(t+1)} > p_1^{(t)} \\ \text{or} \begin{cases} & p_1^{e(t+1)} = p_1^{(t)} \\ \text{and} & q_1^{S(t)} > q_1^{m(t)} \end{cases} \end{cases} \tag{5}$$

where $p_1^{e(t+1)}$ represents the expected price of luxury goods in period $(t+1)$

$q_1^{S(t)}$, the aggregate supply of luxury goods in period (t)

$q_1^{m(t)}$, the aggregate demand for luxury goods in period (t)

$\bar{S}_1$, a given inventory capacity

For the sake of simplicity, we will suppose that inventories cannot survive one single period.

Among luxury goods buyers, speculators are assumed to be the first to be served. Therefore, if $\bar{S}_1$ is not large enough as regards the supply of luxury goods, $S_1^{m(t)}$ represents their inventory holdings at the end of the period. To finance their purchases, speculators are lead to demand some credit from the banks, the amount of which is $S_1^{m(t)} p_1^{(t)}$. They also sell a part of the inventories of the previous period, the receipt of which allows then to reimburse loans conceded by banks in the previous period $(t-1)$. In the case of perfect foresight on which we shall focus here, they are always able to meet their liabilities by selling $\hat{S}_1^t$:[7]

$$\hat{S}_1^{(t)} = \frac{p_1^{(t-1)} S_1^{m(t-1)}}{p_1^{(t)}} \tag{6}$$

Their net speculative gain is then:

$$[p_1^{(t)} - p_1^{(t-1)}]\, S_1^{m(t-1)}$$

which corresponds to the quantity:

$$\frac{[p_1^{(t)} - p_1^{(t-1)}]\, S_1^{m(t-1)}}{p_1^{(t)}}$$

[7] In the case of imperfect foresight, some firms would be lead to face the risk of insolvability of speculators, resulting from failures of expectations.

Productive techniques

The economic system produces two commodities using only one of them as a capital good.

Commodity 1 is the luxury good, consumed by firm-owners and speculators.

Commodity 2 is used simultaneously as a wage-good, as a capital-good and as a final consumption-good by firm-owners. It is, therefore, the only basic good produced by the economic system.

Returns to scale are assumed to be constant. Therefore, the matrix of technological coefficients is A with:

$$A = \begin{pmatrix} 0 & a_{12} \\ 0 & a_{22} \end{pmatrix}$$

In order to produce a net surplus including luxury goods, it is necessary that

$$1 - a_{22} > 1$$

This condition is therefore a physical viability rule.

Productive capacities and the labor force are never full-employed, so that supply changes are not limited by existing resources. Finally, only working capital is considered.

Saving and loans

The financing of firms' production is based on one-period credit. Firms always meet their liabilities and, theefore, no explicit treatment of this kind of financing is offered in the model. However, at the beginning of the period, firm-owners hold money balances $M^{(t)}$ which result from their previous saving decisions, money being the only financial asset in the economy. Then, according to their current consumption decisions, these balances can increase or decrease within the period. At the end of the period, firm-owners thus hold money balances which are now equal to $M^{(t+1)}$.

One the contrary, speculators do not save and, therefore, do not hold money balances at the beginning of the period. They are thus compelled to demand money from te banks. Within the period, they borrow $M_+^{(t)}$ from the banks:

$$M_+^{(t)} = S_1^{m(t)} p_1^{(t)}$$

but they also reimburse their previous borrowings, *i.e.*, in perfect foresight:

$$M_-^{(t)} = S_1^{m(t-1)} p_1^{(t-1)}$$

Then, the accounting part of the model implies the following identity:[8]

$$M^{(t-1)} - M^{(t)} = S_1^{m(t)} p_1^{(t)} - S_1^{(t-1)} \tag{7}$$

[8] Cf. appendix.

In other words, the increase of held money balances is equal to the increase of money advances offered by banks to speculators. Therefore, banks lend to speculators the money they borrow from firms-owners.

The reproducing rules

The determination of produced quantities must respect reproduction rules.

We know that the production of commodity 2 is always technically possible since:

$$1 - a_{22} > 0$$

Reproduction rules can thus also be defined for the production of luxury goods:

$$q_1^{P(t)} a_{12} \leq \hat{q}_2^{P(t)} (1 - a_{22}) - \underline{q}_2^m$$

$$\text{or} \quad q_1^{P(t)} \leq \frac{\hat{q}_2^{P(t)} (1 - a_{22}) - \underline{q}_2^m}{a_{12}}$$

The problem faced here derives from the possible existence of a positive excess demand of commodity 2.

In this case, we can reasonably assume that commodity 2 producers will first supply themselves and thus hold for their future production a quantity equal to $a_{22}\hat{q}_2^{P(t)}$. Then they supply final demandes with $\underline{q}_2^{m(t)}$. Finally, if there remain some quantities of commodity 2, they sell them to producers of industry 1.

Therefore, the quantity of commodity 2 available for commodity 1 producers is equal to:

$$\underline{\hat{q}}_2^{P(t)} - a_{22}\hat{q}_2^{P(t)} - \underline{q}_2^{m(t)}$$

Hence, the above inequality.

Quantity decisions and rates of profit

According to the classical conception of competition, supply changes are connected with prevailing rates of profit and their quantitative differences.

If $\hat{q}_1^{P(t)}$ and $\hat{q}_2^{P(t)}$ represent planned production levels in both industries, we can assume they depend on both rates of profit. Thus, $\hat{q}_1^{P(t)}$ is an increasing function of the rate of profit $r_1^{(t)}$ of the first industry and a decreasing function of the rate of profit $r_2^{(t)}$ of the second industry. Obviously, $\hat{q}_2^{p(t)}$ is an increasing function of $r_2^{(t)}$ and a decreasing function of $r_1^{(t)}$. Then, we can introduce a formulation which facilitates the mathematical treatment of the model:

$$\hat{q}_1^{P(t)} = \gamma' - \gamma \left[\frac{1 + r_2^{(t)}}{1 + r_1^{(t)}} \right]$$

$$\hat{q}_2^{P(t)} = \gamma \left[\frac{1 + r_2^{(t)}}{1 + r_1^{(t)}} \right]$$

In these expressions, $(\gamma' - \gamma)$ may be regarded as the normal degress of utilisation of fixed capital for the production of luxury goods and of basic commodities.

The effective demand

Commodity 1 effective demand is the sum of consumption and inventory demands:

$$q_1^{P(t)} = \frac{\alpha\pi^{(t)}}{p_1^{(t)}} + \frac{\beta M^{(t)}}{p_1^{(t)}} + S_1^{m(t)}$$

Commodity 2 effective demand is the sum of both input and final demands:

$$q_2^{m(t)} = \underline{q}_2^m + q_2^{m(t)} a_{22} + \overline{q}_1^{p(t)} a_{12}$$

The effective supply

The effective supply of firms depends on three components: first, their planned level of production, determined according to a profitability rule; secondly, the reproduction rules which only play an active part in the supply of luxury goods; thirdly, their demand expectations, easily predictible for basic commodities. The demand of commodity 2, as previously defined, results from three elements: the invariable level of final demand $\underline{q}_2^m$, the input demand of industry 2, $q_2^{m(t)} a_{22}$, the input demand of industry 1, $q_1^{P(t)} a_{12}$. If expectations on this last term are formed by firms of industry 2, these firms can fairly well expect their whole demand. The expectation of the last term results from the confrontation of the planned production of industry 1 which is known when markets open, and of the demand with which producers of industry 1 are confronted.

If we call $q_1^{e(t)}$ this quantitative expectation of the effective level of production of industry 1, we have:

$$q_1^{e(t)} = \min [\hat{q}_1^{p(t)}, q_1^{m(t)} - \hat{S}_1^{(t)}]$$

Then, the expected demand from industry 2 for commodity 2 is given by:

$$q_2^{e(t)} = \underline{q}_2^m + q_2^{e(t)} a_{22} + q_1^{e(t)} a_{12}$$

or:

$$q_2^{e(t)} = \frac{\underline{q}_2^m + \min [\hat{q}_1^{P(t)}, q_1^{m(t)} - \hat{S}_1^{(t)}] a_{12}}{1 - a_{22}}$$

Effective supply of commodity 2 is given by:

$$q_2^{S(t)} = \min [\hat{q}_2^{P(t)}, \frac{\underline{q}_2^m + \min [\hat{q}_1^{P(t)}, q_1^{m(t)} - \hat{S}_1^{(t)}] a_{12}}{1 - a_{22}}]$$

Effective supply of commodity *1* depends on its planned component and on the reproduction rule:

$$q_1^{S(t)} = \min \left[\hat{q}_1^{P(t)}, \frac{\hat{q}_2^{P(t)}(1 - a_{22}) - \underline{q}_2^m}{a_{12}}\right] + \hat{S}_1^{(t)}, \tag{8}$$

Markets and disequilibria

Within the period, exchanges take place at fixed prices. The adjustment is made by the short side of the market.

$$\overline{q}_1^{(t)} = \min[q_1^{S(t)}, q_1^{m(t)}] \tag{9}$$

$$\overline{q}_2^{(t)} = \min[q_2^{S(t)}, q_2^{m(t)}] \tag{10}$$

Effective transactions of commodity 2 are then given by:

$$\overline{q}_2^{(t)} = \min\left[\min\left[\hat{q}_2^{P(t)}, \frac{\underline{q}_2^m + \min[\hat{q}_1^{P(t)}, q_1^{m(t)} - \hat{S}_1^{(t)}]\,a_{12}}{1 - a_{22}}\right], \frac{\underline{q}_2^m + \min\left[\hat{q}_1^{P(t)}, \frac{\hat{q}_2^{P(t)}(1 - a_{22}) - \underline{q}_2^m}{a_{12}}, q_1^{m(t)} - \hat{S}_1^{(t)}\right] a_{12}}{1 - a_{22}}\right]$$

Elementary calculus reveals that both terms in the main brackets are always equal. Then, by quantity adjustments within each period, we always have:

$$\overline{q}_2^{(t)} = q_2^{S(t)} = q_2^{m(t)}$$

This identity is no longer true for commodity 1. The difference between supply and demand of luxury goods is equal to:

$$q_1^{m(t)} - q_1^{S(t)} = \frac{\alpha\pi^{(t)}}{p_1^{(t)}} + \frac{\beta M^{(t)}}{p_1^{(t)}} + S_1^{m(t)} - \frac{p_1^{(t-1)} S^{m(t-1)}}{p_1^{(t)}}$$
$$- \min\left[\hat{q}_1^{P(t)}, \frac{\hat{q}_2^{P(t)}(1 - a_{22}) - \underline{q}_2^m}{a_{12}}\right] \tag{11}$$

$\pi^{(t)}$ only depends on prevvailing prices, in period *t*. The influence of prices prevailing in period $(t-1)$ is present through the inventory supply $p_1^{(t-1)} S_1^{m(t-1}/p_1^{(t)}$ and current money balances $M^{(t)}$. Finally, $S_1^{m(t)}$ depends on speculative price behaviors and, in particular, on $p_1^{(t)}$ and $p_1^{(t+1)}$, which is perfectly foreseeable.

To obtain a simpler dynamics, we can discard the influence of past prices, using a specific monetary assumption. The money balances evolution path is given by expression (7): at the beginning of period t, the difference $[M^{(t)} - p^{(t-1)} S^{(t-1)}]$ is equal to the difference between the debt of banks towards firm-owners and the debt of speculators towards banks. This difference thus corresponds to the external money of the economy. Only a part of this money is active; the other part is indeed saved by firm-owners. The external active money is equal to:

$$\beta M^{(t)} - p_1^{(t-1)} S_1^{m(t-1)}$$

We will assume that monetary authorities are able to keep this monetary aggregate constant:

$$\overline{M} = \beta M^{(t)} - p_1^{(t-1)} S_1^{m(t-1)} \tag{12}$$

If we combine (11) and (12) we obtain:

$$q_1^{m(t)} - q_1^{S(t)} = \alpha \frac{\pi^{(t)}}{p_1^{(t)}} + \frac{\overline{M}^{(t)}}{p_1^{(t)}} + S_1^{m(t)} - \min [\hat{q}_1^{P(t)}, \frac{\hat{q}_2^{P(t)}(1-a_{22}) - \underline{q}_2^m}{a_{12}}]$$

Stationary equilibria and dynamics

At the beginning of each period, producers revise market prices. They increase them when effective demand exceeds effective supply. They decrease them when an excess supply appears.

The price adjustment is then given by (13) where i is a given constant:

$$\frac{p_i^{(t+1)}}{p_1^{(t)}} = 1 + \lambda_i [\frac{q_i^{m(t)} - q_i^{S(t)}}{q_i^*}] \tag{13}$$

where q_i^* represents the level of production corresponding to the degree of normal utilisation of fixed capital, obtained when the uniformity of profit rates is reached, *i.e.*, $q_1^* = \gamma' - \gamma$ and $q_2^* = \gamma$.[9]

Since there is no discrepancy between supply and demand in the second industry, the initial market price of the basic good is also its stationary price:

$$p_2^{(t+1)} = p_2^{(t)} = p_2^{(0)}$$

By (13), the long-run price of luxury-goods can be reduced to a first order sequential evolution, the developed form of which is:

$$[\frac{\gamma' - \gamma}{\lambda_1}][\frac{p_1^{(t+1)}}{p_1^{(t)}} - 1] - S_1^{m(t)}$$
$$= \alpha \frac{\pi^{(t)}}{p_1^{(t)}} + \frac{\overline{M}}{p_1^{(t)}} = \min [\hat{q}_1^{P(t)}, \frac{\hat{q}_2^{P(t)}(1-a_{22}) - \underline{q}_2^m}{a_{12}}] \tag{14}$$

The right side of this expression is obtained by the confrontation of the final demand and the supply of good 1. According to (8) and (9), the effective level of transaction of luxury goods may be determined either by planned quantities, or by the reproduction rule, or by demand. The rank of these three terms defines the expression $\pi^{(t)}$ which depends on the level of

[9] The only purpose of this choice is to avoid non linearites coming from the normalisation of excess demands. Another constant may be substitued for q_i^* without any loss of generality.

effective transactions in the two sectors. Then, the form of expression (14) is ultimately related to the value of the short side in the market of luxury goods and to the value of $S_1^{m(t)}$ as defined by (1). A general study might be very complex and would probably reveal some causes of global instability, depending on the value of the parameters.[10] We only concentrate here on the existence of some dynamic paths, initially exhibited by G. Laroque in previously mentioned works, and typicaly associated with speculative behaviors. Let us consider some values of the parameters such that supply and demand are equal when planned quantities are shorter than the level of production of good 1 determined by the reproduction rule. In this case, on one side and the other of this equilibrium, the effective level of transaction depends on demand and from the planned quantities. When deman is shorter than supply and $S_1^{m(t)} = 0$, the evolution is given by the following expression:

$$p_1^{(t+1)} = (1 - a)p_1^{(t)} + b \qquad (15)$$

with:

$$a = \frac{\gamma' \lambda_1}{\gamma' - \gamma}$$

and

$$b = \left(\frac{\lambda_1}{\gamma' - \gamma}\right) \left[\frac{\alpha q_2^m p_2 + \bar{M}}{(1 - \alpha)} + \gamma \left(\frac{a_{22} p_2^{\circ}}{a_{12}}\right)\right]$$

The stationary equilibrium of this evolution is $p^*_1 = b/a$. A necessary and sufficient condition of monotonic stability is $\gamma' > \gamma/(1 - \lambda_1)$. This inequality is always satisfied by our assumptions on γ' and γ. When inventories are $\bar{S}_1$ and demand shorter than supply, the dynamics becomes (16).

$$p_1^{(t+1)} = (1 - a + cS_1^{(t)})p_1^{(t)} + b \qquad (16)$$

$$\text{with } c = \frac{\lambda_1}{(\gamma' - \gamma)(1 - \alpha)}$$

Stationary equilibrium is given by $p^*_1 = b/(a - cS_1^{(t)})$.[11]
Necessary and sufficient conditions of monotonic stability are:

$$\bar{S}_1 < \gamma'(1 - \alpha)$$

[10] Then, one can remark for example that system (14) is instable when the level of transaction is determined by demand and the level of supply by the reproducibility rule.

[11] The positivity of this expression is obtained when $\bar{S}_1 < \lambda_1 \gamma' (1 - \alpha) / (\gamma' - \gamma)$: the interpretation is that the maximal admissible inventory capacity is an increasing function of the normal level of production of luxury goods and a decreasing function of the normal level of production of basic goods.

and

$$\gamma' > \frac{\gamma - \frac{\lambda_1 - 1}{(1-\alpha)}}{(1-\lambda_1)}$$

By these conditions, the maximum admissible inventory capacity is dependent from the normal levels of production of both sectors, according to intuition. When the level of inventories is more than 0 and less than $\bar{S}_1$, the Evolution is similar to (16) after substitution of $\bar{S}_1$ by the relevant level of inventories.

When supply is shorter than demand with $S_1^{m(t)} = 0$, the evolution becomes (17):

$$p_1^{(t+1)} = [1 - (1-\alpha)a]p_1^{(t)} + (1-\alpha)b \tag{17}$$

The stationary equilibrium is still $p^*_1 = b/a$. The necessary and sufficient condition of monotonic stability is then $\gamma' > \gamma/[1-(1-\alpha)\lambda_1]$ which once again is always verified by our assumptions on γ' and γ. When inventories are $\bar{S}_1$, the evolution turns to (18):

$$p_1^{(t+1)} = [1 - (1-\alpha)(a - c\bar{S}_1)]p_1^{(t)} + (1-\alpha)b \tag{18}$$

The stationary equilibrium is still $p^*_1 = b/(a - cS_1)$.

The necessary and sufficient conditions of its monotonic stability are then:

$$\bar{S}_1 < \gamma'(1-\alpha)$$

and

$$\gamma' > \frac{\gamma - \lambda_1\bar{S}_1}{1 - \lambda_1(1-\alpha)}$$

A graphic exposition of these dynamics may be given by selecting relevant variables. We will choose $p_1^{(t+1)}/p_1^{(t)}$ for the horizontal axis and

$$\Delta = \alpha \underline{q}_2^m \frac{p_2}{p_1(t)} + \frac{\bar{M}}{p_1(t)} - (1-\alpha)\left[\gamma' - \gamma\left(\frac{a_{22}p_2}{p_1(t)}\right)\right]$$

for the vertical axis.[12] According to this, relation (15) - (16) - (17) - (18) turn respectively to the following expressions:

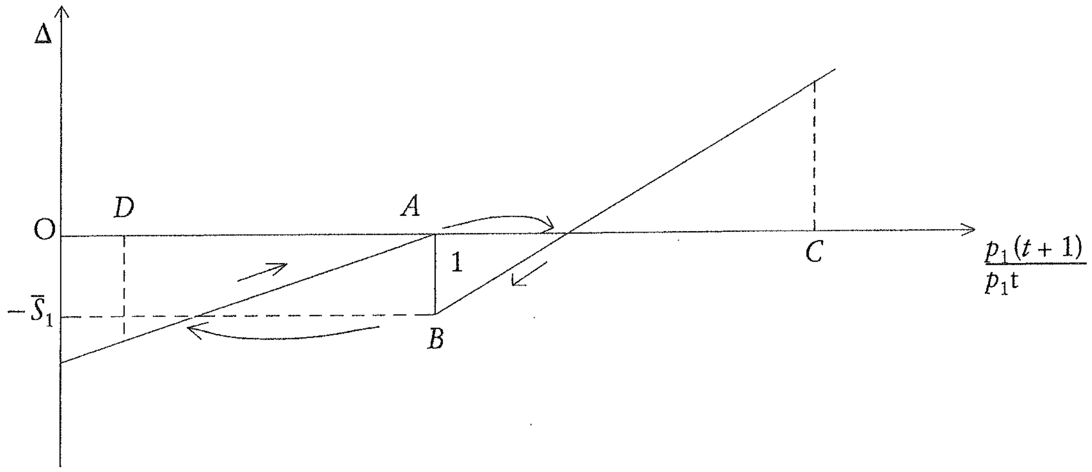
$$\Delta = (1-\alpha)\left(\frac{\gamma' - \gamma}{\lambda_1}\right)\left[\frac{p_1^{(t+1)}}{p_1^{(t)}} - 1\right] \tag{19}$$

$$\Delta = (1-\alpha)\left(\frac{\gamma' - \gamma}{\lambda_1}\right)\left[\frac{p_1^{(t+1)}}{p_1^{(t)}} - 1\right] - \bar{S}_1 \tag{20}$$

[12] This kind of graphic exposition was initially presented by G. Laroque in a previously mentioned paper.

$$\Delta = \frac{\gamma' - \gamma}{\lambda_1}\left[\frac{p_1^{(t+1)}}{p_1^{(t)}} - 1\right] \tag{21}$$

$$\Delta = \frac{\gamma' - \gamma}{\lambda_1}\left[\frac{p_1^{(t+1)}}{p_1^{(t)}} - 1\right] - \bar{S}_1 \tag{22}$$



Four straight lines would correspond in the plane $(p_1^{(t+1)}/p_1^{(t)}, \Delta)$, to the dynamics involved by each of the evolutions (19) - (20) - (21) - (22). However, only a part of these curves are relevant if we consider the dynamics of the economy as a whole, given by (16). When $p_1^{(t+1)}/p_1^{(t)} < 0$, *i.e.*, when the price of luxury goods decreases, the monotonic stability of the evolution grants us that demand is shorter than supply. Moreover, by (5), $S_1^{m(t)} = 0$. Then, at the left side of $(p_1^{(t+1)}/p_1^{(t)} = 1)$ on the horizontal axis, the only relevant evolution is (15). In a similar way, one can deduce that evolution (18) is the only relevant one when $p_1^{(t+1)}/p_1^{(t)} > 1$. Finally, when $p_1^{(t+1)}/p_1^{(t)} = 0$, Δ is given by the set of stationary equilibrium associated with $S_1^{m(t)} \in [0, \bar{S}_1]$.

Starting from some pair $(p_1^{(t+1)}, p^{(t)})$ like C, such that $p_1^{(t+1)}/p_1^{(t)} > 1$, Δ decreases, tending to its stationary value $(-\bar{S}_1)$ corresponding to the stationary equillibrium represented by B. Similarly, starting from some pair $(p_1^{(t+1)}), p_1^t)$ represented for example by D, such that $p_1^{(t+1)}/p_1^{(t)} > 1$, Δ increases, tending to the long run value 0, corresponding to the stationary equilibrium represented by A. Finally, all the points of the segment $[AB]$ are stationary equilibria in the absence of any noise in expectations.

By consideration of (14), it will be noticed that Δ represents a multiple of the excess demand the speculators are faced with before formulating their demand for inventories. If at A speculators expect a positive excess demand, they are induced to increase their demand for inventories from 0 to $\bar{S}_1$, which pushes the economy on the path converging to A. Finally, very small noises in speculators' expectations are sufficient for destabilizing the equilibria standing on [*AB*].

One of the originalities implied by the introduction of speculative behaviors is given by the dynamic typology of the model.

In the market-clearing of gravitation, we indeed only faced mathematical convergences or divergences. Now, it is clear that in the second model, a new type of dynamics is observed. This dynamics indeed corresponds to a cycle including two alternative equilibria. However, this "cyclical" process of gravitation is not entailed by the presence of non linearities, but only by the effects of expectations.

As in the market-clearing model, the notion of reproduction plays here an important role. This role emphasises the necessity for studying more thoroughly the impact of macroeconomic reproduction constraints on microeconomic dynamic processes, within the classical framework. This impact is an additional proof of the dependency of short-run and long-run changes.

Speculative behaviors are taken into account in the model and we have just observed how much they help to characterize the nature of the evolution of economic magnitudes. Errors in expectations appear to be strongly destabilizing. This remark reinforces the interest in a more systematic investigation of microeconomic behaviors in gravitation models.

In the model, inventories are not always involuntary. Therefore, a new type of market adjustment is described which differs from both market-clearing and market-shortage systems of exchange. This confirms the variety of market institutions which may be introduced within a classical framework. It is therefore useful to try to build a typology of these institutions.

3. CONCLUSION

The apparent symetry between both models — one implying market-clearing and the other allowing the occurrence of inventories — might convince the reader that this contribution tries to recover the equilibrium/disequilibrium dichotomy of the traditional general equilibrium analysis, within the classical framework. Such a conclusion would be misleading. We do not indeed claim that our models offer a representative typology of exchanges within the classical approach. Our purpose is rather to use then as significant illustrations of the difficulties and the possible issues of gravitation theory.

On the one hand, both models confirm that the gravitation problem is not self-evident. Gravitation may be impossible to obtain and it may appear under very different forms of dynamics. The debate is therefore still necessary.

On the other hand, both models allow us to understand that classical dynamics may be a more important and interesting topic than gravitation processes. This remark is especially true within the second model. From this point of view, the necessary debate we just mentioned cannot be limited to the gravitation problem. It must concern the whole range of classical dynamics. In this perspective (see R. Arena (1990)), the classical approach can still be useful for understanding to-day's economic problems.

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Appendix

Let us demonstrate that the inventory model, the money-balance increase is always equal to the increment in the speculators' debt, that is identity (7) of the text.

Let $\bar{q}_1^{(t)}$ be the level of transactions in luxury goods within period t.
$q_1^{P(t)}$, the supply of luxury goods produced within period t
$\bar{q}_1^{P(t)}$, the level of transactions of luxury goods produced within period t
$\bar{q}_2^{(t)}$, the level of transactions of good 2 within period t
$\bar{p}_1^{(t)}$, the price of commodity 1 within period t
$\bar{p}_2^{(t)}$, the price of commodity 2 within period t

The value of luxury goods transacted within period 1 can be obtained in the following way, according to the supply side:

$$\bar{q}_1^{(t)} p_1^{(t)} = \bar{q}_1^{p(t)} p_1^{(t)} + \hat{S}_1^{(t)} p_1^{(t)}$$

From the demand side, this value can also be obtained:

$$\bar{q}_1^{(t)} p_1^{(t)} = \underline{q}_1^{m(t)} p_1^{(t)} + S_1^{m(t)} p_1^{(t)}$$

Then,

$$\underline{q}_1^{m(t)} p_1^{(t)} = \bar{q}_1^{P(t)} p_1^{(t)} + \hat{S}_1^t p_1^{(t)} - S_1^{(t)} p_1^{(t)} \qquad (A)$$

The left hand side of this equation represents firm owners' purchases of luxury goods. These purchases are equal to distributed total profits minus the sum of profits spent in industry 2 and of the increase of money balances.

Let $\pi_1^{(t)}$ be the profits distributed in industry 1
$\pi_2^{(t)}$ the profits distributed in industry 2
$\pi_2^{S(t)}$ the profits spent in industry 2 for final consumption
$C_1^{(t)}$ the value of advanced capital of industry 1.

The distributed profits of industry 2 result in the following identity:

$$\pi_2^{(t)} = \pi_2^{S(t)} + C_1^{(t)} \qquad (B)$$

The expenses of firm-owners in industry 2 are equal to their profits minus their expenses in industry 1 for final consumption, minus their money-balance increase, i.e.:

$$\underline{q}_1^{m(t)} p_1^{(t)} = [\pi_1^{(t)} + \pi_2^{(t)}] - \pi_2^{S(t)} + [M(t+1) + M(t)]$$

that is from (B):

$$\underline{q}_1^{m(t)} p_1^{(t)} = \pi_1^{(t)} + C_1^{(t)} + [M(t+1) - M(t)] \qquad (C)$$

Since $\overline{q}_1^{p(t)} p_1^{(t)}$ corresponds to the profits of industry 1 increased by the value of its advanced capital, the right hand side of equation (A) can be re-written:

$$\begin{aligned} &\overline{q}_1^{p(t)} p_1^{(t)} + \hat{S}_1^{(t)} p_1^{(t)} - S_1^{m(t)} p_1^{(t)} \\ &= C_1^{(t)} + \pi_1^{(t)} + \hat{S}_1^{(t)} p_1^{(t)} - S_1^{m(t)} p_1^{(t)} \end{aligned}$$

From (A) and (C), we can deduce:

$$M(t+1) - M(t) = S_1^{m(t)} p_1^{(t)} - \hat{S}_1^{(t)} p_1^{(t)}$$

From expression (6) of the text, identity (7) derives:

$$M(t-1) - M(t) = S_1^{m(t)} p_1^{(t)} - S_1^{m(t-1)} p_1^{(t-1)} \qquad (7)$$

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