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5 **Mauro Caminati** and **Fabio Petri**, Preface

11 **Mauro Caminati**, Gravitation: An Introduction

Part I

45 **Richard Goodwin**, Inaugural Speech

47 **Luciano Boggio**, The Dynamic Stability of Production Prices: A Synthetic Discussion of Models and Results

59 **Marco Lippi**, Production Prices and Dynamic Stability: Comment on Boggio

69 **Ian Steedman**, Questions and Suggestions re Gravitation

73 **Peter Flaschel**, Cross-Dual Dynamics, Derivative Control and Global Stability: A Neoclassical Presentation of a Classical Theme

93 **Michio Morishima**, Comment on Flaschel

Part II

95 **Andrea Salanti**, The Notion of Long-Period Positions: A Useful Abstraction or a "Platonic Idea"?

103 **Alessandro Roncaglia**, Is the Notion of Long-Period Positions Compatible with Classical Political Economy?

113 **Sergio Parrinello**, Some Reflexions on Classical Equilibrium, Expectations and Random Disturbances

125 **Cristian Bidard**, From Arrow-Debreu to Sraffa

139 **Bertram Schefold**, Joint Production, Intertemporal Preferences and Long-Period Equilibrium. A Comment on Bidard

Part III
165 **Richard Goodwin,** Convergence to Strange Long-Period Positions
175 **Ingrid Kubin,** Market Prices and Natural Prices: A Model with a Value Effectual Demand
193 **Willi Semmler,** On Composite Market Dynamics: Simultaneous Microeconomic Price and Quantity Adjustments
221 **Dominique Torre,** On Composite Classical and Keynesian Microdynamic Adjustment Processes: A Comment
229 **Gérard Duménil** and **Dominique Lévy,** Stability in Capitalism: Are Long-Term Positions the Problem? With an Addendum
279 **Jean Cartelier,** The Stability Problem in Capitalism: Are Long-Term Positions the Problem? A Comment on Duménil and Lévy
287 **Richard Arena, Claud Froeschle** and **Dominique Torre,** Gravitation Theory: Two Illustrative Models
309 **Giancarlo Gozzi,** On Gravitation from the Classical Viewpoint: A Comment on Arena, Froeschle and Torre
317 **Ulrich Krause,** Gravitation Processes and Technical Change: Convergence to Fractal Patterns and Path Stability
329 **Pierangelo Garegnani,** On Some Supposed Obstacles to the Tendency of Market Prices towards Natural Prices

"Gravitation: an introduction"*

Mauro Caminati

The papers which are published in this volume, though all related to the main theme of gravitation and classical price theory, touch upon a wide range of issues. The present introduction attempts to provide a unifying framework, with the aim of making connection between these issues more explicit. It may also be of some help to those readers, which are unfamiliar with the more formal literature on gravitation. The essential line of argument is sumarized below.

Section 1 is concerned with the presentation of two main points of divide. Is stability necessary to relevance of production prices? To what extent do mathematical models of short-period behaviour provide solid ground for the derivation of stability, or instability, arguments? On the one hand entrepreneurs' reactions to undesired positions (reactions to undesired positions are often referred to below with the expression "adjustment behaviour") are not purely erratic. On the other hand, our understanding, and to some extent also the very nature, of adjustment behaviour, does not elicit the formulation of "perfect", fully specified reaction rules; for this reason, the stability results obtained often remain partly undetermined. Section 1. is thus concerned with the difficulties posed by the above situation, and with different ways of answering them.

Section 2 contains an intuitive presentation of different mathematical notions of stability, as they are applied to study the stability of prices of production. When, as is mostly the case, notions of local stability are used, the analysis is concerned with the stability of a *given* long-period position (LPP henceforth; for a precise definition, see below, section 1.), and assumes that the state of the system is sufficiently close to it right from the beginning. The limits of this approach, as a foundation for the method of LPPs, are briefly discussed.

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Section 3 is an informal introduction to the structure of gravitation models of classical inspiration. While the classical adjustment process, described by Smith, Ricardo and Marx, emphasizes the stabilizing influence on prices of output reactions to profit differentials, modern formulations of the same process also underline the demand feed-backs which follow from those reactions, and more generally, from the ongoing levels of market prices and outputs. Such feed-backs introduce complications and potential elements of instability.

A promising line of research is offered by the integration, in the classical adjustment process, of adjustment reactions, which are usually qualified as Keynesian. Firms react to excess demand, by changing output (not only prices), and their price decisions also depend on costs (not only on excess demand). If these Keynesian reactions are sufficiently pronounced, then demand feed-backs have a relatively slow influence on prices and profitability, and classical LPPs are shown to have stronger stability properties. One is naturally led to ask if, and which Keynesian reactions are "sufficiently pronounced" in the real world. It is argued that the influence of cost on price formation is likely to increase, relative to the influence of demand, as the deviation of market from normal price increases. This leads quite straightforwardly to the boundedness of deviations of market from natural prices, even in those situations where the latter are locally unstable.

1. LONG-PERIOD POSITIONS AND STABILITY

1.1.

Central to the classical view of competition is the notion of capital mobility:[1] in its search for maximum profitability, capital flows from sectors where the rate of profit is lower, to sectors where the rate of prtofit is higher. Hence, rates of accumulation across industries depend, *ceteris paribus*, on (rates of) profit differentials.[2]

Closely related to this view of competition, is the notion of a classical long-period position (LPP) of the economy.[3] This is, broadly speaking, a state of the system where the driving forces of classical competition are at rest. More precisely, a classical LPP is defined, in this paper[4] as a given productive technique, a given state of distribution (*e. g.*, a given real wage rate), and an associated set of relative commodity prices (production prices),

[1] Cf. EATWELL [23].

[2] A slightly weaker proposition holds true in a fixed capital environment with irreversible investment, if the irreversibility constraint bites.

[3] Cf. GAREGNANI [35].

[4] RONCAGLIA [67] refers to a stronger definition, which includes stability.

such that the rate of profit is uniform across industries. The definition must also include, at least implicitly, a set of given industry outputs, namely, those outputs consistent with the use of the given technique.[5]

Price determination, within classical LPPs, makes no reference to price-elastic demand functions of consumers, but is based on given technical conditions and the exogenous determination of a distributive variable. In this context, price detemination is allowed by the assumption that the given technique, which describes the productive methods *in activity*, is such that the number of processes is equal to the number of (produced) commodities: productive systems are square. The assumption need not be justified in single product systems, but a justification is required with joint production. The problem of squareness is discussed, from opposite stand-points, in the paper by C. Bidard, and in the extensive comment by B. Schefold.[6]

1.2.

But, to what an extent is stability necessary to the relevance of production prices?

According to a first view, existence, and nothing more, is required. The idea is that the pre-eminence accorded to classical competition, makes the notion of production prices necessary in order for the economic process to be intelligible. The argument, as we understand it, is that production prices reflect the influence of the main *systematic forces* at work within capitalistic economies: namely, technology and distribution. These forces act systematically on prices, despite the possibly non-persistent nature of any given state of technology and distribution. We are confronted, here, with a claim which asserts only the *qualitative* persistence of the main forces which act on prices; this is sufficient, according to this view, to the relevance of production prices: technology and distribution are given at any given instant of time, and the corresponding long-period prices can be computed.[7] The long-period nature of such prices should not be identified with their persistence, or with the property of attracting market prices; hence, the issue of their stability is misplaced. This position is re-stated and further

[5] Cf. Sraffa [77], Garegnani [35] and [36]. It may be worth observing that in the neoclassical framework of intertemporal equilibrium, long-period questions can be only adressed by considering those situations where the asymptotic properties (*i. e.* properties holding at dates which are indefinitely far from the initial date) of the equilibrium come to be independent of initial endowments. Quite instructive results have been recently, and not so recently, derived from this type of perspective. One can study, for instance, if, or under what conditions, prices at the terminal date would converge to prices of production (Cf. Duménil and Lévy [16], Dana, Florenzano, Le Van and Lévy [14] and [15]). The same perspective is also adopted, with interesting implications, in Bidard [7] and in Schefold [71]. The latter argues that reswitching and perverse Wicksell effects are not ineffectual in modern neoclassical theory, when the problem of convergence to such long-period states is considered.

[6] Cf. Bidard [7], Schefold [71].

[7] Cf. Roncaglia [66].

clarified in Roncaglia [67]. It may be worth noticing how prices of production become, in this conception, a highly-notional construct, and do not lend themselves to a realistic (in a methodological sense) interpretaion.[8]

According to a second view, the relevance of LPP requires gravitation, either in the form of an axiom, or in that of a verifiable proposition. In both cases (and in contrast with the former view), some *persistence of the data*, such as to allow for gravitation, is involved. The relevant term of comparison is, in this case, the speed, wherewith market magnitudes are attracted by long-period magnitudes. Persistence of the data should not suggest that LPPs must be identified with stationary states, or, more generally, with states of steady growth.[9] Indeed, the attribute of persistence, must be referred to technology and distribution, rather than to normal output. But if the normal level and/or composition of output is not constant, *locally-constant* returns are required to make sure that the given technique has a sufficient degree of persistence. Alternatively, locally constant returns can be regarded as an acceptable abstraction, if changes in long-period output are sufficiently slow.

The attempt to provide theoretical support for the gravitation hypothesis, leads quite naturally to consider the LPP as the *rest point* (*i. e.* equilibrium point, in the language of dynamical systems) of a dynamic adjustment process starting from arbitrary conditions (possibly, close to it). We may observe two quite immediate consequences of this approach.

We note, first, how a specification of the adjustment process entails a specification of long-period output growth. In so far as the stability of an LPP is posed as a problem of the stability of a *rest point*, long-period accumulation must be identified with balanced growth. This is quite obvious, since the adjustment to an LPP involves changes in quantities (beside prices), and, by definition, the rest point of the adjustment process is stationary (time invariant), under the forces which drive the adjustment. Under a suitable formulation, the vector of long-period quantities is allowed to change, but only up to a multiplicative scalar, which identifies the rate of balanced growth. As was observed before, this identification of LPPs with steady states is unnecessary, from an economic point of view (and, to some extent, also from a mathematical one; see Krause [48]). On the one hand, the stability of LPPs would be more correctly posed as a problem of path-stability, where short-period magnitudes adjust towards a moving position. On the other hand, balanced growth can be regarded as a first-step approximation; this would be legitimate only to the extent that changes in long-period *proportions* are sufficiently slow, if compared to the speed of convergence of short-period magnitudes.

Our second point is that mathematical models of gravitation are

[8] Cf. Salanti [68].

[9] The point is made quite explicitly in Schefold [71], which is published in this volume.

confronted with a puzzling dilemma between the scope of the analysis and the definiteness of its conclusions. The dilemma arises because of the difficulties faced by economists, when they try to identify solid foundations for a specification of adjustment rules, *beyond a certain degree of approximation*. It is mostly the case that reaction rules, which are only broadly specified, are consistent with quite different stability results. Unumbiguous results depend, then, on more severe restrictions, which may lack a convincing economic justification. It is also the case that there are relevant characteristics of the real-world (*e. g.* storable *versus* non-storable goods), which lead to different forms of adjustment, with different stability properties.[10]

In view of the above problems, and of others we shall mention below, most contributors seem to agree that what is at stake is *not* a *perfectly general*, mathematical foundation of the idea of gravitation. B. Schefold suggests that, in so far as gravitation is necessary to the relevance of LPPs, it should be taken as an *axiom*.[11] When considered in this perspective, the proper aim of the on-going theoretical research on gravitation, is not to prove or disprove, but to "influence the... credibility" of the axiom.[12] The same remark would apply, of course, to any empirical test of gravitation. Significant convergence to the above conclusion, should be regarded as a first, and non-negligible, outcome of the workshop.

Hypothetically definitive proofs of convergence to LPPs, are also questioned on a different ground; Parrinello [62] (also quoted with approval in Roncaglia [67]) points out that they would lead to the following dilemma: in so far as a "perfect" short-period model is at all available, LPPs would be a redundant theoretical construct, *i. e.* a non-necessary approximation.

Leaving aside the quest for simplicity and analytical tractability,[13] the need for approximation still remains, in view of the presence of accidental disturbances. These are only abstracted from by deterministic models of gravitation, but must be introduced, at a later stage, at least. When considered in this perspective, the stability (in a sense to be specified below) of the systematic (deterministic) component of adjustment behaviour, simply implies that deviations from long-period magnitudes remain bounded all the time, if accidental disturbances are bounded. It may be worth adding that, even where accidental disturbances satisfy some regularity assumptions (for instance, their mean value is zero), the average value of a variable, over time intervals of arbitrary length, does not, in general, coincide with its long-period value. This is true also for infinitely long time intervals,

[10] See below, sections 3 and 4.

[11] For an explicit statement in this sense, cf. SCHEFOLD [69] and [70].

[12] Cf. BOGGIO [12].

[13] We refer here to the fact that approximations may still be useful, and in some case necessary, with the aim of reducing the complexity of higher-order problems.

and is a consequence of the non-linearity of the classical adjustment process,[14] in its general form. It would seem that stable deterministic models of gravitation, however "perfect" they may be, are complements of, rather than substitutes for, long-period models.

The explicit consideration of accidental disturbances, joined to a pessimistic attitude as to the possibility that deterministic models of gravitation may find a way out of the dilemma "generality versus definiteness and/or uniformity of results", leads S. Parrinello to suggest that the whole issue of gravitation should be approached in a quite different way. The suggestion is to abstain from any formulation of adjustment paths; a LPP should *only* be conceived as a statistical concept, defined in terms of a *long-period* (stationary) *distribution* concerning the states of the economy, much in the same way that equilibrium is defined in statistical, rather than in classical mechanics. In other words, and in sharp contrast with our previous observation, the long-period values of prices and quantities would be defined by their mathematical expectation corresponding to a given distribution: the underlying assumption is, of course, that short-period price-quantity realizations, *under normal conditions*, would obey the given distribution.[15]

The emphasis on a normal distribution of short-period positions corresponds to the economically plausible intuition that there may be deviations from LPPs which would be regarded as part of a *normal state of affairs*. Ths notion leads S. Parrinello to elucidate an important point. namely, a certain degree of randomness is already embodied in the economic fundamentals (technical conditions, for instance), which characterize a LPP (*e. g.*, productive methods in use would take into account that a certain pattern of output fluctuations is recurrent).

One may well agree with the notion of a normal pattern of accidental disturbances, while rejecting at the same time the idea[16] that short-period behaviour is purely erratic. The observed volatility of this behaviour may rather be the joint outcome of systematic and of random components. As is argued in Garegnani [34], Smith's formulation of the classical adjustment process suggests that price and quantity reactions to market conditions are subject to definite sign restrictions, but elude any exact quantitative specification, in so far as they depend partly upon accidental factors. The relation between market conditions at time t and those at time $t + 1$, namely, $\mathbf{x}_t$, $\mathbf{x}_{t+1}$, is then expressed, in mathematical language, by a generic function $f()$, whose argments are x_t and the random variable u_t, summarizing the impact of all accidental factors: $\mathbf{x}_{t+1} = f(\mathbf{x}_t, \delta u_t)$, where

[14] Cf. Franke [31].

[15] In this way, questions concerning the determinants, characteristics and stability of LPPs should be more correctly referred to the underlying long-period distribution (technology and the exogenous distributive variable would, presumably, represent structural parameters of the distribution). In particular, the issue of gravitation is posed by asking whether, after the influence of *abnormal* shocks, a distribution would converge to its long-period configuration.

[16] There is no implication, here, that the idea in question is shared by S. Parrinello.

the real parameter δ determines the intensity of accidental shocks. If considered in this perspective, deterministic models of gravitation correspond to the particular case, where $\delta = 0$. They are concerned with a preliminary, but necessary, analysis of the systematic components of adjustment, and pave the way for a more complex analysis, where systematic and random components are simultaneously dealt with.

2. LPPS, ADJUSTMENT BEHAVIOUR AND STABILITY: SOME PRELIMINARY ISSUES

2.1.

Following Arena and Torre [5], it is useful to distinguish between at least two different ways of approaching the overall issue of stability of LPPs.

2.1.a. On the one hand, one can consider the stability of *given* LPPs; in this case, long-period magnitudes (in particular, the methods of production in use at the long-period output and the real wage) feature among the data of the analysis.

The possibility of *changes* in long-period magnitudes is not denied, but is made harmless, through the assumption that these changes, if continuous through time, are "sufficiently slow", relative to the speed of convergence of the variables to their normal configuration;[17] where these changes are drastic, the assumption, then, is that they are infrequent (*una tantum*), so that they do not normally interfere with gravitation.[18]

In so far as the approach *2.1.a.* is concerned with the stability of *given* LPPs, the possibility of changes in long-period magnitudes, which are endogenous to the process of gravitation, is ruled out. This is not equivalent to denying any influence of adjustment behaviour on long-period magnitudes. To see this, let us assume a given distribution of accidental shocks (e. g. on output: the outcome of production decisions is partly random); as explained above, the long-period distribution concerning the states of the economy would be the joint outcome of the following three groups of factors: the given long-period data (*e. g.*, thechonology and the real wage), the given distribution of shocks and entrepreneurs' adjustment rules. The point is that there should be no implication that these groups of factors are mutually independent. For instance, if we admit adaptive forms of learning, desired stocks per unit of output (hence, the desired technique) may not be independent of the observed pattern of output,[19] and the determinants thereof. Still, no *changes* in long-period magnitudes

[17] Cf. GAREGNANI [35].

[18] Cf. *ibid.*

[19] In this case, the desired technique may not be cost-minimizing in the standard sense (a point which is also insisted upon by Parrinello's paper in this issue).

are necessarily involved here; what is involved is rather a *consistency requirement* of the long-period data, among themselves, and with the other determinants of the observable, normal state of affairs. This consistency requirement, at least in the form af an assumption, appears to be a prerequisite of approach *2.1.a.*[20]

2.1.b. If, instead, we consider this consistency as an object of theoretical investigation, we must pose the more complex issue concerning the formation of LPPs. In this broader context, it may be highly misleading to abstract from changes in long-period magnitudes, which are endogenous to the process of gravitation.

A first source of these changes is related to the influence of factors, such as endogenous technical progress.[21] It is a widely held opinion, that there are processes of learning by experience, which tend to establish a link between the *cumulative* level of output and the improvement of best-practice techniques. Different adjustment paths may then give rise to persistent differences in technical conditions; in this way, the rest point of the adjustment (in our case, the LPP) may change, depending on which adjustment path is actually followed (*i. e.*, LPPs may become *path dependent*).

A second source of the changes referred to above can be identified in the possible dynamic interactions among long-period magnitudes. An exogenous change in one long-period magnitude, through one, or more, of the adjustment reactions, which are implied in the process of gravitation, may trigger changes in other long-period magnitudes. Mainwaring [53] refers to a change of the normal real wage, which, via modified demand conditions, affects normal output and methods of production. The latter influence may follow from non-constant returns to scale, as in Mainwaring [53], or from endogenous technical progress, or, eventually, from the fact that modified demand conditions give rise to a different pattern of normal fluctuations in output, hence, to a different *desired* ratio of capacity utilization (or of inventories to output).

When these types of phenomena are admitted, gravitation cannot be thought any more as a process which takes place around given magnitudes, which can be determined independently of the process itself. A theoretical investigation concerning the centre of gravitation of economic magnitudes seems to require, in this case, an explicit, and global consideration of short-period adjustments.

[20] As will be observed below, the suggestion seems to be that it can be misleading to consider the effects of *arbitrary* changes in long-period data, within the limits of this approach. For instance, a change of the normal real wage may bring about a persistent change in (the pattern of) output, and a consequent change of the *desired* technique; the effects on the normal rate of profit could not be ascertained within the traditional comparative-static framework, which takes the initial *desired* technique as given and unchanged.

[21] For a similar remark, cf. HARRIS [42].

2.2.

Almost invariably, mathematical models of gravitation can be located within approach *2.1.a.* above. The economy is assumed to be "sufficiently close" to its LPP at the initial date, a fact which makes the neglect of endogenous changes in long-period magnitudes quite natural. The strictly local character of the analysis is often already embedded in the very formulation of the adjustment process, when this assumes one or more of the following conditions: stocks of inventories are "sufficiently large", as compared to short-period excess demands; the technology is linear; entrepreneurs know the normal rate of profit r^* and the normal rate of growth g^*. In this situation, there seems to be no *further* loss of generality, when local methods and concepts are employed in the mathematical investigation.

Two main stability concepts appear in this context: that of *local stability* and of *local asymptotic stability*.

With reference to the deterministic dynamics generated by entrepreneurs' adjustment rules (*i.e.*, abstracting from accidental disturbances), the *local stability* of natural values $(\mathbf{q}^*, \mathbf{p}^*)$, with $\mathbf{q} \equiv$ output vector, $\mathbf{p} \equiv$ price vector (both $\mathbf{p}$ and $\mathbf{q}$ are row vectors), implies that market values $(\mathbf{q}, \mathbf{p})$ will stay arbitrarily close to the rest point of natural values $(\mathbf{q}^*, \mathbf{p}^*)$, provided they are "sufficiently close" to them at the initial date.[22]

It is worth observing that, as time proceeds, a bounded increment of the distance between market and natural values is allowed by the definition. In other words, dynamic stability alone, does not reflect the idea of convergence; the implication is that market prices, initially close to the sable (but not asymptotically stable) production prices, may end up indefinitely far, after the system has been repeatedly perturbed by accidental disturbances.[23]

Local asymptotic stability adds the requirement of (local) convergence to that of (local) stability.

The local nature of the above definitions lies obviously in the fact that initial conditions must be "sufficiently close" to the rest point $(\mathbf{q}^*, \mathbf{p}^*)$. The main reason, for using such local concepts, is that the analysis does not usually consider the original non-linear adjustment process, but a linear approximation thereof, which holds in a sufficiently small neighbourhood of the dynamic equilibrium.

It is quite clear that local asymptotic stability does not give unconditional foundations to long-period theorising.

[22] More precisely, for any neighbourhood V of (q^*, p^*), there is a neighbourhood U of (q^*, p^*), with U contained in V, such that, if the initial condition (q_0, p_0) belongs to U, then (q_t, p_t) is in V for all $t \geq 0$; cf. HIRSH and SMALE [43].

[23] Cf. LASALLE [51].

In the first place, if random disturbances are taken into account, we must make sure that their influence is so small that the economy is never displaced outside the region of attraction of the given LPP.

In the second place, we may wonder about the effects of *exogenous* changes in the long-period data of the theory. As is well known, the long-period method deals with such changes, by means of comparisons between the corrensponding LPPs. The local dimension of stability, then, requires that only sufficielty small, if not continuous, changes of exogenous data should be allowed in the analysis. For otherwise, after a drastic change in one long-period datum, the state of the economy may not be sufficiently close to the new LPP, and gravitation around this position would be at question.

Last, but not the least, we recall the existence of phenomena,[24] which do not make it possible to regard long-period magnitudes as determined prior to, and independently of, the process of gravitation. As was observed above, in all such cases, it is quite illegitimate to proceed by means of local approximations around *given* LPPs: the analysis must then outstep the limits of approach *2.1.a.*, in the direction of approach *2.1.b.* A suggestion of this type can be read between the lines, and occasionally quie above the lines, in a number of contributions; see, for instance, Arena, Froeschle and Torre [4], and Gozzi [40].

2.3.

Having just stressed some limitations of local-stability concepts, we now observe how relevant difficulties are, nevertheless, encountered in the proof of the local stability of classical LPPs. Section 3 will give an intuitive account of what appears to be the main source of these difficulties.

For the moment, we observe that these limitations and difficulties lead some authors to propose alternative stability requirements for the notion of LPP. In synthesis, and with great simplification, the suggestion (coming, more or less explicitly, from Steedman [79], Flaschel [28], Semmler [74], Goodwin [38], Krause [48]) is to allow for *local instability* of the LPP, provided that the state of the system is attracted by a *bounded* set (of states), which contains the LPP. In the local version, initial conditions are requied to be close to the bounded set. Of greater economic interest is, perhaps, the global version, which does not impose restrictions on initial conditions, and which is referred to below as global stability "in the large" (*i.e.*, concerning a bounded *set*, rather than a *point*)[25]. Most relevant is the fact

[24] See above, paragraph *2.1.b.*

[25] To be slightly more precise, the LPP would be required to belong to a *bounded* set W, such that deterministic trajectories, starting inside W, never leave W (W is *invariant*), and, deterministic trajectories, starting in a sufficiently small neighbourhood $U(W)$ of W, reach W in finite time. The global version requires, in addition, that *all* deterministic trajectories reach

that, contrary to what may be the case with local asymptotic stability, random disturbances, which are bounded, but not "small", would not inhibit gravitation, under global stability "in the large".[26]

Economic behaviour, leading to global stability "in the large", is sketched, with differences in emphasis, in Goodwin [38], Flaschel [28], Kubin [50] and Semmler [74].[27] In an extreme attempt at simplification, the basic common intuition is that there are reaction mechanisms which come plausibly into place, or start being effectively stabilizing, only when the system is somewhat far from its state of rest.[28]

The problem of gravitation is generally understood to refer to the dynamic properties of relative magnitudes: relative prices and relative quantities. In a decentralized economic system, however, entrepreneurs' decisions on prices and quantities act directly only upon the absolute magnitude of these variables; indeed, entrepreneurs do not (normally) have direct cotrol over relative magnitudes. For this reason, adjustment processes should be formulated, at a first stage at least, in terms of absolute magnitudes.

Indeed, the dynamics of absolute magnitudes is generally not without consequence for the dynamics of relative magnitudes: the stability of the latter may not be independent of the behaviour of the former. This implies that arbitrary restrictions on absolute magnitudes should be avoided, even if they take the apparently innocent form of a normalization,[29] possibly

W in finite time; this version corresponds to the mathematical notion of *strict set stability*; cf. K. G. Nishimura [61]. It may be worth warning the non-mathematical reader that the local version of the above notion, although reminiscent of the, equally local, notions of *attracting set* and of *attractor*, does not correspond to any of them. Broadly speaking, trajectories converge to an attracting set A, only as time goes to infinity. Hence, a sufficiently small neghbourhood of A, but not A itself, meets the conditions on the set W, stated above. An attractor is defined, instead, as an attracting set, which is indecomposable in the (broad) dynamic sense that there is a trajectory, which comes close to every point in the set. Cf. Guckenheimer and Holmes [41], pp. 33-37 and 255-7.

[26] Referring to the case of global orbital asymptotic stability it has been shown that, if random perturbations obey some regularity assumptions, deviations from the rest point of the unperturbed (deterministic) system are still bounded all the time, with probability one; cf. Klein and Preston [46], Kosobud and O'Neil [47]. The result does not require that disturbances be "sufficiently small", but only that their variance be bounded. Intuition suggests that these properties can be extended to other, more general, cases of global stability "in the large".

[27] A similar view was advocated by L. Punzo in his intervention at the workshop.

[28] A different mechanism, still leading to stability "in the large", is at work in Krause [48]: in this case, technical progress, or non-constant returns to scale, prevent convergence to a point; still, all trajectories end up arbitrarily close to each other, *i. e.*, there is convergence to the same motion, within a bounded set. In this as in many other situations of stability "in the large", deterministic trajectories within the stable set are highly irregular, in that they may exhibit a completely aperiodic motion. Situations where this motion is also highly sensitive to initial conditions, are often characterized as *chaotic*, a feature which makes such deterministic settings resemble the stochastic ones. A case of this type is considered in Goodwin [38].

[29] This may take the form of an invariance condition, which restricts prices and quantities to remaining within certain sets; cf. Fisher [26], Flaschel and Semmler [29].

justified by the purpose of directing the attention to the relative dynamics.[30]

The above remark also suggests that the macroeconomic dimension of adjustment processes is not irrelevant to the issue of gravitation. Indeed, what has been said concerning the possible distortion of stability properties caused by arbitrary invariance conditions, holds true, *a fortiori*, when the imposition of arbitrary macroeconomic conditions is considered. The assumption of Say's law is a case in point: in so far as gravitation is concerned with stability in relative magnitudes, macroeconomic dynamics is often "simplified" through the assumption of Say's law. Although this simplification may have its merits, it may also bring in distortions:[31] the stability of LPPs must also be (and has been, in part) investigated under more general macroeconomic assumptions.

Recent contributions by G. Duménil and D. Lévy, including the paper published in this issue,[32] are illuminating in this respect. The overall issue of stability of capitalistic market economies is split into a problem of proportions, as described, in particular, by the behaviour of relative prices and outputs, and a problem of dimension, as described, for instance, by the behaviour of the actual rate (as opposed to the target rate) of capacity utilization.

When Ricardo and Marx claimed the long-run stability of natural prices, they had clearly in mind a problem of proportions; indeed, they had conflicting views in so far as the other issue is concerned (*i. e.*, dimension).

The suggestion that comes from G. Duménil and D. Lévy is that, while there are parameter configurations which allow for the overall stability of the system, there are other, perhaps more plausible, configurations, which lead to stability in proportions and to instability in dimension.

3. CLASSICAL ADJUSTMENT PROCESSES

Reduced to their bare bones, modern reconstructions of the adjustment process described by Smith, Ricardo and Marx are as follows: changes in prices depend on excess demand (hence, on output), intersectoral changes in output depend on rate of profit differentials (hence, on prices).[33] An extreme case of the above descrption arises when price changes are

[30] To avoid arbitrariness, dynamic interactions between the normalized and the original system should be considered.

[31] On this ground one can agree with SCREPANTI [72], which contrasts the different premises to the analysis of gravitation that where offered by Smith and Ricardo on one hand, and by Marx on the other. The main reason is, of course, that Marx, unlike Smith and Ricardo, rejected Say's law.

[32] Cf. DUMÉNIL and LÉVY [21], [22].

[33] Here, the word "change" should be generally interpreted as "proportional change per unit of time".

instantaneous, and current *levels* of market prices are determined by a condition of zero excess-demand for commodities.

In so far as market-price formation is concerned, a crucial difference introduced by modern formulations of the classical adjustment process, concerns the way of dealing with demand. The classical economists did not provide precise rules for market-price *determination*; they confined themselves to stating that, *ceteris paribus*, a direct relation holds between the excess of the natural over the market price of a commodity, and the excess of the quantity of the same commodity, which is brought to the market, over the effectual demand.[34] This is defined by Smith as the quantity demanded at the natural price.

In the modern formulations, the supply brought to the market is not compared to *effectual demand*, but to *actual demand*. This can be defined as the demand which is generated, at the existing combination of prices and outputs, by the systematic forms of demand formation at work in the economy (that is, abstracting from accidental disturbances on demand). When the economy is not in its LPP, effectual and actual demand would generally differ, for a least two reasons:

In the first place, if market prices differ from natural prices, supply reactions to profit differentials would affect current flows of gross investment, and hence actual demand.

In the second place, the ongoing levels of prices and outputs tend to affect actual demand, because of income effects.

Some authors add, to the demand feed-backs just considered, others, which take place through substitution effects in the consumption component of actual demand.

The classical economists could not, and did not, fail to see that supply and effectual demand do not actually determine market prices.[35] The stated relation between supply and market price, given effectual demand, rests, so it would seem, on the implicit assumption, that the mechanisms, which rule the formation of actual demand on the market, are consistent with, and do actually enforce that relation. In this sense, classical economists came close to *assuming* the stability of natural prices.[36]

As we shall see, modern studies on gravitation do not share the same unconditionally optimistic view. By and large, this can be related to the above differences in the consideration of demand feed-backs.

An interesting unification of the (in)stability results, which follow from different specifications of the classical adjustment process, is offered in the paper by L. Boggio, presented at the Workshop, and in the abridged version,

[34] Cf. Smith [75], Book I, ch. VII; Ricardo [65], p. 119.

[35] This is also shown by the fact that effectual demand is defined as a point, rather than as a function (of prices) Cf. Garegnani [34].

[36] See below, 3.2.1.

published here.[37] The above presentation can be usefully integrated by, and in part juxtaposed to, that offered, in a highly condensed form, by G. Duménil and D. Lévy, in the addendum to their article;[38] the latter also contains an overview of the different integrations of the classical adjustment process.

For the benefit of those readers, who are unfamiliar with the mathematical literature on gravitation, the following section offers a synoptic and very informal introduction to the architecture of gravitation models of classical inspiration.

3.1.

The classical adjustment process, with gradual adjustment of prices and quantities, implies that price changes depend on output and output changes depend on prices; hence, this form of adjustment is usually called "cross-dual" in the literature, but the definition is to some extent misleading, because strict cross-duality holds true only if simplifications are introduced. In the general case, since present demand also depends on net investment, and this depends on the planned change in output, (proportional) price changes come to depend on both prices and quantities. The same holds true for (proportional) changes in output, if rate of profit differentials are computed as deviations from the *average* rate of profit, which depends, in its turn, on the composition of output.

Before proceeding with our presentation, ti may be worth reminding the reader of a previous remark, which stressed how, in the context of a stability analysis, technical reasons lead to the identification (as a first step approximation) of long-period paths with states of balanced growth. The long-period growth of output is then described by the uniform rate of growth g^*, which is mostly assumed to be strictly positive.

The classical adjustment process is sometimes simplified, by assuming that capitalists know the uniform, long-period rate of profit r^*, and that the rate of profit differentials are computed as deviations from r^*. The assumption makes proportional changes in output depend only on prices.[39] A further crucial simplification is obtained if current net investment (and hence current excess demand) is not related to the planned change in output (which depends on prices), but consists of a proportional increment of inputs, according to the steady-state rate of growth g^* (supposedly known). Proportional price changes come to depend only on current output. We may note that, in this way, market prices do not have an *immediate* demand feed-back, through net investment.[40]

[37] Cf. Boggio [11] and [12].

[38] Cf. Duménil and Lévy [22].

[39] This would not be the case, if profit differentials were computed as deviations from the average market rate of profit, which also depends on the composition of output.

[40] This aspect is quite crucial to the stability properties of these models; cf. Boggio [9].

With these simplifications, the adjustment process takes on a simple, strictly cross-dual form; for ease of reference, in the present paper this form is labelled *simplified cross-dual.*[41]

In addition to the simplifications just described, different specifications of the classical adjustment process arise as a consequence of the way in which the relation between plans and realizations is dealt with.

There are at least two aspects of this relation, which are important in this respect:

a) whether or not plans are always realized;

b) whether prices, at each date, are determined according to some slow-adjustment reaction (based on excess demand), or whether they are instantaneously adjusted, so as to enforce a condition which states that the full absorption of the existing output is partly for consumption, the remainder for the implementation of the *realized* increase in output (between the present and the next relevant date). In discrete-time models, if the production interval and the interval which separates two subsequent market dates are set equal to h, the condition is: $\mathbf{x}_t = \mathbf{A}\mathbf{x}_t + \mathbf{A}[\mathbf{x}_{t+h} - \mathbf{x}_t] + \mathbf{c}_t$, where $\mathbf{A}$ is the input-coefficients matrix, $\mathbf{x}$ is output vector and $\mathbf{c}$ the vector of consuption. In the above expression, $[\mathbf{x}_{t+h} - \mathbf{x}_t]$ is the realized increase of output, which may or may not coincide with the planned increase in output. If it coincides, then $[\mathbf{x}_{t+h} - \mathbf{x}_t]$ is a function of prices $\mathbf{p}_t$; also $\mathbf{c}_t$ is function of prices $\mathbf{p}_t$ (and of outputs $\mathbf{x}_t$), at least in so far as these affect capitalists' incomes, and a pattern of consumption expenditure out of these incomes is assumed. Prices may enforce the full absorption of output with or without the intervention of a rationing procedure: only in the latter case can they be unambiguously regarded as market-clearing prices; at these prices, intended demand coincides with supply, and plans with realizations.

In models of slow price adjustment, the price ruling at the next date[42] is set at the previous date, on the base of the existing excess demand. To the extent that plans are always realized, this is allowed by "sufficiently large" stocks of inventories, an assumption which is partly justified by the local character of the analysis. With some remarkable exceptions,[43] the presence of stocks of inventories is only implicit: their dynamics is not formalized and, most significantly, does not affect entrepreneurs' decisions.[44] For this reason, it is sometimes argued that the idea of plan realization, in disequilibrium models which abstract from inventories, is incorrect. These models should not be referred to actual disequilibrium processes, where entrepreneurs take decisions in a fully decentralized and sequential way.

[41] Cf. Flaschel and Semmler [29].

[42] Obviously enough, in continuous-time models current decisions are concerned with time-rate of changes in variables.

[43] Cf. Franke [32], Dumenil and Levy [19], [20].

[44] Models of this type include, for instance, those by Boggio ([9], [10], [11], [12]) and by Flaschel and Semmler ([29]).

They should be regarded, instead, as virtual processes, *i. e.*, a sort of *classical tâtonnement.*

Whatever the way of dealing with inventories (either explicit,[45] or implicit), the absence of any short-period full-absorption contraint, which acts through prices, makes it possible to abstract from direct substitution effects in demand.

In other models price formation is accomplished instantaneously, so that the full-absorption contraint is met (notice that abstraction from inventories is less arbitrary, in this case); this is allowed by the presence of direct substitution effects in demand. These models differ in the way plans, and investment plans in particular, are formed.

In so far as commodity prices respond *instantaneously* to intended-demand and supply conditions, and the commodity market is assumed to be stable, present prices represent short-period equilibrium prices:[46] they make sure that present planned demand for each commodity equals supply.

The path followed by the economy, starting from an arbitrary initial condition, can be now regarded as a sequence of short-run equilibria on the commodity market. Short-run-equilibrium prices at time t, are determined by output at time t (as decided at the previous relevant date), and induce (via profit differentials) decisions on the output to be produced at the next relevant date; in this way, output decisions provide the link between two successive short-run equilibria.[47]

Alternatively, realizations do not coincide with plans-except by a fluke: prices, at any date, do not respond instantaneously to the potential gap between the current supply and the demand, which is intended at these prices. A rationing procedure implements the full absorption constraint, with or without the intervention of an instantaneous price change. The rationing procedure is, to some extent, arbitrary: some models, notably, two models by H. Nikaido,[48] assume that intended consumption is always realized, and realized net investment is determined by net saving; in other

[45] We may observe, in passing, that the explicit introduction of stocks of inventories, allows for a form of adjustment, where current decisions depend, not only on current disequilibria, but also on disequilibria oserved in the past. Cf. DUMENIL and LEVY [22], *addendum.*

[46] Cf. ARENA, FROESCHLE and TORRE [3] and [4]; FRANKE [32].

[47] This occasionally suggests a certain analogy wih the neoclassical tatonnement in quantities (cf. MAS COLLEL [55]); in both cases, equilibrium is short-run in so far as the prevailing price structure determines an incentive to revise output decisions: rates of profit are not uniform, in the classical case; prices do not equal marginal costs in the neoclassical case. The analogy, however, is only very partial. This is not only because such neoclassical short-run equilibria are intertemporal, or because they involve a clearing of the markets for commodities and primary factors; a further, quite instructive difference is the following: non-notional interpretations of the sequence of short-run equilibria, do not raise particular problems in the classical framework, as the certainly do, on the other hand, in the neoclassical one, where the equilibrium, whose stability is to be ascertained, is defined with respect to a given initial endowment. The difficulty is, of course, that endowments would change in the course of a non-virtual adjustment process.

[48] Cf. NIKAIDO [59] and [60].

models, planned net investment is always realized, and workers' consumption absorbs the residual supply.[49] Prices behave so that plans, which are not realized in physical terms, are realized in money terms.[50]

The above presentation should make clear how demand feed-backs (*e.g.*, investment in "simplified" and "non-simplified" cross-dual models), and assumptions on market-price formation (*e.g.*, models with slow price adjustment, with market clearing, with rationing) give rise to different specifications of the classical adjustment process. Two strictly related problems arise, in this connection. In the first place, the different specifications do not always find a compelling theoretical justification.[51] In the second place, the results of the analysis have often proved to depend crucially on the choice (of specification) that had been made; hence, the need to obtain a better understanding of how the different results are related to the different choices, and to single out the more promising lines of research. A most significant and systematic contribution in this direction comes from Boggio [11] (also synthesized in Boggio [12]), and from the *addendum* to Duménil and Lévy [22].

3.2.

The introductory remarks of part 3 of this paper, drew attention to the fact that the explicit consideration of demand feed-backs bring potential elements of istability into the classical adjustment process. The present section is mainly devoted to a more detailed, but intuitively clear illustration of this fact.

3.2.1. The transfers of capital triggered by profit differentials have both supply and demand effects, but these effects do not fall upon the same date. In so far as current profit differentials affect current investment, current demand is also affected, while current output is obviously not. Thus, before having any stabilizing influence by increasing the supply of those idustries yielding higher than average (rate of) profits, transfers of capital have a demand effect, and there is no reason to expect this effect always to be stabilizing.

[49] Cf. Kubin [49] and [50].

[50] The latter type of rationing procedure displays better stability properties than the former, as is clarified by the work of I. Kubin, including the paper published in this issue (cf. Kubin [50]). I. Kubin offers a convincing intuitive explanation for this result: in Nikaido's models, the full absorption constraint interferes with the classical principle which makes capital transfers depend on profit differentials. One may add that, in Kubin's models, the full-absorption constraint operates through a substitution effect in consumption: the composition of workers' expenditure on consumption is fixed in money terms (see below). I. Kubin suggests that this view of rationing is more realistic than the alternative one (where investment plans are rationed), because the access to short term finance is easier for firms than for consumers (and workers in particular).

[51] The remark applies, in particular, to models with rationing and with market clearing.

We should also notice that, if we abstract from consumtpion,[52] relevant demand effects are related to gross investment; thus, they depend partly on the pace of aggregate accumulatin, partly on intersectoral capital transfers and partly on replacement investment. In so far as the long-period rate of growth g^* is assumed to be strictly positive, and the analysis is local (hence, market prices are close to natural prices), one can safely confine attention to the case where, despite capital transfers, net investment is strictly positive.

To show how demand feed-backs may bring potential elements of instability, we consider a system with two commodities (both basics), one circulating-capital, and one wage commodity, and where workers' consumption is included in the input matrix **A** (*i.e.*, the wage rate is fixed in physical terms and is payed *ex-ante*; the ith row of **A** shows the unit input requirements for the prduction of commodity $i = 1,2$); let $r_i(\mathbf{p})$ be the rate of profit on replacement costs in sector $i = 1,2$; $p \equiv p_1/p_2$ and $q \equiv q_1/q_2$ identify the price ratio and the output ratio, respectively, their long-period values being p^* and q^*.

Since there are only two commodities, $p - p^* > 0$ implies $r_1(\mathbf{p}) > r^* > r_2(\mathbf{p})$.

We may notice, in passing, that with three, or more, commodities, the analogue condition does not hold true, in general, and the deviation of market price from natural price does not provide unambiguous information on the deviation of market profitability from normal profitability. Speculations on the dynamic implications of this fact were first offered in Steedman [78]: is stability more problematic when the number of commodities is large? Steedman's arguments are critically discussed in Garegnani [37].[53]

In our two-commodities world, instead, the price ratio gives immediate information on (rate of) profit differentials, hence on the direction in which the output ratio must change: q increases, whenever $p > p^*$.

Each possible non-negative combination of p and q can be identified with a point in the non-negative orthant of a Cartesian plane (see fig. 1). The LPP is here represented by point (q^*, p^*); obviously enough, this identifies also Smith's point of effectual demand.

[52] Consumption is only implicit in models where capitalists do not consume and workers' consumption is incuded in the input matrix.

[53] This author observes how the ambiguity referred to above cannot arise for the commodity yielding the minimum market rate of profit: the market price of this commodity is necessarily lower than the natural price. Thus, the monotonic rise of the minimum market rate of profit is a sufficient condition for convergence to prices of production. The attempt to relax this very strong condition meets, however, with the problem of coming to grips with the demand feed-backs of supply decisions. To this end, the author does not resort to a specific dynamic model; he proposes, instead, a generalization of A. Smith's postulate concerning the effect on market price of a deviation between hte output brought to the market and the effectual demand for it. The nature of Smith's postulate, and its limits, will be considered below, in the simplified context of a two-commodity world.

Classical economists' ideas on market-price formation imply that, if $q_t = q^*$, then $p_t = p^*$, while, if $q_t > (<) q^*$, then $p_t < (>) p^*$. Occasionally, they state something more, namely, that, in addition to the above conditions, the absolute value of $(p_t - p^*)$ increases with the absolute value of $(q_t - q^*)$. We may notice, in passing, that these conditions are not equivalent to a full determination of market prices.

Let us consider an arbitrary initial condition $t = 0$, where $q_0 > q^*$, and, according to the above rules, $p_0 < p^*$. In so far as q tends to decrease, whenever $p_t < p^*$, and if we postulate that the time path of q is continuous, then q will eventually come arbitrarily close to q^*, so that p will converge to p^*. This shows that classical economists' ideas on market price formation come *close* to being a postulate on the stability of natural prices (see fig. 1).

The qualification is made necessary by the above proviso, on the continuity of the time path of q. If, on the contrary, only discrete changes in q occur, we cannot rule out the possibility that the discrete fall in the output ratio causes a jump from (q_0, p_0) to a point in the region I of fig. 1. Although this behaviour may still lead to convergence, through a sequence of jumps, of decreasing length, from region I to region III, and *vice versa*, it also opens the way to the possibility of divergence, through an analogous sequence of jumps, but of increasing length (see fig. 2.b). Thus, already in this simple setting, and abstracting from demand feed-backs, we can observe how stability may be more problematic, if only discrete changes in variables are allowed (see below).

In the same simple setting, demand feed-backs are now introduced. In order to preserve a closer analogy with the approach just considered, an instantaneous reaction of market prices to supply and demand conditions is assumed, first. The analogy consists in the fact that current market prices are formed as a consequence of the supply, which is currently brought to the market.[54] Irrealistic as it may be, the case of instantaneous market-price formation is worth considering in this introduction, because it allows for a more intuitive presentation of the role of demand feed-backs. With this aim in mind, we also hold to the assumption of market-clearing market prices, which is adopted in a numbere of influential papers on gravitation. The understanding of how demand feed-backs work, in this context, will then show when the market-clearing assumption is unwarranted, and why. In the latter case, the behaviour of the market-clearing price ratio $p(q)$ will be interpreted as a simple indicator of the nature of demand feed-backs.

[54] Thus, they are not *directly* influenced by market prices ruling at previous dates, as is the case with slow price adjustment.

The classical economist saw, however, that supply may not coincide with output, if goods are storable, and entrepreneurs are prepared to hold stocks of inventories (cf. SIMTH [75], BOOK I, ch. VII). We abstract, here, from this distinction.

Fig. 1

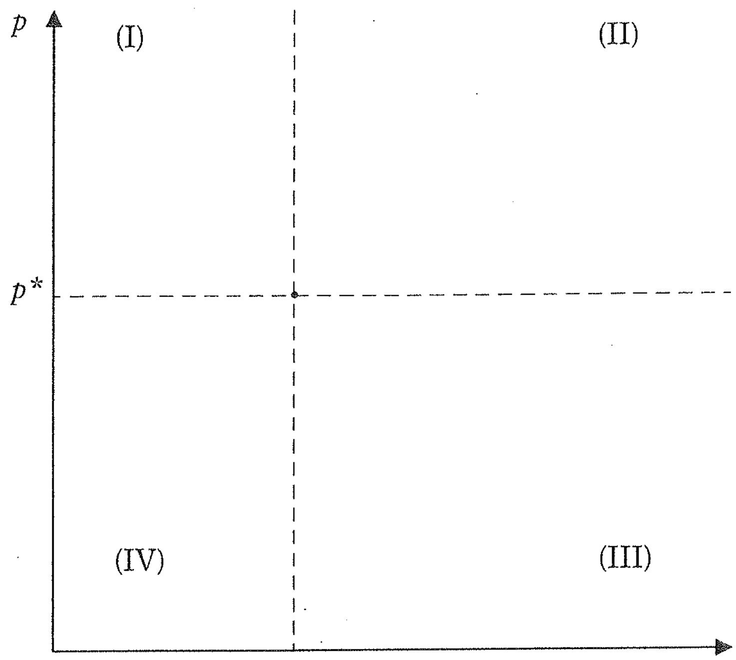


Fig. 2.a

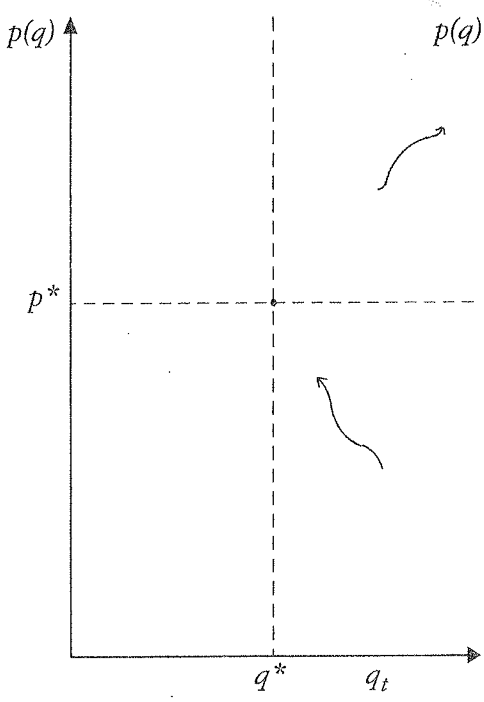
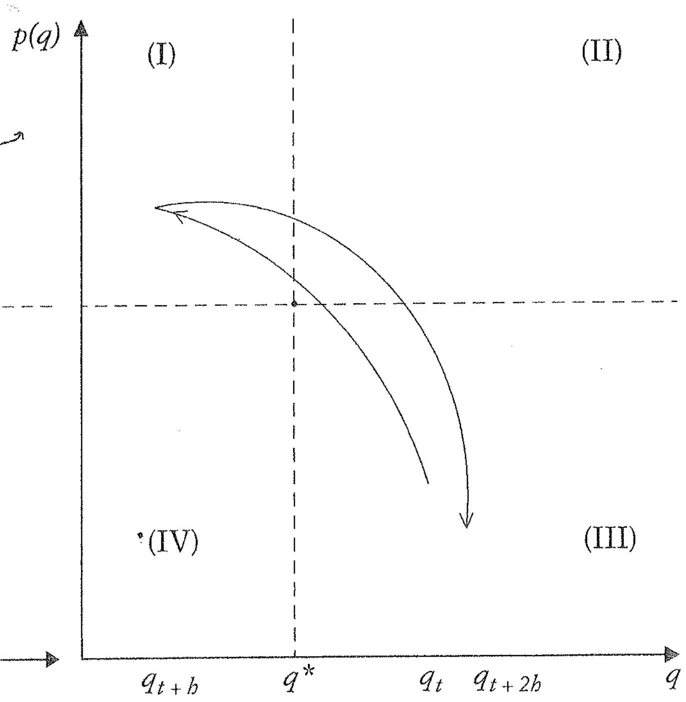


Fig. 2.b



Given an arbitrary output vector $\mathbf{q}_t$, prices $\mathbf{p}_t$ are such that, if capitalists do not consume, net output $\mathbf{q}_t$[I-A] equals *intended* net investment. This investment provides the inputs which are required by the desired change in output, hence it depends on current market prices, which determine profit differentials. But, in so far as prices $\mathbf{p}_t$ are market clearing, intended and realized investment coincide, and the same applies to desired and realized changes in output. The discrete and continuous time formulations of net investment at time t are then $[\mathbf{q}_{t+h} - \mathbf{q}_t]\mathbf{A}$, and $\dot{\mathbf{q}}_t\mathbf{A}$, respectively (where h is the length of the production interval, $\dot{\mathbf{q}}_t$ is the row vector of time derivatives $\delta q_i(t)/\delta t$ $i = 1, 2$, and both $[\mathbf{q}_{t+h} - \mathbf{q}_t]$ and $\mathbf{q}_t$ are functions of $\mathbf{p}_t$). In this way, $\mathbf{p}_t$ comes to be uniquely determined as a function of $\mathbf{q}_t$, and, likewise, p_t as a function of q_t: $p_t = p(q_t)$. Obviously enough, $p^* = p(q^*)$.

Assuming that the value of profits and net investment coincide, the market clearing prices $\mathbf{p}_t$ can be shown to exist, provided that: a) the sensitivity of the desired change in output with respect to price is more than infinitesimal; b) the output ratio q_t belongs to the interval, which is delimited by the input proportions in the two industries, namely a_{11}/a_{12}, a_{21}/a_{22}. The reason behind this restriction is, of course, that, since demand coincides with gross investment, the proportion in which the two commodities are demanded in the economy as a whole, namely, the gross investment ratio, would necessarilly belong to that interval, whatever is the desired change in activity, induced by prices. Provided that a_{11}/a_{12} and a_{21}/a_{22} are not equal, the function $p(q)$ can be shown to exist, to be well defined and continuous, at least in a *neighborhood* of q^*.

It is worth stressing that, if, as was the case above, $p_t < (>) p^*$, whenever $q_t > (<) q^*$, and if there are no (production or other) lags, then, again, p must eventually converge to p^*, for q would necessarily converge to q^*, following a time-continuous path. If, on the contrary, $p_t > (<) p^*$, whenever $q_t > (<) q^*$, prices induce the output ratio to move in the "wrong" direction; hence, *no matter whether there is, or there is not a production lag*, q would necessarily diverge from q^*, and p from p^*.

As it turns out, either the former, or the latter situation may occur, depending on the nature of demand feed-backs at work in the economy; in this simplified setting (recall that workers' consumtpion is included in A, and capitalists do not consume) they can be traced back univocally to the relative input requirements in the two sectors. In what follows, q is always assumed to be in a neighbourhood of q^*.

Proposition I: If $q > (<) q^*$, then $a_{11}/a_{12} > a_{21}/a_{22}$ implies $p(q) > (<) p^*$. If $q > (<) q^*$, then $a_{11}/a_{12} < a_{21}/a_{22}$ implies $p(q) < (>) p^*$ (see fig. 2.a, which refers to a continuous-time adjustment of q).

It is now time to emphasize, following Garegnani [37], how the market-clearing assumption may turn out to be arbitrary, in this setting. If $a_{11}/a_{12} > a_{21}/a_{22}$, the ratio between the demand for commodity 1 and the demand for commodity 2 is an increasing function of the price ratio p; except by a fluke, the market-clearing price ratio $p(q)$ would *not* be reached, in this case, as a result of competition between buyers and sellers. In other words, the virtual instantaneous tâtonnement (which determines market prices) would be unstable under standard tâtonnement rules.

The above remark questions the plausibility of the behaviour of $p(q)$, which is suggested by *Proposition I*; we should notice, however, that the concomitant instability of the LPP (p^*, q^*) is not called into question: the slightest deviation of the output ratio from its long-period level q^*, would instantaneously bring about (if $a_{11}/a_{12} > a_{21}/a_{22}$) a large (certainly non-infinitesimal) deviation of the price ratio from its long-period value p^*. A quick look at the definitions of paragraph 2.2. shows that the LPP (p^*, q^*) would be unstable (*i.e.*, would not be locally stable). This conclusion is confirmed if we consider another polar case of market-price formation, according to supply and demand, which is suggested by the literature. This is the case of a *slow* adjustement of market prices according to supply and demand. It is worth stressing how the slowness of the adjustment means that the current market price was decided upon at the last relevant date, through a correction of the market price ruling at that date, where the sign and size of the correction depended on the excess demand at the same date. Thus, Smith's postulate that $q_t > q^*$ implies $p_t > p^*$ is generally inconsistent with slow price adjustment.

The alternative just mentioned is examined, for instance, by L. Boggio ([9], [11] and [12]) and R. Franke ([32]). They show how, if profits + investment and workers' consumption is fixed in physical terms, the condition $a_{11}/a_{12} > a_{21}/a_{22}$, in systems with two base commodities, or an equivalent condition in systems with n base commodities, leads to the instability of production prices.

The above discussion suggests to interpret function $p(q)$ of *Proposition I* as a simple indicator of the demand feed-backs of supply decisions. The finding that $p(q) > p^*$ if $q > q^*$ means that, given the reaction of planned output to profitability, technical conditions of production cause the relative demand for one commodity to increase with the relative price of the same commodity. If this is the case, factors of instability are introduced by the assumption that output and demand conditions react only on prices (no role being allowed for Keynesian effects on quantities), and, conversely, that market prices are adjusted only according to the state of output and demand (no role being allowed for target-profit considerations).

Proposition I draws the attention on the possibly "ill-behaved" nature of demand feed-backs (of output decisions), which follow from technical conditions of production. The next proposition underlines, instead, the elements of local instability, which may follow from a discrete-time representation, even if demand feed-backs are "well behaved" (in which case the market-clearing price ratio can be stable).

It is worth stressing that the (local) instability result, which is stated in the following proposition, carries over to the case where market prices are non-instantaneously adjusted according to supply and demand conditions (cf. Boggio [9], [11] and [12]; Franke [32]; Duménil and Lévy [17] and [18]).

Proposition II: If $a_{11}/a_{12} < a_{21}/a_{22}$, and the adjustment takes place in discrete time, the output ratio q_t would diverge from q^*, and p_t from p^*, through a sequence of diverging oscillations (see fig. 2.b).

The choice of a discrete-time formulation of the dynamics in sometime related to the presence of a production lag; but the discrete-time formulation assumes more than a simple production lag (cf. Boggio [11]). This, in itself, implies only that the time rate of change of output, at any date, depends upon decisions taken some time before.

What is crucial to a *pure* discrete-time formulation is either that economic variables are defined only at discrete time instants, or that, although variables may be defined at any instant, changes in variables take place only at discrete time instants (this type of situation is considered in Boggio [11]).

For our purposes, it is useful to refer to the latter (more general) situation. The relavant discrepancy between output and demand in pure discrete-time systems, that is, the discrepancy which acts on prices, can then be thought of as the result of a cumulation of discrepancies between the instant flows of output and demand, within the interval which separates two subsequent market dates. (This interval can be chosen as short as one likes, so that there may be arbitrarily many, but still *finite*, market dates within a production interval; cf. Boggio [11]). In pure diffrential models, the instant flows of output and demand act directly on market prices, and thus trigger a prompt revision of output decisions; in discrete-time models the instant disequilibria must cumulate, before they can trigger the adjustment of output.

In the light of the above considerations, it is not too surprising that, unlike its time-continuous analogue, the discrete-time formulation of the classical adjustment process may lead to instability, even when demand feed-backs are well behaved.

The framing of a production lag within a differential model (thus leading to a mixed difference-differential dynamic system) would elicit: *a*) a continuous reaction of prices to the discrepancy between the instant flows of output and demand; *b*) a continuous reaction of decisions concerning *future* output to price signals. (The full continuous-time model proposed in Boggio [11] retains feature *a*), but not feature *b*)). Despite its formal complication, this choice may be perhaps more appropriate. If market prices are hindered from responding to any excess-demand signal, and/or decisions cocnerning future output are hindered from responding immeditely to any price signal, the full operation of the cross-dual adjustment is, somewhat arbitrarily, restricted. In the light of the above considerations, it would seem that the dynamic implications which follow, when one outsteps the limits of pure differential models, may not be invariably characterized by the results synthesized in Proposition II.

The above remark completes our long discussion of the significance of the results stated in Propositions I and II.

These results, which are recurrent in mathematical models of gravitation, are here established by means of simple arguments, which lend themselves to an immediate economic interpretation.

Let $\mathbf{Y} \equiv \mathbf{q}[\mathbf{I} - \mathbf{A}]$ denote the net output of the economy, and $y \equiv y_1/y_2$ the net-output ratio; we may notice that y is a function of q, which is expressed as:

$y = y(q) \equiv [(1 - a_{11})q - a_{21}]/[(1 - a_{22}) - a_{12}q]$.

Also let $\mathbf{i}(\mathbf{q}, \mathbf{p})$ be the net investment which would be planned at prices $\mathbf{p}$ and outputs $\mathbf{q}$; this investment may be only notionally defined, because p need not be market clearing at $\mathbf{q}$. Again, if we define $i \equiv i_1/i_2$, i can be unambiguously expressed as function of p and q: $i = i(q, p)$. Since the rate of profit on replacement costs is uniform at p^*, the (notional) desired change in output would be $\mathbf{q}g^*$, so that $\mathbf{i}(\mathbf{q}, \mathbf{p}^*) = \mathbf{qA}g^*$. It follows that:

$i = i(q, p^*) = (a_{11}q + a_{21})/(a_{12}q + a_{22})$.

Remark I: If $q > (<) q^*$, at the non-market-clearing price ratio p^*, the (notional) net-investment ratio $i = i(q, p^*)$ is lower (higher) than the gross-output ratio q and the net-output ratio $y(q)$. Synthetically: if $q > (<) q^*$, then $i(q, p^*) < (>) y(q)$; $i(q, p^*) < (>) q$.

To prove this, we observe: $i(q^*, p^*) = y(q^*) = q^*$, for $\mathbf{q}^*[\mathbf{I} - \mathbf{A}] = \mathbf{q}^*\mathbf{A}g^* = \mathbf{q}^*(g^*/1 + g^*)$; the vitality of the economic system implies that the derivative $\delta(y(q) - q)/\delta q$, taken at $q = q^*$, is positive, and the derivative $\delta(i(q, p^*) - q)/\delta q$, also taken at $q = q^*$, is negative.

If $a_{11}/a_{12} > (<) a_{21}/a_{22}$, the higher the planned time rate of change in q, the higher (lower) would be the investment ratio i. More precisely, if $a_{11}/a_{12} > (<) a_{21}/a_{22}$, $i(q, p)$ is an increasing (decreasing) function of p, at $p = p^*$. These facts, together with *Remark I*, yield *Proposition I*.

We are left with the task of proving *Proposition II*, which is concerned with the case where $a_{11}/a_{12} < a_{21}/a_{22}$, while only discrete changes in output are allowed.

In the first place, we may observe that, at the market-clearing prices p_t, the following equality holds true:[55] $\mathbf{q}_t = \mathbf{q}_{t+h}\mathbf{A}$ (gross output equals intended gross investment). Since $[1/(1 + g^*)]\mathbf{q}^* = \mathbf{q}^*\mathbf{A}$, at the notionally defined planned output[56] $\beta\mathbf{q}^*$ the gross-investment ratio would be q^*, and would be too low (high) to be market clearing, in so far as $q_t > (<) q^*$.

[55] Recall that h is the length of the production interval, which coincides in this case with the interval which separates two subsequent market dates.

[56] βq^* is only notionally defined, because capitalists may not *intend* to plan this output at prices p_t, and the corresponding gross investment $\beta q^* A$ may not be market clearing. The positive scalar β can be so chosen, that the restriction profits = net-investment expenditure holds at p_t.

In the second place, since $a_{11}/a_{12} < a_{21}/a_{22}$, relative increase in the planned output of comodity 2 is required to bring about a relative increase of the present demand for commodity 1 (the ratio $(a_{11}q + a_{21})/(a_{12}q + a_{22})$ is a decreasing function of q).

It follows that $q_t > (<) q^*$ implies $q_{t+h} < (>) q^*$; more generally, $q_t > (<) q^*$ implies: $q_{t+nh} < (>) q^*$, if the integer n is odd; $q_{t+nh} > (<) q^*$, if the integer n is even.

As above, at the market-clearing prices $\mathbf{p}_{t+h}$, $\mathbf{q}_{t+h} = \mathbf{q}_{t+2h}\mathbf{A}$; hence, $\mathbf{q}_t = \mathbf{q}_{t+2h}\mathbf{C}$, with $\mathbf{C} \equiv \mathbf{A}^2$; it can easily be checked that $a_{11}/a_{12} < a_{21}/a_{22}$ implies $c_{11}/c_{12} > c_{21}/c_{22}$, so that the ratio $(c_{11}q + c_{21})/(c_{12}q + c_{22}) \equiv c(q)$ is an increasing function of q. Using again a notionally defined planned output $\beta'\mathbf{q}_t$(for some scalar $\beta' > 1$) as a term of comparison, and observing that $\delta(c(q) - q)/\delta q < 0$, at $q = q^*$, we are led to conclude that $q_t > (<) q^*$ implies $q_{t+2h} > (<) q_t$. This completes the proof of *Proposition II*.

3.2.2. Leaving aside, for the reasons already explained, the elements of instability which seem to follow from the choice of a pure discrete-time representation, the main conclusion which can be drawn from the above discussion can be re-stated and synthesized as follows. In models of the classical adjustment process where *i*) in the *aggregate* capitalists invest all their profits, technical conditions of production may be such that the ratio between the demand for and the output of any commodity increases with the relative price of the same commodity. If this is the case, factors of (*local*) instability are introduced by the assumption that output and demand conditions react only on prices, and, conversely, that market prices are adjusted only according to the state of output and demand.
The italicized qualification in parenthesis does not only refer to the local nature of the (in)stability concept under discussion, or to the fact that the mathematical results of the models referred to above are obtained by means of linear approximations which hold true only in a neigbourhood of the LPP. Not less important is the following fact, which was already observed and will be further stressed below: the behavioural equations of the original non-linear version of these models are likely to change, as soon as the system departs significantly from the LPP.[57]

[57] Assumption *ii*) above does not cover "simplified-cross-dual" processes (as defined in paragraph 3.2. above), even where they assume $g^* = r^* = R \equiv$ maximum rate of profit. Clearly, except in dynamic equilibrium, profits are not automatically invested in these models. "Simplified" cross-dual processes in continuous time, and with $g^* = r^* = R$, can be shown to yield stability of the LPP, under otherwise general assumptions on technical conditions (including joint production) and for economies with any finite number of commodities. If capital transfers do not only depend upon profit differentials, but also upon the change thereof (derivative control), asymptotic stability is obtained; cf. FLASCHEL and SEMMLER [29]. Unfortunately, the above nice results do not carry over to the more general case where $g^* \leq r^* \leq R$; cf. FLASCHEL and SEMMLER [30].

3.2.3. One may object that the above conclusion is not very disturbing, after all. It is a fact that profits are not automatically invested in real-world economies. Indeed, less drastic assumptions, concerning investment behaviour, open the way to positive results, even when instantaneous production and arbitrary restrictions on input proportions are ruled out.

Still, the models yielding such "positive results" are exposed to other objections. As it turns out, asymptotic stability seems to require *ad-hoc* demand feed-backs, from the side of consumption. So long as the "right" consumption feed-backs are assumed, and investment reactions to price changes are *sufficiently slow*, the "right" demand feed-backs will then dominate the dynamics of adjustment.

Obviously enough, there are two ways in which market prices may affect the composition of consumption: they may act directly, in which case we are faced with proper substitution effects, or they may act through income effects.

The former type of influence is assumed in models where, in the aggregate, capitalists' relative consumption of any two commodities is a decreasing function of their relative price.[58]

As was observed before, another way to obtain direct substitution, through prices, occurs in models where workers, who are paid (*post-factum*) a constant nominal wage, plan the composition of their consumption in nominal, rather than in physical terms.[59]

It is worth stressing that the generality of the above substitution effects is highly questionable. Even where the mechanism may seem plausible at a microeconomic level, there is no ground for assuming that the same mechanism holds, in the aggregate, after price and income effects are simultaneously dealt with. A consideration of the literature on excess-demand functions in Walrasian exchange economies is quite instructive, in this sense.[60]

A different consumption feed-back occurs, via income effects, if the composition of workers' consumption differs from the composition of capitalists' consumption (while both are fixed in physical terms).[61] Unfortunately, a result of Boggio [11] suggests that not all differences in composition will serve to the purpose of stability: for a given composition of workers' consumption, there exists a continuum of a priori plausible compositions of capitalists' consumption, such that the classical adjustment process in discrete time is unstable.

[58] See, for instance, Boggio [9].
[59] Cf. Kubin [49] and [50].
[60] Cf. Kirman [45].
[61] See, for instance, Duménil and Lévy [17] and [18].

4. COMPOSITE FORMS OF ADJUSTMENT

4.1.

It is worth supplementing the overview on classical adjustment processes, which was given in section 3., with a few general remarks.

i) The idea that in a competitive environment firms are price-takers and "the market" makes prices, is clearly related to the neoclassical notion of perfect competition;[62] the idea that firms make prices, is not only much closer to real world conditions, but is also consistent with the classical view of competition.[63]

At the same time, the difficulties which *necessarily* arise in the modelling of the adjustment to a neoclassical intertemporal equilibrium as an actual (as opposed to virtual) disequilibrium process,[64] do not arise for classical LPPs. This way of modelling the stability of production prices is particularly advocated in the work of G. Duménil and D. Lévy. Since firms' and consumers' plans may not be realized in this setting, Cartelier [13] argues, quite convincingly, that money holdings and monetary institutions should be explicitly introduced.

ii) In so far as price formation is concerned with the money value of the output already in existence, rather than with the money value of future output (*i.e.* price formation takes place *after* production), and output is perishable, it may be legitimate to assume that the market price depends only on supply and demand, and is not *directly* influenced by cost.[65]

Things are quite different, if price decisions are concerned with the market price of the output to be produced, rather than of the output in existence. In this case, in so far as firms are allowed to make prices, it is highly plausible that price formation is not independent of cost considerations. This would be all the more true if goods are storable and firms hold stocks of inventories.

Indeed, if the stocks of inventories are not too far from their desired level, firms may find it profitable to place the burden of the adjustment to a given excess demand, on inventories, more than on prices. Output decisions will then seek to bring the stocks of inventories to their desired level. In this case, output decisions are influenced by demand, not only through its effect on prices and profitability.

[62] Cf. STIGLER [80], MCNULTY [56].

[63] Cf. EATWELL [23].

[64] We refer here to those difficulties arising because equilibrium is defined with respect to a given initial endowment; cf. FISHER [26], ZAGHINI [81].

[65] The view of price formation, which has just been described, prevails in the writings of the classics, in so far as the organization of their ideas on production and circulation were clearly influenced by the observation of the productive cycle in agriculture. Thus, Smith suggested that the market price is determined as a result of the output *brought to the market*, and of effectual demand. Interestingly enough, he distinguished between the price reaction of durables and of non-durables to a given excess supply. Cf. SMITH [75], BOOK I, ch. VII.

4.2.

The above remark hints at a composite adjustment process, which retains both classical and Keynesian elements.

There exist in the literature stable dynamic models which retain such Keynesian elements. They define dual systems where price changes depend on cost (hence on prices), given sectoral target rates of profit r_i ($i = 1,\ldots,n$), while changes in output depend on the demand generated, via multiplier effects, by investment decisions[66] (following exogenous growth expectations, or other rules). These models have been shown to be stable in relative variables.[67]

In the particular case, where the planned rates of growth and the target rates of profit are assumed to be uniform across industries, relative prices converge to the (unique) prices of production p^*, relative outputs converge to the (unique) steady growth relative outputs q^*.[68] By analogy with our former definition of a "simplified cross-dual" adjustment,[69] we define this particular case "simplified-dual" adjustment.

In the more general case, where the target rates of profit are not uniform across industries, prices converge to a generalized production-price vector.[70] Within this type of model, the formation of target rates of profit is not explained, and rate of profit differentials do not feed back into investment decisions. This fact can be justified, as is argued in Boggio's contribution to this issue, if one interprets the Keynesian dual adjustment as a short-run phenomenon, which coud be combined with a long-run, classical adjustment.[71]

The existing seminal attempts to integrate Keynesian and classical adjustment processes emphasize different Keynesian reactions; thus, they do not arrive at the same model structure. Still, their results are very instructive, in so far as they give convergent indications. Semmler [74] and Flaschel and Semmler [30] examine the stability properties of a composite, "simplified-cross-dual" and "simplified-dual" dynamics. An earlier study by L. Boggio[72] considers a composite form of adjustment, which combines

66 Investment decisions are either determined by exogenous (and uniform) normal-growth expectations, as in Filippini [25], or by maximum (uniform) growth compatible with available productive capacity, as in Aoki [1]. In this paper investment decisions are also affected by price consideration, but these do not follow classical threads of thought.

67 They are explicitly derived from the early formulations of the dynamic Leontief system, from which they differ in two main respects: in the first place, the assumption of perfect foresight is dispensed with; in the second place, full utilization of productive capacity is not assumed. As is well known, the early formulations of the dynamic Leontief system yielded dual instability.

68 Cf. Flaschel and Semmler [30], Semmler [74].

69 See above, paragraph 3.1.

70 Necessary conditions are specified in Boggio [11].

71 Cf. Boggio [11] and [12].

72 Cf. Boggio [10].

a mark-up process of price formation, with a classical adjustment, where target rates of profit are revised according to excess demand, and intersectoral transfers of capital respond to profit differentials. In models, by G. Duménil and D. Lévy,[73] the classical, cross-dual, adjustment is supplemented by a Keynesian effect of excess demand on output, which takes place through a changing level of stocks per unit of output. In all of these studies, asymptotic stability requires that the adjustments of the classical type proceed at sufficiently low speed. So long as excess demand is not allowed to have a rapid influence on prices (either because its immediate influence falls mainly on output, or because prices depend mainly on cost), and capital transfers respond slowly to profit differentials, the possibly "perverse" demand effects of entrepreneurs' decisions do not feed back rapidly into the system dynamics. This fits quite well with the above economic interpretation, in terms of short-run and long-run, of the Keynesian and of the classical adjustment processes.

4.3.

The stability results obtained for the composite adjustment processes do not require *ad-hoc* restrictions on input proportions, on technology (fixed capital and joint production are allowed in some models), on the number of commodities, or on consumption; thus, there is an important indication that composite forms of adjustment can support the gravitation hypothesis. Still, there are at least two aspects which seem to deserve a close consideration.

In the first place, the existing literature does not give a clear indication concerning the integration of the classical adjustment process, which is most realistic and/or theoretically appropriate:[74] short-run mark-up processes are emphasized in Boggio's work; the stabilizing influence of a short-run relation between excess demand and output (in the presence of stocks) is highlighted by Duménil and Lévy.[75] It would seem that these integrations are more complementary than conflicting, as will be further stressed below.

In the second place, we observed before how the stabilizing influence of these integrations can be traced back to a common feature, *i.e.* to a weaker effect of demand on prices and profitability. Asymptotic stability requires the effect in question to be arbitrarily weak. One may wonder whether this is a general feature of capitalist economies under (classical) competition. Indeed, evidence of relatively fast price reactions to slack demand conditions is not infrequent, even in modern capitalism.

[73] Cf. DUMENIL and LEVY [19], [20], [22].

[74] The issue regarding the relative speed of price and quantity adjustment by firms, is a highly general one, and goes well beyond the scope of the present introduction. Some of the relevant recent literature on this topic is quoted in Semmler [74].

[75] Cf., in particular, the *addendum* to DUMENIL and LEVY [22].

Some light may be shed on the above two related points, if we consider the problem of gravitation in a more *global* perspective.

If market prices are sufficiently close to long-period prices, as is assumed in local stability analyses, they necessarily fulfil some minimum profit requirement; *e.g.*, they cover input costs; if the market rate of interest is lower than the long-period rate of profit, these costs can be inclusive of interest. The same condition would not be fulfilled in one or more industries[76] if market prices were far from long-period prices. It is not at all obvious that, in the latter situation, firms in these industries would be prepared to increase their output, to meet a higher demand, and would not drastically increase the price of output. Indeed, it is doubtful whether firms producing a storable output would let the price fall below the short-run lower bound correspoding to a minimum-profit requirement. If the condition is not fulfilled, production should come to a stop. The money rate of interest plays, plausibly, a crucial role in the determination of some short-run lower bound for the rate of profit on replacement costs, which is implicit in the pricing decision.

The above remark has two immediate and quite obvious implications.

First, in the presence of quantity adjustment in response to excess demand conditions, price making cannot be totally independent of cost considerations. A shift from excess-demand to cost may occur in the pricing decision, as the system moves a long way from states where the conditions of profitability are sufficiently uniform.

Second, if cost considerations set a short-run lower bound to the movement of market prices (and if commodities are all basics), the dynamics of *relative prices* is necessarily limited to a bounded region in price space. In so far as interest costs help to determine the short-run lower bound of prices, the amplitude of those deviations comes to depend crucially upon the behaviour of the market rate of interest. This argument, and other arguments hinging upon asset reevaluation through financial investment,[77] suggest that financial phenomena are highly relevant to the issue of gravitation.

[76] Suppose there are n goods (all basics), and the real wage is fixed in physical terms. Let σ_i be the ratio between the market price and the long-period price of commodity i ($i = 1,\dots,n$) in terms of an arbitrary commodity numeraire. The industry with the lowest σ_i would certainly make losses, if some σ_i is sufficiently far from 1.

[77] Cf. Gozzi [40].

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