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REAL WICKSELL EFFECT, DEMAND FOR CAPITAL AND STABILITY

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ABSTRACT

The aim of this paper is to study the relationship between reverse capital deepening and instability of the equilibrium between investments and savings. It is shown for a model with n commodities, infinitely many linear technique of production, and overlapping generation that a badly-behaved real Wicksell effect, as in the case of a "reswitching of techniques", can involve instability.

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1. INTRODUCTION

The equilibrium considered here is the one Keynes described as follows in discussing the neo-classical theory of the rate of interest[1]:

> this tradition has regarded the rate of interest as the factor which brings the demand for investment and the willingness to save into equilibrium with one another. Investment represents the demand for investible resources and saving represents the supply, whilst the rate of interest is the 'price' of investible resources at which the two are equated. Just as the price of a commodity is necessarily fixed at that point where the demand for it is equal to the supply, so the rate of interest necessarily comes to rest under the play of market forces at the point where the amount of investment at that rate of interest is equal to the amount of saving at that rate. (Keynes 1973 [1936]: 175)

Stated in formal terms, both the (gross) investments and the (gross) savings per worker are regarded in neo-classical theory as functions of the rate of interest and denoted respectively by $v(r)$ and $v^s(r)$. Because of the usual market mechanism, the rate of interest r is assumed to increase when $v(r) > v^s(r)$ and to decrease when the opposite is true. Therefore, given a sign-preserving function h(.), we have the following differential equation:

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[1] Keynes complained about the difficulty of finding an explicit statement of this theory in the "leading treatises" of neo-classical economics. There are, however, some revealing passages, which he himself quotes, in Marshall's *Principles*, where the role of the interest rate as the price of capital is quite clearly shown. In particular, as regards savings, Marshall wrote that "[a] rise in the rate of interest offered for capital, i.e. in the demand price for saving, tends to increase the volume of saving" (Marshall 1920: 236). On the investments side he wrote:

> the demand for the loan of capital is the aggregate of the demands of all individuals in all trades; and it obeys a law similar to that which holds for the sale of commodities: just as there is a certain amount of a commodity which can find purchasers at any given price. When the price rises the amount that can be sold diminishes, and so it is with regard to the use of capital. (Marshall 1920: 521)

The combination of the two sides of the savings-investments market, according to Marshall, means that:

> interest, being the price paid for the use of capital in any market, tends towards an equilibrium level such that the aggregate demand for capital in that market, at that rate of interest, is equal to the aggregate stock forthcoming there at that rate. (Marshall 1920: 534)

A similar mechanism also appears to be implicit in the Ramsey-Solow models: on the one hand, savings depend explicitly on the rate of interest; on the other, the equality between the marginal product of capital and the rate of interest reveals that the latter affects the technique in use and thus the employment of capital per worker by firms. The rate of interest therefore keeps investments equal to savings in every period.

$$\dot{r} = h\left[v(r) - v^s(r)\right]. \tag{1}$$

Given the above equation, in accordance with classical mechanics, we shall call equilibrium an interest rate level r* such that $\dot{r} = 0$, i.e. $v(r^*) = v^s(r^*)$. The purpose of the present paper is to study this kind of equilibrium and more specifically its local asymptotic stability in order to shed light on the phenomena capable of preventing it. In particular, our discussion will concern the role played by phenomena like "reverse capital deepening" and "reswitching", since there still appear to be some ambiguities in this regard (see for example Bloise and Reichlin 2009: 56–9).

We shall begin in sec. 2 by introducing a model of production and considering the choice of technique and the consequent demand for capital goods, which will give rise to the (gross) investment decisions when transferred into value terms.

As regards saving decisions (sec. 3), these will be based on an overlapping-generation model in which every individual has a lifetime of two periods and is required to save part of the income earned during the first period in order to consume during the second, when she/he will no longer be able to work.

Having defined equilibrium and its local (asymptotic) stability for the case under consideration (sec. 4), we shall then go on to consider the phenomena that can determine equilibrium stability or instability. In particular, under some clear conditions, we shall argue (sec. 5) that a badly-behaved real Wicksell effect, as in the case of a "reswitching of techniques", can involve instability.

Finally, some special assumptions will be made both on the description of technological knowledge and on saving behaviour. These assumptions, as readers will see, are designed to make the connection between the sign of the real Wicksell effect and the stability of equilibrium more transparent. Removing them would make the analysis more general, but would also cloud the vision of what we intend to show. As Walras asked, "[w]hat physicist would deliberately pick cloudy weather for astronomical observations instead of taking advantage of a cloudless night?" (Walras 1977, p. 86).[2]

2. PRODUCTION AND INVESTMENT DECISIONS

2.1 Some assumptions

We consider an economy with n products. The commodity labelled [1] is both a consumption good and a circulating capital good, while the other commodities – labelled [2], [3], ..., [n] – are pure (circulating) capital goods.

A continuum of possible techniques of production is available. Each technique is characterized by an n×n matrix $A(\theta)$ and an n-vector $\ell(\theta)$, for every $\theta \in \Theta$, with $\Theta = \{\theta \in R : 0 \leq \theta \leq 1\}$, such that $a_{ij}(\theta) \geq 0$ and $\ell_i(\theta) > 0$ are, respectively, the quantity of commodity [j] and the amount of labour employed in the production of one unit of commodity [i].[3]

Assumption 1. *The functions* $a_{ij}(\theta)$ *and* $\ell_i(\theta)$ *are continuous and at least twice differentiable on the set* S*, with* $S = \{\theta \in R : 0 < \theta < 1\}$.

[2] Walras put forward this argument to justify the study of price determination under the assumption of free competition instead of considering the various forms of imperfect competition encountered in real life.

[3] It is worth pointing out that while our way of representing a continuum of possible production techniques is not very familiar to economists, it is more general in character than the common production functions (C.E.S., Cobb-Douglas, etc.). Specifically, our formalization of technology is compatible with the possibility of the reswitching of techniques, while the usual production functions are not.

It does, however, involve some peculiar features. In particular, given two techniques θ' and θ'', the techniques obtained by selecting some methods from θ' and others from θ'' do not necessarily belong to the set of possible techniques Θ. This limitation appears, however, to have no particular effect on the argument presented in this paper.

For linear production models with a continuum of techniques, see also Bellino (1993).

For the possibility (or impossibility) of reswitching in models with well-behaved production functions see in particular Hatta (1976), Steedman (1990) and Schefold (2008).

Given a (row) vector of activity levels (gross products) $b \geqq 0$, $b \cdot A(\theta)$ is the corresponding vector of demand for capital with technique θ, $b \cdot [I - A(\theta)]$ is the vector of net products and $b \cdot \ell(\theta)$ is the demand for labour.

Our argument will be developed by focusing on self-replacing states of the economy in which commodity [1], the consumption good, is the only net product. For each technique θ, there exist a scalar $y(\theta)$ and a (row) vector $q(\theta)$ that are respectively the net product of commodity [1] per worker and the vector of activity levels generating it. These can be obtained by solving the following equations[4]:

$$y(\theta) \cdot e_1 = q(\theta) \cdot [I - A(\theta)] \tag{2}$$

$$1 = q(\theta) \cdot \ell(\theta). \tag{3}$$

From equation (2) it is easy to obtain[5]:

$$y(\theta) \cdot e_1 \cdot [I - A(\theta)]^{-1} = q(\theta) \tag{4}$$

and the substitution of equation (4) in equation (3) gives:

$$1 = y(\theta) \cdot e_1 \cdot [I - A(\theta)]^{-1} \cdot \ell(\theta) \tag{5}$$

which implies:

$$y(\theta) = \frac{1}{e_1 \cdot [I - A(\theta)]^{-1} \cdot \ell(\theta)}. \tag{6}$$

Assumption 2. *No technique in Θ is dominated: for every technique $\theta \in \Theta$, there exists no other technique $\widehat{\theta}$ such that $y(\widehat{\theta}) > y(\theta)$, $q(\widehat{\theta}) \cdot A(\widehat{\theta}) \leq q(\theta) \cdot A(\theta)$ and $q(\widehat{\theta}) \cdot \ell(\widehat{\theta}) \leq q(\theta) \cdot \ell(\theta)$.*

Assumption 3. *The techniques in Θ are labelled in such a way that $y(\widehat{\theta}) > y(\breve{\theta})$ whenever $\widehat{\theta} > \breve{\theta}$.*

[4] The (row) vector e_1 is [1, 0, ..., 0].
[5] Needless to say, the matrix $[I - A(\theta)]$ is assumed to be invertible for every $\theta \in \Theta$. Satisfaction of the usual productivity conditions is also assumed.

Assumption 3 clearly implies that $dy/d\theta > 0$ for every $\theta \in S$.

2.2 Choice of technique

For an interest rate r, a wage rate w and a price vector $p \in R_+^n$, the unit full cost $c \in R_+^n$ of the n products with technique θ is defined by the following equation:

$$c = (1+r)\cdot A(\theta)\cdot p + \ell(\theta)\cdot w. \tag{7}$$

Let us assume commodity [1] as the numéraire and denote with p(θ,r) and w(θ,r) respectively the price vector and the wage rate that make the unit cost vector equal to the price vector with the technique θ and the interest rate r. We thus have:[6]

$$w(\theta,r) = \frac{1}{e_1 \cdot [I-(1+r)\cdot A(\theta)]^{-1}\cdot \ell(\theta)} \tag{8}$$

and

$$p(\theta,r) = \frac{[I-(1+r)\cdot A(\theta)]^{-1}\cdot \ell(\theta)}{e_1 \cdot [I-(1+r)\cdot A(\theta)]^{-1}\cdot \ell(\theta)}. \tag{9}$$

The function w(θ,r) is known as the wage-interest function or curve for technique θ. Moreover, according to a well-known result, given an interest rate r (taken within a certain interval), the technique θ° is optimal if and only if w(θ°,r) ≥ w(θ,r) for every θ ∈ Θ.[7] In other words, solving the following maximisation problem:

$$\begin{cases} \max\limits_{\theta} & \dfrac{1}{e_1 \cdot [I-(1+r)\cdot A(\theta)]^{-1}\cdot \ell(\theta)} \\ \text{s.t.:} & \theta \in \Theta \end{cases} \tag{10}$$

[6] When c equals p, equation (7) implies $p = [I-(1+r)\cdot A(\theta)]^{-1}\cdot \ell(\theta)\cdot w$. If commodity [1] is the numéraire commodity, we therefore have $1 = p_1 = e_1 \cdot [I-(1+r)\cdot A(\theta)]^{-1}\cdot \ell(\theta)\cdot w$, which directly implies equation (8). And by substituting this for w in the first equation, we obtain equation (9).

[7] In other words, as a corollary of the Non-Substitution Theorem, given a certain level of the interest rate, the optimal technique is the one that makes it possible to pay the maximum wage rate. See Burmeister (1980: 102-11) for a detailed analysis.

with r considered parametrically between 0 and a certain maximum, makes it possible to express the optimal technique as a function of the rate of interest: $\theta° = \theta(r)$.

Once the function $\theta° = \theta(r)$ is known, all the magnitudes that were formerly functions of the technique in use now become functions of the rate of interest. To be precise, while $y(\theta)$, $q(\theta)$, $w(\theta,r)$ and $p(\theta,r)$ are the net and gross product, the wage rate and the price system with a possible technique θ, $y(r) = y[\theta(r)]$, $q(r) = q[\theta(r)]$, $w(r) = w[\theta(r),r]$ and $p(r) = p[\theta(r),r]$ are the corresponding variables with the optimal technique.

2.3 Investment decisions

In accordance with the considerations developed above, the vector of the employment of capital goods per worker with a certain technique $\theta \in \Theta$ is defined as:

$$k(\theta) = q(\theta) \cdot A(\theta). \tag{11}$$

It thus follows that $k(r) = k[\theta(r)]$ is the vector of capital per worker with the optimal technique. The (gross) investment decisions (per worker) can be therefore defined as follows:

$$v(r) = k(r) \cdot p(r). \tag{12}$$

According to well-known terminology, the change in the investment decisions – which can also be called the "demand for capital" in our model[8] – per worker due to a variation of the interest rate can be decomposed into a real Wicksell effect and a price Wicksell effect. A great deal has been said about these effects.[9] In a nutshell, the former reflects the change in the vector of capital goods due to the change in the technique in use, and the latter the change in the prices of capital goods due to change both in the technique in use and in distribution variables.

[8] It is worth noting that in the model considered here, where there are only circulating capital goods, the gross investment of each period coincides with the demand for capital in value terms. The entire amount of capital employed must be replenished by gross investment in each period.

[9] The literature on Wicksell effects is so vast that would be impossible to recall every single paper. We shall therefore mention just a few: Swan (1957), Robinson (1958), Bhaduri (1966), Burmeister (1975) and (1980), Garegnani (1984).

Considering an infinitesimal change in the interest rate and denoting by $dk/dr \in R^n$ and $dp/dr \in R^n$ respectively the changes in the (row) vector of employment of capital goods per worker and in the price vector it entails, we have:

$$\frac{dv}{dr} = \underbrace{\frac{dk}{dr} \cdot p(r)}_{\text{real effect}} + \underbrace{k(r) \cdot \frac{dp}{dr}}_{\text{price effect}}. \tag{13}$$

As regards the sign of the variation dv/dr, this is generally unpredictable. At least in the case considered here, the two effects can be either positive or negative and the same or opposite in sign. In particular, on the one hand, the investment schedule can be monotonically increasing even in the case of a well-behaved choice of technique and, on the other, a monotonically decreasing investment schedule can emerge in a case with reswitching of techniques (cf. Fratini 2010 and, for a generalization, Bidard 2010).

2.4 Reswitching and capital deepening

Reswitching can occur in the model considered here. It is in fact possible for a technique to be optimal for two different levels of the interest rate but not for the levels between them.

It is well-known that reswitching implies a non-monotonic shape of the function y(r) (see, inter alia, Burmeister & Turnovsky 1972, Burmeister 1976, Mas-Colell 1989 and Fratini 2010). This is particularly evident in our model, where assumption 3 means that there is a one-to-one correspondence between the technique in use and the amount of net product per worker.

When $r = 0$, the optimal technique is $\theta = 1$, i.e. the one with the greatest net product per worker.[10] This implies that an increase in the rate of interest initially entails a decrease in the net product per worker.

[10] If it is posited that $r = 0$, the argument to be maximised in equation (10) becomes the net product per unit of labour $y(\theta)$ (cf. equation (6)). Since $y(1) > y(\theta)$ for every $\theta \in \Theta \backslash \{1\}$ because of assumption 3, then $1 = \theta(0)$.

The inverse relationship between the rate of interest and the net product per worker appears to be consistent with the neo-classical idea that an increase in the rate of interest brings about the use of techniques that are “less capital intensive” or “less roundabout”. If it were possible to measure aggregate capital in “technical units”, the techniques that employ less capital per worker would in fact give a smaller net product per worker.[11] Needless to say, however, such a technical measurement of aggregate capital is generally impossible.

After the initial decreasing stretch, the pattern of the function y(r) becomes unpredictable. Since every technique can be brought back into use several times, the function y(r) can exhibit alternatively decreasing and increasing stretches. Moreover, the sign of the variation in the net product per worker due to a rise in the rate of interest is closely linked to that of the real Wicksell effect.

This can be easily proved. Let us consider an interest rate r and the associated optimal technique $\theta° = \theta(r)$. Denoting by y° and k° the net product and the vector of capital goods, both per worker, with technique θ°, then – because of equations (8) and (9) – we have:

$$y° - r \cdot [k° \cdot p(r)] - w(r) = 0 \tag{14}$$

which means that technique θ° entails zero (extra)profits for r, p(r) and w(r). Moreover, as technique θ° is profit-maximising for r, p(r) and w(r), a slight change in the technique in use cannot alter the amount of (extra)profits per worker, which are still zero:

$$\left(y° + \frac{dy}{d\theta}\right) - r \cdot \left[\left(k° + \frac{dk}{d\theta}\right) \cdot p(r)\right] - w(r) = 0 \tag{15}$$

Therefore, substituting equation (14) in equation (15), we have:

[11] In the hyper-simplified models with just one commodity serving both for consumption and for capital, as in the case of the standard neo-classical growth model, the net product per worker is usually an increasing function of the employment of capital per worker.

$$\frac{dy}{d\theta} - r \cdot \frac{dk}{d\theta} \cdot p(r) = 0 \tag{16}$$

Bearing in mind that y(r) = y[θ(r)] and k(r) = k[θ(r)], this clearly implies:

$$\frac{dy/d\theta}{d\theta/dr} = r \cdot \frac{dk/d\theta}{d\theta/dr} \cdot p(r) \tag{18}$$

Therefore, for $r > 0$, the real Wicksell effect is negative if and only if $dy/dr < 0$. As a result, the reswitching of techniques entails a positive, i.e. anti-neo-classical, Wicksell real effect.

3. SAVING DECISIONS

In every neo-classical model, savings are a function of the rate of interest. There are, however, many different justifications for this. In the present paper, for the sake of simplicity, we will consider an overlapping-generation model with identical individuals whose life lasts for two periods.

During the first period of life, each individual is a worker and inelastically supplies one unit of labour. In the second period, the individual becomes unable to work, and therefore her/his consumption depends on the part of the wage rate saved during the first period. Denoting by $u(x_1, x_2)$ a consumer utility function with the customary properties, we have the following utility maximisation problem:

$$\begin{cases} \max\limits_{x_1,x_2} \ u(x_1,x_2) \\ \text{s.t.:} \ w = x_1 + \dfrac{x_2}{1+r} \end{cases} \tag{19}$$

whose solution is $x_1(w,r)$ and $x_2(w,r)$.

The optimal savings decision per worker is therefore:[12]

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[12] It is easy to show that v^s, as defined in equation (20), is the amount of gross savings per worker. In every period, the total gross income of households is made up of the sum of the wages of the younger generation plus the gross interest on the savings of the older, and is therefore Lw +

$$v^s(r) = w(r) - x_1[w(r),r]. \tag{20}$$

There is not much to say about the saving function $v^s(r)$. Because of the usual substitution effect, an increase in the rate of interest should cause the consumption of the first period x_1 to decrease with respect to the consumption of the second period x_2, but since there is also an income effect, x_1 may very well increase when the rate of interest increases. Moreover, the wage rate is inversely proportional to the rate of interest, and an increase in the latter will therefore decrease the income out of which savings are made. As a result, the saving function can have any shape,[13] with the only restriction that $v^s(r) \leq w(r)$.

4. EQUILIBRIUM AND STABILITY

As already stated at the beginning, given the dynamic process initially described by equation (1), an equilibrium is an interest rate r* such that $v(r^*) = v^s(r^*)$.[14] For the kind of model considered here, the existence of at least one equilibrium depends on the initial data, i.e. on the shape of the functions A(.), ℓ(.) and u(.). As we are interested in stability, however, existence is assumed here.

$Lv^s \cdot (1+r)$, where L is the number of individuals in each generation. The total consumption expenditure is instead $Lx_1 + Lx_2$. As a result, the total gross savings are $L\,[w + v^s \cdot (1+r) - x_1 - x_2]$. Now, since equation (20), together with the budget constraint in the utility maximization problem (19), implies $x_2 = v^s \cdot (1+r)$, the gross savings per worker are $w - x_1 = v^s$.

[13] For a discussion of the shape of the saving function in a two-period overlapping-generation model, see also Fratini (2007).

[14] It may be noted in passing that when $v(r^*) = v^s(r^*)$, the value of the gross product per worker is equal to the gross income per worker, as suggested in the macroeconomic textbooks. At the equilibrium interest rate r*, the value of the gross product per worker is in fact $q(r^*) \cdot p(r^*) = (1+r^*) \cdot q(r^*) \cdot A[\theta(r^*)] \cdot p(r^*) + q(r^*) \cdot \ell[\theta(r^*)] \cdot w(r^*)$. Therefore, since $q(r^*) \cdot A[\theta(r^*)] \cdot p(r^*) = v(r^*)$ because of equations (11) and (12), and $q(r^*) \cdot \ell[\theta(r^*)] = 1$ because of equation (3), we have: $q(r^*) \cdot p(r^*) = (1+r^*) \cdot v(r^*) + w(r^*) = (1+r^*) \cdot v^s(r^*) + w(r^*)$, where the last expression is the gross income per worker as previously defined.

Following the standard argument,[15] let r* be an equilibrium. We say that it is locally (asymptotically) stable if:

$$\left.\frac{dv}{dr}\right|_{r^*} - \left.\frac{dv^s}{dr}\right|_{r^*} < 0. \tag{21}$$

Now, the investment function derivative can be always decomposed into a real effect and a price effect, as shown in equation (13). On the contrary, savings manifest themselves as a pure amount of value with no specified physical shape, and can thus take every possible form. As a result, the change in the amount of savings due to a variation in the rate of interest cannot generally be decomposed. When an equilibrium is reached, however, savings are and must be converted into a precise system of (real) assets: the equilibrium vector of capital goods k* = k(r*). The reason why savings do not have generally a specified physical form is in fact precisely what allows them to take the one required by firms in equilibrium.[16]

For the purpose of analysing local equilibrium stability in a small neighbourhood of r* only, we can therefore decompose the variation of savings into an "asset value effect", which represents the variation in the value of the equilibrium vector of assets k* due to the change of the interest rate, and a residuum z:

$$\left.\frac{dv^s}{dr}\right|_{r^*} = \underbrace{k^* \cdot \left.\frac{dp}{dr}\right|_{r^*}}_{\text{asset value effect}} + z. \tag{22}$$

[15] Let us call the difference v(r) – $v^s(r)$ "excess demand for capital". The equilibrium r* is locally stable if the excess demand for capital and r vary in opposite directions in a neighbourhood of r*.

[16] According to Walras, for example, agents convert their savings directly into capital goods in order to lend them to firms, which demand their use (cf. Walras 1977 [1926]: 479-482). This mechanism has recently been taken up by Garegnani in order to argue that "capital goods are demanded by savers as elements of a single commodity, 'perpetual net income'" (Garegnani 2011: 46). Alternatively, we can imagine that savings are used in order to buy securities issued by firms and representing the value of their capital. In this case too, in equilibrium, savings per worker are converted into the vector of capital goods k*, albeit indirectly.

Once this distinction is made, we see that the local equilibrium stability is affected by the real Wicksell effect and by the residuum only, while the price Wicksell effect has no relevance because it is compensated for exactly by the asset value effect:

$$\left.\frac{dv}{dr}\right|_{r^*} - \left.\frac{dv^s}{dr}\right|_{r^*} = \underbrace{\left.\frac{dk}{dr}\right|_{r^*} \cdot p(r^*)}_{\text{real effect}} + \underbrace{k^* \cdot \left.\frac{dp}{dr}\right|_{r^*}}_{\text{price effect}} - \underbrace{k^* \cdot \left.\frac{dp}{dr}\right|_{r^*}}_{\text{asset value effect}} - z = \underbrace{\left.\frac{dk}{dr}\right|_{r^*} \cdot p(r^*)}_{\text{real effect}} - z \tag{23}$$

5. WICKSELL EFFECTS AND STABILITY

We are now in a position to discuss the relevance of reswitching or, in more general terms, of the positive real Wicksell effect, for the instability of the equilibrium between investments and savings.

Even though stability depends on the real Wicksell effect, a positive real Wicksell effect is in general neither a necessary nor a sufficient condition for instability. This is due to the presence of the residuum z, which can determine stability or instability independently of the real Wicksell effect. The relevance of the latter for equilibrium stability can, however, be studied by imposing some restrictions on the residuum.

In particular, if we assume a non-negative residuum – i.e. $z \geq 0$, which means that the change in savings due to an increase in r is equal to or greater than the asset value effect – then the positive real Wicksell effect becomes a necessary, but not sufficient, condition for instability.

We can go further and assume that $z = 0$, i.e. that savings vary exactly as much as the value of the equilibrium vector of assets, at least in a small neighbourhood of r*. In this case, the equilibrium can be locally unstable if and only if the real Wicksell effect is positive.

The shape of the investments or demand-for-capital curve – i.e. of the function v(r) – is instead irrelevant for stability. In fact, as stated above, this shape is affected both by the real and by the price Wicksell effect, and since it is possible for the latter to prevail over the former, we can have a monotonically increasing demand for capital curve associated with a well-behaved real effect or a monotonically decreasing curve in a case with reswitching. As a result, we can have stability with an increasing investments curve and instability with a decreasing one.

6. CONCLUSIONS

The possibility of an increasing demand for capital schedule, at least in a certain stretch, which emerged as a result of the capital debate of the 1970s, has been viewed as a possible cause of instability for the equilibrium on the capital market. Authoritative examples of this view include Garegnani (1990: 61-71), Kurz & Salvadori (1995: 447, 8) and Petri (2004: 226, 7).

This argument was mainly presented in terms of Wicksell's theory, where the exogenously given supply of capital[17] ultimately proves misleading, however, for two reasons. First, since the supply of capital is apparently independent of the interest rate, the property of stability seems to depend on the shape of the demand-for-capital curve alone rather than the shape of the excess-demand curve, as is usually the case. Second, as is well-known, the value of any bundle of commodities cannot be consistently taken as given before income distribution and relative prices are determined and, as a result, numerical solutions of Wicksell's equations cannot be regarded as an economically meaningful equilibria. Both these objections were indeed raised (see for example Potestio 1999 and Bloise & Reichlin 2009).[18]

Here we have tried to put the possibility of instability arising from capital paradoxes on a different basis.[19] We have con sidered a model in which demand for capital comes from investment

[17] In accordance with the analysis of Kurz & Salvadori (1998) and (2001), the given amount of capital in value terms found among the data in the theory of value and distribution of Jevons, Böhm-Bawerk and Wicksell, for example, is to be seen as the value of a specified basket of commodities, the wage goods, taken in the proportions in which workers consume them.
Kurz & Salvadori make use of this in order to show the irrelevance of the choice of the numéraire for equilibrium stability. According to our argument, however, the same fact can also be used in order to determine what we call the asset value effect.

[18] For an assesment of the objection raised by Potestio (1999) and Bloise & Reichlin (2009) see also Kurz & Salvadori (2001) and Fratini (2007).

[19] An initial attempt to show the risk of instability arising from capital paradoxes for the equilibrium between investments and savings is in Garegnani (1970: 425, 6). Garegnani's argument is, however, not very clear. Starting from an equilibrium position, he imagines an increase in savings that

decisions and its supply from saving decisions. In this model, where saving decisions vary with the rate of interest, the local stability of the equilibrium on the capital market depends on the sign of the derivative of the "excess demand for capital", i.e. the difference of the derivatives of the functions of investments and savings.

With a view to identifying the phenomena that can affect this local stability, we have noted that the amount of savings, which generally presents itself as a pure amount of value, is converted in equilibrium into a vector of real assets: the capital goods. This has allowed us, in a small neighbourhood of the equilibrium rate of interest only, to decompose the change in savings due to a change in the interest rate into an "asset value effect" and a residuum.

On the investment side, we know that the change in the demand for capital when r varies can always be decomposed into a real and a price Wicksell effect. And since the price Wicksell effect compensates exactly in equilibrium for the asset value effect on savings, the local stability of the equilibrium ultimately depends on the real Wicksell effect and on the residuum.

In cases where the residuum is well-behaved or even negligible, instability is therefore possible only if the real Wicksell effect is positive, as happens in the case of reswitching, for example. The shape of the investments curve appears instead to be not particularly revealing for stability, as it is also affected by the price Wicksell effect.

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appears to be due to a change in consumers' behaviour. This increase in savings tends to lower the rate of interest, but if the investment schedule is not well-behaved, the demand for capital in value terms decreases together with the interest rate, so that the gap between saving and investments, according to Garegnani, does not disappear and can even increase. His conclusion is as follows:

> we are forced to the conclusion that a change, however small, in the "supply" or "demand" conditions of labour or capital (saving) may result in drastic changes of r and w. That analysis would even force us to admit that r may fall to zero or rise to its maximum, and hence w rise to its maximum or to fall to zero, without bringing to equality the quantities supplied and demanded of the two factors. (Garegnani, 1970, p. 426)

It is therefore unclear whether, in Garegnani's argument, the problem is one of comparative statics, of equilibrium instability, or even of equilibrium existence, as Bliss (1970) understood it to be.
More recently, Schefold (2005) discussed the possibility of instability of the Arrow-Debreu equilibrium associated with reswitching in the case of the readjustment after a change in the economy's endowment of labour. The possibility of instability of the Arrow-Debreu equilibrium due the adjustment on the saving market is also addressed in Parrinello (2011). Here, on the contrary, we do not refer to the Arrow-Debreu model.
A discussion of the stability of the market of savings and investments very close to the one here presented is given in Petri (2011), where an early version of the present work is also mentioned.

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