Centro di Ricerche
e Documentazione
"Piero Sraffa"

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Centro Sraffa Working Papers
n. 40

May 2020

ISSN: 2284-2845
Centro Sraffa working papers
[online]

EXHAUSTIBLE RESOURCES AND CLASSICAL THEORY

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Abstract: Smith, Ricardo, Marx and Sraffa made no theoretical distinction between exhaustible resources and lands. The very notion of exhaustibility, however, can be opposed to that of 'indestructible powers of the soil' (Ricardo) and calls for a specific analysis distinct from that of rent. The diversity of the contemporary attempts to deal with that question in a classical framework shows how varied are the understandings of the main methodological features of classical theory. Three crucial points emerge: first, the treatment of prices, which are invariant in classical theory but, according to the Hotelling rule, are changing through time for exhaustible resources; second, the notion and the measure of the rate of profits; and, third, the relationship between economic analysis and a more historical and sociological approach stressing the balances of power between classes. Our own approach starts from a very simple model, called the corn-guano model, where guano is the exhaustible resource, and examines the dynamics of such an economy on the physical side and the value side. These lessons serve as a basis for an extension to multisector models. We provide a critical assessment of a few alternative approaches developed by Sraffian scholars.

Keywords: exhaustible resources; classical theory; Hotelling rule; Sraffian approach

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Acknowledgements: We are grateful to the reviewers and editors of *Œconomia* and to a reviewer of the Centro Sraffa Working Papers for their helpful comments, which have allowed us to ameliorate the paper.

1 A THEORETICAL CHALLENGE

When classical economists discussed natural resources, they mostly thought of land. Although they did occasionally refer to what we now call exhaustible resources, e.g., when dealing with mines, they did not perceive these as sufficiently different from land to deserve specific theoretical attention. A typical example is David Ricardo, who defined rent as “that portion of the produce of the earth, which is paid to the landlord for the original and indestructible powers of the soil” (Ricardo, [1817] 1951: 67). He criticised Adam Smith’s inconsistent views on rent (*ibid.*: Chapter XXIV) but argued that the principle of differential rent which he applied to the case of lands was “precisely the same” (*ibid.*: 85) for the case of mines. In the same vein, Marx ([1867] 1962) and later Sraffa (1960) paid little attention to exhaustible resources in their economic theories. Sraffa, for instance, referred to “[n]atural resources which are used in production, such as lands and mineral deposits, and which being in short supply enable their owners to obtain a rent” (1960: 74), seemingly ignoring the theoretical specificity of exhaustible resources which are not produced but, unlike land, are destructible in the long run.

For a long time, therefore, those working in the tradition of classical political economy acknowledged the existence of exhaustible natural resources, but considered them to be insufficiently distinct from land to merit special attention. This changed in the 1980s. The first to squarely address the question of the integration of exhaustible resources into classical theory was Sergio Parrinello (1983). His aim was to examine the compatibility of Sraffa’s formalisation of classical theory and the Hotelling rule with regard to exhaustible resources, which is a result associated with neoclassical theory. The integration of exhaustible resources is an important methodological question: since the Hotelling rule obliges us to go beyond the Sraffian framework, one must identify the main characteristics of classical theory in order to preserve them in an enlarged approach. In that respect, we retain three main features of classical theory:

(i) The primacy given to production. Classical theory ignores the notion of utility, and it is often assumed that final demand is given independently of prices.

(ii) The theory of distribution is linked with a vision of a capitalist society divided into classes: capitalists own the means of production and receive profits, workers supply labour and receive wages, and landlords make their lands available for production and receive rents. If landlords are ignored for simplicity, the distribution of the net product between these two classes results from a balance of power. While Ricardo and Marx took the level of the wage as a historical datum, Sraffa chose the rate of profits as the exogeneous distribution variable.

(iii) A long-term view of the economy. The Classicals did not ignore the influence of demand on prices, but considered that the relevant economic question is that of the determination of long-run prices, once production is adapted to the ‘normal’ level of demand. The price of a commodity is then explained by its difficulty of production. The most well-known form of that conception is the labour theory of value, modified by Sraffa into a theory of prices of production, in which relative prices and wages are constant from period to period and which takes into account the influence of distribution on prices. Departing from Sraffa himself, most Sraffians have interpreted the notion of long run in a restrictive sense and identified it with that of a steady state or of a regular growth path.

Taking into account exhaustible resources leads us beyond the traditional Sraffian framework for two reasons: from a physical standpoint, the intertemporal path cannot be the same before and after exhaustion of a resource and, as far as prices are concerned, the Hotelling rule entails an intertemporal change in the price of the resource, and therefore a change in the prices of all commodities directly or indirectly produced by it. As a result, the dynamics of quantities and prices are more complex than those considered by Sraffa and his followers. The innovative (or is it explosive?) nature of Parrinello's question becomes apparent now: how can classical theory be adapted to the study of exhaustible resources?

Following Parrinello's pioneering article, several authors inspired by Sraffa's ideas have attempted to come to grips with the issue of exhaustible resources. A first wave of contributions was made by Neri Salvadori (1987), Bertram Schefold (1989) and Heinz Kurz and Neri Salvadori (1995, 1997, 2000). In 2001 a peak was reached when *Metroeconomica* published a 'Symposium on exhaustible natural resources and Sraffian analysis', edited and introduced by Ian Steedman. This included papers by Christian Bidard and Guido Erreygers (2001a, 2001b), Eiji Hosoda (2001), Heinz Kurz and Neri Salvadori (2001), Christian Lager (2001), Sergio Parrinello (2001) and Bertram Schefold (2001). Since then, Marco Piccioni and Fabio Ravagnani (2002), Heinz Kurz and Neri Salvadori (2002, 2009, 2011, 2015), Sergio Parrinello (2004), Fabio Ravagnani (2008) and Biao Huang (2018) have further refined the analysis. We will return to some of these contributions later in the paper. The 2001 symposium found its origins in the debate sparked by the corn-guano model, a pedagogical device which we developed in order to deal with the issue of exhaustible resources in a simple and transparent way. We remain convinced that a simple economic model can be useful as a first step in the process of analysing a complex problem and, in this respect, our approach differs from the one followed by all other authors.[1] Of course, further steps are required to verify which properties remain valid in a more general framework (see also Bidard and Erreygers, 2020).

We begin by explaining the background and properties of the corn-guano model (Section 2). Next, we discuss what in our view constitute major obstacles for the generalisation of the model (Section 3). We then develop a few ideas for the multisector version of the model (Section 4). We also provide a critical account of alternative answers given by other economists of classical inspiration, as the point at stake behind a seemingly rather technical question is the very understanding of classical theory (Section 5). We end with a few concluding remarks (Section 6).

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[1] Kurz and Salvadori (1995: 366-368) however studied an energy-oil model close to the corn-guano model but with different properties (e.g., the method using the exhaustible resource and the backstop method are used simultaneously during several periods, a phenomenon excluded in the corn-guano model). We shall not examine that model, as the authors have changed their formalisation since and work on multisector models directly.

2 THE CORN-GUANO MODEL

Corn models have been used frequently by authors of classical inspiration (Skourtos, 1991). Sraffa famously attributed a ‘corn-ratio’ theory of profits to Ricardo (Sraffa, 1951: xxxi-xxxiii), an interpretation which has been challenged. Whether or not Ricardo had such a simplified economic model in mind, we find it appropriate to start with a brief reminder of the corn model (see chapter 1 of Bidard, 2004, for a more extensive presentation). We then introduce exhaustible resources and the corn-guano model.

2.1 The Corn Model

Let us assume an extremely simple economy in which there exists only one commodity, corn, which is produced by means of itself and labour. Let us furthermore assume that the same production process is used year in, year out, and that it can be described schematically as follows:

$$a \text{ (corn)} \oplus l \text{ (labour)} \to 1 \text{ (corn)} \tag{1}$$

Two economic properties can be enunciated:

(C1) Maximum rate of profits: Even if labourers ‘could live of the air’ (Marx, [1894] 1964: Ch. 15, section II, 257), corn must be invested in order to produce corn, and therefore the rate of profits r is finite. Its maximum level R can be interpreted in physical terms:

$$1+r < 1+R = 1/a \tag{2}$$

(C2) Ricardian trade-off: In the interval $[0, R]$ the variations of the rate of profits are inversely related to those of the real wage.

Formally, these properties follow from the price equation associated to the operation of process (1), which is written as:

$$(1+r)(ap+lw) = p \tag{3}$$

where p stands for the price of corn and w for the wage.[2] One way of studying this equation is to take corn as numeraire ($p=1$), in which case equation (3) determines the relation between the rate of profits and the real wage expressed in units of corn (i.e., w/p). Alternatively, we can choose the wage as numeraire ($w=1$). We then obtain a relation between the rate of profits and the price of corn expressed in labour (i.e., p/w, the inverse of the wage expressed in corn). Whatever the numeraire – a mixed corn/labour numeraire would yield exactly the same result – we always find the same relation between the 'real rate of profits' and the 'real wage' and the above-mentioned two properties hold.

If several processes of type (1) are available, the operated process is the least expensive of all, given the rate of profits. We have the following choice-of-technique property:

(C3) Wage-maximisation: For a given rate of profits, the operated process coincides with the process that maximises the real wage.

2.2 The Corn-Guano Model (I)

We now introduce exhaustible resources by transforming the corn model into the corn-guano model. This model is conceived as a methodological tool: its analytical simplicity allows us to shed light on the distinct economic features linked to the introduction of exhaustible resources. Its structure is rich enough to initiate the reader to the study of the dynamics of models with exhaustible resources. The dynamics of the more general models which we will explore in Section 3 are certainly more complex, but some of the intricacies are already present here.

The study of exhaustible resources is inexorably linked to the 'Hotelling rule', a seminal result first derived by Harold Hotelling (1931). Basically, an exhaustible resource owner faces a choice between selling one unit of his resource at date t, or letting it lie idle and sell it at date $t+1$. In the first case the sale gives the owner an immediate revenue $z(t)$, i.e. the price (or royalty) of one unit of the resource at date t, which can then be invested at the rate r. In the second case the owner simply waits and obtains revenue $z(t+1)$ at date $t+1$. If one of the two options were more profitable than the other, *all* resource owners would have an incentive to follow the most profitable course of action, which would mean that either the *whole* supply of the resource would be exploited at date t, or that *none* of it would. But since in any period of time before exhaustion, part of the supply is exploited and part of it is conserved, the two options must be equally profitable in equilibrium. As the first option allows the owner to obtain a rate of return r, then so must the second. This implies that the royalty $z(t)$ must increase at the rate r, a result known as the Hotelling rule.

[2] Equation (3) is in line with the classical tradition according to which wages are advanced (we adopt the same hypothesis later for royalties). By contrast, Sraffa's convention is that the wage is paid *post factum*. All formulae are easily adapted to one or the other hypothesis, and the qualitative properties of the models are not affected by these choices.

In the corn-guano model, there is only one produced commodity, called corn, and one exhaustible resource, called guano. Corn can be produced in a one-period time either by means of the 'guano method':

$$a_1 \text{ (corn) } \oplus \ l_1 \text{ (labour) } \oplus \ 1 \text{ (guano)} \rightarrow 1 \text{ (corn)} \tag{4}$$

or by means of the 'backstop method' (which will be necessarily used after the exhaustion of the stock of guano):

$$a_2 \text{ (corn) } \oplus \ l_2 \text{ (labour)} \rightarrow 1 \text{ (corn)} \tag{5}$$

A unit of guano not used up to date t remains available at date $t+1$, which we represent by means of the 'guano conservation method':

$$1 \text{ (guano)} \rightarrow 1 \text{ (guano)} \tag{6}$$

These three processes admit constant returns.

As is usual in Sraffian models, we treat the rate of profits r as exogenously given. If, during period t, which starts at date t and ends at date $t+1$, process i is operated (i.e., its activity level is strictly positive), that process breaks even at the given rate of profits. Non-operated methods, by contrast, pay extra-costs.

We assume that if guano were free, the guano method would be less costly than the backstop method. Otherwise, none would ever want to use guano for the production of corn. If in a given period the two corn methods are operated simultaneously, the price paid for the use of guano, or royalty, can be interpreted as a kind of differential rent: it is equal to the difference in the production costs of the two methods when guano is ignored. For simplicity, we assume that the period T when the stock of guano becomes exhausted is known. The underlying hypotheses may be that the initial stock and the demand for corn are exogenously given and that the guano method is continuously used until exhaustion.[3]

In this section we analyse the first version of the model (Bidard and Erreygers, 2001a), characterised by the choice of corn as numeraire: $p(t)=1$ for any t. Since the royalty and the

[3] *Ex post*, one must check that the last assumption is consistent with the analysis of prices, i.e. one must check that, up to date T, the guano method is cheaper that than the backstop method.

wage change with time, time indications are added. The following properties can be established easily:

(CG1) Maximum rate of profits: The rate of profits has a finite upper bound.

(CG2) Ricardian trade-off: In every period the rate of profits and the real wage are inversely related.

(CG3) Maximum royalty: In the period of exhaustion T, the royalty $z(T)$ is equal to the differential rent between the two methods of corn production.

(CG4) Hotelling rule: The royalty at date t is equal to $(1+r)^{t-T} z(T)$.

(CG5) Continuous use of guano: For any $t < T$, the guano method is cheaper than the backstop method. Therefore the guano method is used first, until exhaustion of the stock of guano.

(CG6) Translation principle: Let us consider two economies which only differ by their initial stock of guano. When the exhaustion dates are chosen as origins of time, the dynamics of prices and quantities are identical, except that the past is truncated in the economy with the smaller stock.

(CG7) Fall of the real wage: For a given rate of profits, the real wage decreases from date 0 to T and reaches its long-term level at date T.

(CG8) In a corn-guano model with several alternative guano methods, the cost-minimising method at each date before exhaustion coincides with the wage-maximising method, given the rate of profits and the level of the royalty at that date.

A few comments are in order:

(i) The upper bound of $1+r$ is equal to $\min[1/a_1, 1/a_2]$. Property (CG1) is in accordance with property (C1) of the corn model: the rate of profits cannot exceed the maximum level which can be sustained by either of the two methods of corn production.

(ii) Property (CG2) is an immediate generalisation of property (C2).

(iii) A simple economic argument for (CG3) is: "At the moment of exhaustion [...] we expect the backstop method to be used alongside the guano-method. Only by fluke would the then remaining supply of guano be sufficient to satisfy the whole demand for corn by means of the guano process: normally the remaining quantity will be too low, and the backstop process must be operated to fill the gap." (Bidard and Erreygers, 2001a: 249)

The coexistence of the two processes in the period of exhaustion requires that they are equally costly at that time. That condition determines the royalty at the date T of exhaustion, when it is equal to the differential rent between the two corn methods. In this basic version of the corn-guano model, the simultaneous operation of the two methods occurs in the period of exhaustion, but not in other periods.

(iv) As long as guano is not exhausted, the preservation process (6) is operated, which implies that the price of guano rises at a rate equal to the rate of profits. Property (CG4) is nothing but the Hotelling rule.

(v) Property (CG5) justifies *ex post* a simplifying assumption made at the beginning.

(vi) According to property (CG6), once the period of exhaustion is known, everything that precedes it can be determined using the principle of backward induction.

(vii) The distribution of the net product between profits, wages and royalties changes over time as a result of the non-constant price of guano. At a given rate of profits, it is therefore not surprising that the increase of the price of guano entails a decrease of the real wage. From a theoretical point of view, the important thing to note is that the real wage $w(t)$ changes from period to period for no other reason than the future exhaustion of guano:

$$w(t) = w(T) + \frac{1-(1+r)^{t-T}}{l_1} z(T) \quad (t < T) \qquad (7)$$

This is an unexpected property from a classical perspective, and property (CG7) constitutes the most significant difference between the corn model and the corn-guano model.

(viii) Property (CG8) in an extension of property (C3) of the corn model, the difference being that, for a given rate of profits, the wage-maximising method is uniquely defined in the corn model, whereas it changes with the level of the royalty in the corn-guano model.

3 MULTISECTOR MODELS AND MEASUREMENT OF PROFITS

Our aim is to examine whether the properties of the corn-guano model carry over to multisector models with one exhaustible resource. Up to now, we have identified the corn model with a one-good model, and the corn-guano model has been obtained by introducing one exhaustible resource into it. In a more faithful historical interpretation, a corn model is a model in which corn is the only basic commodity. Sraffa (1951) pointed out that the move from the corn model to general multisector models introduces the question of values, i.e. prices. In a multisector setting with an exhaustible resource, the Hotelling rule and the fact that the exhaustible resource enters directly or indirectly in the production of other goods implies that the relative prices of goods will also change over time, and that evolution is the source of conceptual problems which need to be tackled first. That issue is not exclusively linked to the Hotelling rule: if relative prices change for another reason, the same problems occur. Here, we highlight three intimately connected aspects: the measurement of the rate of profits, price effects, and real effects.

3.1 The Numeraire and the Rate of Profits

Consider a multisector model and suppose that at date t we invest a basket of inputs equal to a and obtain at date $t+1$ a basket b of outputs. If a and b are proportional $(b=\lambda a)$, the rate of profits can be interpreted in physical terms and amounts to $\lambda-1$. Similarly, if the relative prices p at dates t and $t+1$ are the same (a hypothesis retained in the classical theory of value), the rate of profits is calculated by means of these prices and amounts to $r=b'p/a'p-1$.[4] But outside a steady state or a regular growth path, be it for the study of out-of-equilibrium paths ('gravitation problem', with different sectoral rates of profits at date t) or that of equilibrium paths (uniform rates but with varying relative prices, as in the presence of exhaustible resources), the definition and measurement of the rate of profits sets a problem. From now on, we add time labels to distinguish relative price vectors at different moments of time, i.e. $p(t)$ at date t, $p(t+1)$ at date $t+1$, etc. Since inputs and outputs may also vary from period to period, we write $a(t)$, $b(t+1)$, etc.

For a multinational firm which trades in dollars and euros, with a flexible exchange rate between the two currencies, the yearly rate of profits differs according to the currency chosen as numeraire (it is higher when calculated in terms of the one which depreciates). A similar phenomenon occurs in multisector models for intertemporal production with changing relative prices: the rate of profits of the firm is not the same according to the commodity or basket of commodities chosen as numeraire, and therefore the references to a 'rate of profits' and to its uniformity across industries depend on the prior definition of a numeraire. The natural numeraire is money. In non-monetary models, a commodity or a basket n of commodities is used as numeraire, which means that the prices $p(t)$ and $p(t+1)$ at dates t and $t+1$ are normalized by setting $n'p(t)=1$ and $n'p(t+1)=1.$ The *apparent* rate of return r_n, i.e. the one that appears when using the given numeraire n for purposes of valuation, compares the nominal value of the investment $a(t)$ to the nominal value of the outcome $b(t+1)$:

$$r_n=\frac{b'(t+1)\,p(t+1)}{a'(t)\,p(t)}-1 \tag{8}$$

That rate is independent of the numeraire if relative prices are constant, a common hypothesis in long-run models. If they change, there are as many apparent rates of return as numeraires, and one may wonder if some numeraire would be more significant than the others and could be used

[4] All vectors represent column vectors; transposition is indicated by a prime.

to define an absolute rate of return. This question of measurement and the search for the definition of 'absolute values' pervades the history of economic thought: Ricardo, for instance, expected that gold could be such a standard by means of an appropriate management of money, and his very last writings were about the search of a commodity whose difficulty of production would be constant and, therefore, could be used as an invariant standard (Ricardo, [1823a] 1951, [1823b] 1951). In neoclassical theory, production is conceived as being oriented towards the satisfaction of needs, and the representative consumption basket is a quite natural standard. In any case, one would like to take "what really counts for us" as a standard of measure. If we trade coal but consume only corn, then it would certainly interest us to compare the units of corn we sacrifice at time t by buying coal to the units of corn we earn at time $t+1$ by selling coal. For this both the investment and the outcome must be expressed in corn, which in this case represents what matters.

Let us assume there exists a specific basket s, called the standard of value, which captures what really counts for us. At time t, the *real* (or *absolute*) rate of return r^*, based upon a comparison of the real investment and the real outcome, is defined by formula (8) assuming that the values $p(t)$ and $p(t+1)$ are those corresponding to the specific numeraire $n = s$. Alternatively, the real rate of return can also be derived from the apparent rate of return by means of the factor of appreciation of the standard of value. Let $1+\theta_n$ be the factor of appreciation of the standard of value s in terms of the prices defined by the numeraire n, i.e.

$$1+\theta_n = \frac{s'p(t+1)}{s'p(t)} \qquad (9)$$

The relationship between the real and the apparent rates of return is then such that:

$$1+r^* = \frac{1+r_n}{1+\theta_n} \qquad 10)$$

The fact that the value of the rate of return depends on the numeraire was pointed out long ago by Irving Fisher: "the rate of interest is always relative to the standard in which it is expressed" (Fisher, 1930: 41). We assume here that all investors adopt the same standard of value, although we acknowledge this should not to be taken for granted, as Keynes (1936) observed. In that case, the analysis is simplified by assuming that the numeraire is equal to this shared standard of value, implying that the apparent rate of return is also the real rate of return. The intertemporal price equations of a system of production (A,l,B) can then be written as $(1+r^*)(Ap(t)+lw(t)) = Bp(t+1)$. It is possible to choose a different numeraire, but then the intertemporal price equations must be written as $(1+r^*)(1+\theta_n)(Ap(t)+lw(t)) = Bp(t+1).$ In what follows, we assume that the numeraire coincides with the standard of value.

We end this discussion on the influence of the numeraire by drawing attention to an unexpected phenomenon which occurs when labour is chosen as numeraire, the rate of profits r being given. In a corn model with changing prices, the dynamics of prices are then defined by $(1+r)(ap(t)+l) = p(t+1),$ with $0 < a(1+r) < 1.$ Therefore, the price of corn at date t amounts to

$$p(t) = p^* + \left[a(1+r)\right]^t (p(0) - p^*) \tag{11}$$

where p^* is the long-run price defined by equality $(1+r)(ap^*+l) = p^*.$ When time passes, the sequence of prices tends towards its long-term level. But let us look at the past ($t < 0$): if $p(0) < p^*,$ the prices must have been negative at some point; if $p(0) > p^*,$ they were positive but arbitrarily high, so that the real wage was negligible. But then the effective rate of profits is close to $R = 1/a - 1,$ when it is supposed to be equal to r. This shows that the choice of labour as numeraire, which is quite natural when studying long-term positions, is problematic in a dynamical framework.

3.2 Price Effects

For given but different price vectors $p(t)$ and $p(t+1)$, the rate of profits in an industry depends on the numeraire used to measure it. A dual aspect of the same phenomenon is that, for a given rate of profits across industries, the relative prices depend on the numeraire. The following calculations illustrate that point.

Let there be a multisector model with a unique technique (A, l, I), where A is the square matrix of material input coefficients (constant returns are assumed) at date t, l the labour vector and I the output matrix at date $t+1$. Let us choose a basket d $(d > 0)$ as numeraire. For a given rate of profits r, the price vector and the wage evolve according to the rule:

$$(1+r)[Ap(t)+lw(t)] = p(t+1) \tag{12}$$

After pre-multiplication by d', we get:

$$(1+r)(d'l)w(t) = d'p(t+1) - (1+r)d'Ap(t) \tag{13}$$

and since $d'p(t) = d'p(t+1) = 1$, we obtain:

$$(1+r)(d'l)w(t) = d'\left[I - (1+r)A\right]p(t) \tag{14}$$

Formula (14) shows that, for a given rate of profits, the price vector $p(t)$ at date t determines the wage $w(t)$, hence the next price vector $p(t+1)$ by formula (12):

$$p(t+1) = (1+r)\left[A - \frac{ld'A}{d'l}\right]p(t) + \frac{l}{d'l} \tag{15}$$

Formula (15) makes the influence of the numeraire on the evolution of relative prices explicit. The sequence of relative prices is stable in the special case $p(t) = p^*$ where p^* is defined by equalities $p^* = (1+r)(Ap^* + w^*l)$ and $d'p^* = 1$. But the conditions of convergence of the sequence $p(t)$ towards p^* are modified. In the present framework, that condition is written:

(H) The dominant eigenvalue of matrix $\bar{A} = A - \dfrac{ld'A}{d'l}$ is smaller than $(1+r)^{-1}$.

Condition (H) differs from the usual hypothesis which refers to the input matrix A only and, for instance, the eigenvalue with maximum modulus of $\bar{A}$ may be complex, as matrix $\bar{A}$ is not semipositive.

3.3 Real Effects

In the presence of several methods of production in the same industry, the very fact that the numeraire affects prices implies that the relative costs of two alternative methods may change with the numeraire. As a consequence, the cost-minimising method may also change, and therefore the competitive intertemporal path: it is easy to build a one-period example with such a switch of method according to the numeraire.

Moreover, an indeterminacy problem arises. Given the numeraire and knowing the prices $p(t)$, two phenomena occur: on the one hand, the operated technique determines the wage of the period by (14); on the other hand, the prices and the wage determine the costs of production which, in a competitive framework, are minimum. Because of that cross determination, it is unclear whether the real and the value dynamics are well determined (existence and uniqueness). The following result provides a positive answer:

Wage-maximisation property. Given the rate of profits, the numeraire basket and the price vector $p(t)$ at date t, the operated technique is the wage-maximising technique.

That property (Bidard and Erreygers, 2020) is a non obvious extension to the dynamic framework of a standard result when relative prices are constant through time. It is also an extension of property (CG8) of the corn-guano model. A significant gap with the standard result is that the wage is maximum in terms of the numeraire, which may differ from the workers' effective consumption basket.

4 DYNAMICS AND THE NATURAL PATH

4.1 The Corn-Guano Model (II)

The question of the measure of profits arises in multisector models as soon as the relative price of two commodities changes through time. We know that, in the corn-guano model, the wage evolves with time. Therefore, a similar problem of measure is met in the corn-guano model when a given combination corn and labour is chosen as numeraire. Hence the second version of the model (Bidard and Erreygers, 2001b), which is a pedagogical device allowing us to maintain a very simple analytical framework (only one produced commodity) and to explore the difficulties met when one tries to extend properties (CG1) to (CG8) to multisector models. The emphasis is on the dynamics.

It is now assumed that the numeraire is a given combination of d units of corn and f units of labour, with both d and f positive (by contrast, in our first version we assumed $d = 1$ and $f = 0$). Hence, we have $dp(t) + fw(t) = 1, \forall t$. There is no harm in assuming $f = 1$. The dynamics of prices and wages until exhaustion are defined by the equations:

$$p(t+1) = (1+r)[a_1 p(t) + l_1 w(t) + z(t))] \quad (t = 0,...,T-1) \tag{16}$$

$$z(t) = (1+r)^{t-T} z(T) \qquad (t = 0,...,T-1) \tag{17}$$

$$a_1 p(T) + l_1 w(T) + z(T) = a_2 p(T) + l_2 w(T) \qquad 18)$$

$$dp(t) + w(t) = 1 \qquad (t = 0, ..., T) \qquad (19)$$

That is, there are $3T+2$ equalities to determine $3T+3$ unknowns ($T+1$ prices, $T+1$ wages and $T+1$ royalties). It thus appears that there now exists one degree of freedom in the determination of the price dynamics: it might be the inital price of corn, the initial royalty, the final royalty, etc.

When corn was the numeraire ($p(t)=1$), equality $1=(1+r)(a_2+l_2 w(T))$ held at date T and showed that the wage and the price at the exhaustion date T immediately reached their long-run values (w^{**}, p^{**}) associated with the backstop method. It may be tempting to introduce a similar hypothesis in the new framework, and that additonal condition would serve as the missing equation for a full determination of the dynamics. The following argument shows that this cannot be the case.

Let the final price $p(T)$ be arbitrarily given. The forward and the backward dynamics are then fully determined. Let us look at the backwards dynamics: the wage $w(T)$ is determined by the numeraire equation (19), then the royalty $z(T)$ by (18). According to the Hotelling rule (17), the royalties $z(t)$ at any date ($t<T$) are known. And, once the price and the wage at date t are known, those at date $t-1$ are obtained by solving the price equation (16) and the numeraire equation (19). The price of corn τ periods before exhaustion is given explicitly by formula:

$$p(T-\tau) = p^* + (1+r)^{-\tau}(1-\alpha_1)^{-1} z(T) + [(1+r)\alpha_1]^{-\tau} C \qquad (20)$$

where p^* would be the long-run price of corn associated with the guano method *if* guano were free, $\alpha_1 = a_1 - l_1 d$ and C is a constant calculated in order that, for $\tau = 0$, formula (20) fits with the already known values of $p(T)$ and $z(T)$. It is assumed that $|\alpha_1| < (1+r)^{-1}$ (the condition is necessary to have $p^* > 0$). Suppose that the initial stock of exhaustible resource is great enough, so that τ can take large values. If C is negative, the same holds for the value of $p(0)$; if C is positive, the value of $p(0)$ exceeds d^{-1} and therefore that of $w(0)$ is negative. The conclusion is that the path defined by the value $C=0$ is the only one compatible with any initial stock of guano. Let us call it the 'natural path'.

On the natural path at date T, the price of corn, the wage and the royalty are uniquely defined by the three equalities (18), (19) and (20) with $\tau = 0$ and $C = 0$. There is no reason why $p(T)$ should be equal to the long-run price determined by the backstop method. As a consequence, it will continue to evolve according to the forward dynamics defined by the backstop method: in simple words, the price of corn evolves initially because guano will become exhausted, and eventually because guano has been exhausted! Property (CG7) does not hold any longer for a mixed corn-labour numeraire.

For a given stock of guano, the natural path is only one of the infinitely many feasible paths obeing the dynamics. However, when the stock of the exhaustible resource is large, all feasible paths tend to converge to the natural path, as shown by formula (20) (see Bidard and Erreygers, 2001b, for more details).

4.2 The Dynamics of Multisector Models

We now consider a true multisector model with $n \geq 2$ commodities, with one exhaustible resource (guano) used in one sector (agriculture). The wage-maximisation property (CG8) has a general validity, independently of the question of exhaustible resources. The translation principle also holds because, if one knows the prices, the wage and the royalty at the exhaustion date, the forward and the backward dynamics of values are uniquely defined up from that date, and this also holds in the presence of several guano methods: suppose that, for some stock, a guano method 1 is used for 35 years, then another guano method 2 for 17 years, guano being then exhausted; then, if the economy starts with a lower stock of guano sustaining production for 30 years only, method 1 will be used for 13 years, followed by method 2 for 17 years, with the same dynamics on a lower time interval.

The question we examine in the present section is whether the long-run prices are reached at the exhaustion date (as in the first version of the corn-guano model) or not (as in the second version) when the numeraire is either one commodity or a basket of commodities, labour being excluded.[5]

Let d be the numeraire basket, which may consist of a single commodity. The n prices $p(T)$ at date T, the wage $w(T)$ and the final royalty $z(T)$ are linked by two equalities: the numeraire equation $d'p(T) = 1$ and the property that the royalty at date T can be considered as a one-shot differential rent, the level of which equalises the costs of production of corn by the guano and backstop methods. There remain n degrees of freedom. Let us assume, for simplicity, that the guano method is continuously operated before exhaustion. With (A_1, l_1, I) representing the operated technique, the value dynamics are defined by the equations:

$$p(t+1) = (1+r)[A_1 p(t) + l_1 w(t) + z(T)(1+r)^{t-T}] \qquad (21)$$

[5] Note that property (CG8) then holds, but not for the corn-guano model with a mixed numeraire.

$$d'p(t+1) = 1 \tag{22}$$

In equality (21), $z(T)$ is now a vector whose first component – which corresponds to the agricultural sector – is the royalty at date T and the other components are zero. We can now combine the calculations made above in Sections 3.2 and 4.1: for a value of the royalty at date T considered as a parameter, equality

$$1 = d'p(t+1) = (1+r)[d'A_1 p(t) + d'l_1 w(t) + d'z(T)(1+r)^{t-T}] \tag{23}$$

allows us to determine the level of $w(t)$. Inserting that value in (21) allows us to define $p(t+1)$ as a function of $p(t)$. If corn is not a part of the numeraire (e.g. because another commodity is chosen as numeraire), we have $d'z(T) = 0$ and formula (14) holds exactly. If corn is the numeraire or a part of the numeraire basket, the affine relationship (14) is modified by the presence of a term including the royalty at date $t-T$, but that term is negligible as long as guano is far from being exhausted. Consider now the backward dynamics: the value of $p(t)$ is obtained from that of $p(t+1)$ and, by induction, one obtains a more complex formula analogous to (20)

$$p(T-\tau) = p^* + (1+r)\bar{A}_1^{-\tau} C + \varepsilon(T-\tau) \tag{24}$$

In that formula, matrix $\bar{A}_1$ is equal to $A_1 - \dfrac{l_1 d'A_1}{d'l_1}$, C is a vector and $\varepsilon(T-\tau)$ another vector which tends to zero when $T-\tau$ increases to infinity. Assumption (H) of Section 3.2 ensures the convergence of the long-run dynamics of values of the guano technique, when guano is free. Condition $C = 0$ is the way to ensure the positivity of prices at any date before exhaustion of guano, when the initial stock of guano is large. It defines the natural path, and all feasible paths converge to it. Since there is no reason for the prices at date T to be equal to the long-run prices associated with the backstop technique, the prices will continue to fluctuate after exhaustion. To sum up, the origin of the difficulties mentioned in Section 4.1 does not lie in our adopting a rather bizarre numeraire in that section: it is a general phenomenon in multisector models. From that point of view, the corn-guano model with corn as numeraire is the exception.

5 ALTERNATIVE APPROACHES

In the preceding sections we have explored how exhaustible resources could be integrated in classical theory starting from a simple economic model. However, not everyone working in the classical tradition has embraced our approach. In this section we reflect on some of the alternatives that have been put forward and assess how they have tackled the issues. The presence of exhaustible resources poses difficult challenges to which various responses have been given. Instead of writing an exhaustive survey of the literature, we examine the alternative models with three questions in mind: What are the critiques addressed to the corn-guano and its extensions? What is the conception of classical economics underlying the alternative construction? Is that construction consistent from a mere logical point of view?

5.1 Parrinello: Abandoning Perfect Foresight

Exhaustible resources would not be a problem for economic theory if their exhaustion were of no concern to economic agents, for instance if they foresaw that the supply of these resources would be forever sufficient to cover demand (e.g., because the resource becomes obsolete after a certain date). So the interesting case arises when economic agents do worry about exhaustion. Two points of view may be adopted here. Either one assumes that agents acknowledge that exhaustion will be on the agenda some time in the future, but do not have a clue about the date at which exhaustion will occur. Or, alternatively, one assumes that agents know the date of exhaustion exactly. Let us designate these hypotheses as those of 'imperfect' and 'perfect foresight', respectively. There is no doubt that the hypothesis of imperfect foresight is more realistic. From a theoretical point of view, however, it has the disadvantage of making the determination of prices subject to fragile hypotheses on expectations. By contrast, the hypothesis of perfect foresight is certainly heroic, but it allows us to determine prices.

Following Hotelling (1931) and a large part of the literature on exhaustible resources, we have assumed perfect foresight in our corn-guano model. In his 2001 contribution, Parrinello explicitly rejected this hypothesis and assumed that the date of exhaustion is unknown. In order to close the model he needed to come up with an alternative assumption. The trouble is that Parrinello's assumption – a rank condition – is of a purely mathematical character, and may be in conflict with other assumptions of the model.

Parrinello's oil-corn model is many respects similar to the corn-guano model. As in our model, Parrinello assumed that the rate of profits is given, that corn serves as the numeraire, and that before exhaustion, two processes are available for the production of corn. (In Parrinello's model the processes can change over time, but this is a non-essential variant.) It is easy to show that, in order to arrive at a determinate solution, in exactly one period two processes must be operated simultaneously while in all others only one process (the cheapest of the two available) is used. If two processes were operated in more than one period, the system of prices would be over-determined; and if in no period two processes were used, it would be under-determined. In our corn-guano model a simple economic argument is invoked to state that the two-process period must be the period of exhaustion. Parrinello did not address this economic argument and, in his oil-corn model, the period of exhaustion is unknown and the period of coincidence may be any period before exhaustion. Towards the end of the article, Parrinello seemed to opt for the solution that the two-process period must be the initial period, on the grounds that "[t]he future cannot affect the past" (Parrinello, 2001: 311). But that leads to a problem of time inconsistency: since the period he refers to is 'today', which moves as time passes by, Parrinello's rule (coincidence in the present period) would be wrong tomorrow if it were true today. That contradiction does not occur when the period of coincidence is defined as that of exhaustion.

In 2004 Parrinello proposed a revision of his theory of exhaustible resources. He abandoned the rank condition, and instead introduced the notion of the 'effectual supply' of an exhaustible resource, i.e. the limited quantity of the resource available for production in every period. He claimed that if both the path of effectual demand for goods and the path of effectual supply of exhaustible resources were known, it would be possible to extend the classical theory of prices, and in particular Sraffa's equations for the determination of rent, to the case of exhaustible resources. Parrinello's intent is to put forward Sraffa's idea of given quantities: "No changes in output are considered" (Sraffa, 1960, Preface). One may however notice that this principle is incompatible with any dynamics (and the Ricardian dynamics is undoubtedly a part of the Classicals' inheritance), and that Sraffa himself is unfaithful to it in his study of intensive rent: "The existence side by side of two methods of production can be regarded as a phase in the course of a progressive increase on the land. The increase takes place through the gradual extension of [the more productive method] […]. In this way the output may increase continuously, although the methods of production are changed spasmodically." (Sraffa, 1960, Section 88).

5.2 Schefold on Long-Period Analysis

The corn-guano model is a theoretical tool developed in order to shed light on the problems that arise when one tries to integrate exhaustible resources in a classical approach. It proceeds by building a bridge between the corn model, which belongs to the Ricardian tradition, and Hotelling's seminal model on exhaustible resources. Like its basic bricks, it is an economic abstraction and its ambition is methodological. Its main feature is to proceed by mixing the simplest characteristics of two models: three equations are sufficient and their treatment is transparent. The substantial differences between the solution of the corn-guano model and that of the standard corn model can be attributed unambiguously to the presence of an exhaustible resource. For instance, in the corn-guano model the relationship between the wage and the rate of profits is not time invariant, despite the fact that the same production process remains in use as long as the stock of guano is not exhausted. This result is at odds with the 'objective' point of view defended by the classical economists and Sraffa, according to whom the knowledge of the operated methods and the real wage suffices to infer the level of the rate of profits.

Once it is acknowledged that the introduction of exhaustible resources leads to qualitatively different results, a second step consists of examining the degree of generality and the robustness of the laws derived from the simple model (for instance: is the exhaustible resource always used continuously until exhaustion?), and of questioning key concepts (how is the notion of rate of profits defined in the presence of changing prices?). This justifies the analysis of more complicated multisector models. In our mind, models of exhaustible resources are simple cases of models characterised by time-varying prices, with the cause of changing relative prices lying in production (as opposed to psychological motives, such as the consumer's impatience). Therefore, the study of the corn-guano model is the first step in the elaboration of a research program of classical inspiration. It is not at all meant as an attempt to describe a 'Peruvian' economy. When Schefold (2001) criticised our model for its unrealistic features, he was obviously right, since our aim is theoretical consistency rather than empirical accuracy.

A related, but more specific, criticism raised by Schefold concerns the lack of distinction between guano *in situ* and guano extracted. For reasons of simplicity, our model assumes that guano *in situ* can be used without further processing or effort in the production of corn. According to Schefold this is nonsense: guano can be used as a fertiliser in the production of corn only after it has been extracted and transformed. Hence he stressed the need to make a distinction between exhaustible resources in the ground and exhaustible resources above the ground.[6] The issue at stake is whether the distinction makes a significant difference. It does not: a simple extension of the corn-guano model with an additional process describing the extraction of guano is basically all that is needed. The price of *in situ* guano still follows the Hotelling rule, whereas the price of extracted guano follows a slightly modified Hotelling rule.

Schefold also stresses the differences of quality across mines. Again, there is no denying that this is a well-established empirical fact, but what matters from a conceptual standpoint is to underline the specificity of exhaustible resources in comparison to land and to identify the notion of royalty as distinct from that of rent. In that respect, a model which ignores the heterogeneity of mines is suitable and justified.[7]

A puzzling feature of Schefold's alternative formalisation concerns the way prices change: production by means of exhaustible resources is presented as akin to production by means of lands of different fertility, in the sense that the normal prices of produced commodities "will rise and fall in steps, as in Sraffa's rendering of Ricardo's theory of rent" (Schefold, 2001: 320). More specifically, Schefold divided time in successive 'long periods' – 'decades' in his terminology – during which prices of produced commodities remain at their normal levels. Normal prices change spasmodically at the instant of time which separates one decade from the next. Schefold does not explain, however, why such changes are necesary and why they occur only between two decades. In the theory of rent, a change of normal prices follows an increase in demand which meets a scarcity constraint and requires the introduction of a new marginal method of production. As no criterion of that type holds in Schefold's model, it is unclear why prices are frozen for long periods of time, and then change suddenly. In his formalisation, the

[6] It should be noted that for an unknown reason Schefold shifted terminology and considered the extracted resource ('above the ground') rather than the *in situ* one ('in the ground') to be the exhaustible resource of his own model.
[7] Kurz and Salvadori (2009: 5) attribute to Ricardo the following implicit hypothesis: "For each exhausted deposit of the resource another one with exactly the same characteristics is discovered [...]". That interpretation, which is not sustained by a precise textual reference, is another way to assume away the specificity of exhaustible resources.

precise length of a decade is an essential characteristic, the theoretical determination of which is left open.

Another issue which lends itself to contention is that according to Schefold, different rules apply to the prices of commodities (including extracted guano) on the one hand, and to the price of the *in situ* resource on the other. Commodity prices remain at their normal levels during each period, but "an essential change in the price of the resource takes place within each period" (*ibid.*). No economic reason is given for this asymmetrical treatment. In the absence of a strong argument to the contrary, we believe that all prices should be allowed to change within a period, not only those of the resources in the ground.

Schefold's alternative model can be criticised on several points. It is worth mentioning that if the prices of produced commodities (including the produced guano) are stable for a decade while that of *in situ* guano changes, numerous opportunities for arbitrage are open, be they between guano *in situ* and the other commodities during a decade, or between commodities before and after the periodic break. Schefold's implicit thesis is that a competitive economy cannot adapt itself smoothly to the presence of exhaustible resources and suffers a dramatic crisis at the end of every decade. The definition of a decade, which is essential for the determination of the resulting intermittent dynamics, remains unclear.

5.3 Kurz and Salvadori on the Concept of Profit

For many years Kurz and Salvadori have worked on a theory of exhaustible resources of a Sraffian inspiration or, they claim, 'with classical features'. Their early contributions (Salvadori, 1987; Kurz and Salvadori, 1995, 1997, 2000) have been instrumental in our motivation to develop the corn-guano model, and the number of pages they have devoted to the topic justifies a detailed analysis of their framework, with which we shall explain the reasons of our disagreement. Their work is also the alternative attempt whose formalisation is the most developed, which can therefore be submitted to a more precise criticism than the others.

Let us begin by Kurz and Salvadori's (2001) treatment of the corn-guano model. Instead of assuming that the (real) rate of profits is given, as we did, they started from the assumption that the real wage is given. We have argued that the two cases can be examined just as easily (Bidard and Erreygers, 2001a: 251-2). Our position is that in the case of a given real wage we also need to specify a standard of value, otherwise the rates of profits that will be determined by the model have no 'real' meaning. Let corn be the numeraire, i.e. let us take one quarter of corn as the unit of prices:

$$\forall t \quad p(t) = 1 \tag{25}$$

The model then determines a unique sequence of royalties $\{z(t)\}$ and of real profit rates $\{r(t)\}$, just as the original corn-guano model determined a unique sequence of royalties and real wage rates (see section 2.2). By contrast, rather than specifying the standard of value, Kurz and Salvadori followed a different route: "The sequence of nominal rates of profit $\{r_t\}$ is assumed to be given" (Kurz and Salvadori, 2001: 284). Implicitly, this procedure – choosing the nominal profit rates without specifying the numeraire – boils down to using a sequence of *changing* numeraires, so defined that they yield the desired rates of profits. This is how it goes. Let $\{r(0), r(1), r(2), ...\}$ be the sequence of real profit rates obtained by taking a quarter of corn as the numeraire, and let $\{r_0, r_1, r_2, ...\}$ be another arbitrary sequence. Suppose now that at time t the numeraire consists of $\alpha(t)$ quarters of corn, i.e. $\alpha(t)p(t) = 1$. Then, if the sequence of numeraires $\alpha(t)$ evolves through time according to the rule $\alpha(0) = 1$ and

$$\alpha(t+1) = \frac{1+r(t)}{1+r_t}\alpha(t), \ t = 0,1,2,... \tag{26}$$

we obtain the sequence $\{r_0, r_1, r_2, ...\}$ as nominal profit rates of the model. As we have argued before (Bidard and Erreygers, 2001a: 246), we do not believe that assuming given rates of profits without specifying the numeraire is the right choice.

We now examine Kurz and Salvadori's own approach and assess their claims concerning the classical features of their construction. Their formalisation has evolved, but the basic equation attached to the working of the technique $(A, l) \rightarrow I$ is always written

$$p(t+1) = (1+r)Ap(t) + lw(t) \tag{27}$$

(the equality is replaced by an inequality for a non-operated method). When applied to an exhaustible resource, the Hotelling rule

$$z(t+1) = (1+r)z(t) \tag{28}$$

is obtained as a particular case. The significant gap with our approach is that Kurz and Salvadori do not set the numeraire equation $d'p(t) = 1$. Kurz and Salvadori (1995) followed the standard interpretation of Sraffian models and considered the rate of profits as a real magnitude but, in

1997, they realised that, by changing $p(t)$ into p_t and $w(t)$ into w_t defined by the transformation formulae

$$p_t = \beta(t)\,p(t) \tag{29}$$

$$w_t = \beta(t-1)\,w(t) \tag{30}$$

where the sequence $\beta(t)$ is defined by $\beta(0) = 1$ and

$$\beta(t+1) = \frac{1+r_t}{1+r}\beta(t) \quad t = 0,1,2,... \tag{31}$$

equality (27) becomes

$$p_{t+1} = (1+r_t)Ap_t + lw_t \tag{32}$$

Thanks to the conversion formulae (29) and (30), a sequence of price-and-wage vectors sustaining the rate of profit r is thus transformed into another sequence sustaining the rates r_t. The change in the rates of profit has no effect on the relative prices at each date and there is no upper bound to the rates of profits. These two phenomena are unexpected in a Sraffian framework and led them to reinterpret the magnitude r of equality (27) as a 'nominal rate of profit', an idea they have since defended. The transformation allows the study of prices to be decomposed into two steps. First, by setting $r = 0$ in (27), the simplified equality

$$p(t+1) = Ap(t) + lw(t) \tag{33}$$

is obtained. Second, once it has been shown that some problem admits a solution $\{p(t), w(t)\}$ for a 'zero nominal rate of profit', the formulae (29)-(30) provide a solution to the general problem corresponding to any positive nominal rate of profits r, and even to any arbitrary sequence $\{r_t\}$ of nominal rates.

Kurz and Salvadori (2000) make use of the simplified equation (33) (since they assume that the real wage basket is given and incorporated in the input matrix, the equation they consider is even reduced to $p(t+1) = Ap(t)$) to establish their main result, which is an existence property. That result can be stated as follows. Consider a multisector model with one or several exhaustible resources and, possibly, extraction costs and capacity constraints. Let there be a given sequence $\{r_t\}$ of nominal rates of profits and a basket d. Then, there exist a sequence of prices and a sequence of activity levels with the following three properties: (i) the operated methods at date t do yield the nominal rates, whereas the others pay extra-costs; (ii) they produce a final demand basket γd $(\gamma > 0)$ at each date; and, (iii) the scarcity constraints on the initial endowments are met. The scalar γ is endogeneously determined and is defined by a maximality property.

Rather than examining the proof, let us return to the remarkable equality (33), because it is immediately recognised by theoreticians: it coincides with Walras' ([1874] 1988: 284) "ni bénéfice ni perte" condition, as written in the neo-Walrasian approach of intertemporal production (Malinvaud, 1953; Arrow and Debreu, 1954; Debreu 1959). More explicitly, let $p_i(t)$ be the *present* price of the dated commodity i, i.e. the price paid today for the delivery of one unit of commodity i at date t, and $w(t)$ the *present* wage for one unit of labour available at date t. Equality (33) is a non-arbitrage condition for entrepreneurs, under the constant returns hypothesis. In that framework, the Hotelling rule is that the present value of the royalty is constant.

Because there is no harm in assuming that the 'nominal rates of profit' are zero and that Kurz and Salvadori's equations cannot then be distinguished from neo-Walrasian equations, the formal properties established in a neo-Walrasian framework hold for the other. In the Appendix, we show that the results established by Kurz and Salvadori follow immediately from neoclassical theory, thus reducing the number of required equations from sixty in their 2000 paper (and up to a hundred-and-thirty in Huang, 2018) to zero.

The point, however, concerns the economic interpretation of equality (33). Walras and his followers distinguished the entrepreneur, who combines factors of production he does not own, the owners of capital goods and the workers. The no-profit condition (33) means that the entrepreneur is left with no income once he has paid workers (who receive wages) and the owners of the capital goods (the capitalists, in the classical sense, who receive profits). The vanishing of 'pure profits', which are similar to a rent, has no relationship with that of profits, and equality (33) can in no sense be interpreted as a zero rate of profits equality, be that rate either 'real' or 'nominal'. The same for the more general equality (27) which, when isolated from the reference to a numeraire, is based on a conceptual confusion between the classical notion of profits and the neoclassical notion of pure profits.

The reader is invited to look at Chapter 16 of Kurz and Salvadori (2015) for quite different analytical and historical views on the same story and on the nature of Classical economics.

5.4 Ravagnani: The Role of Socio-Historical Factors

Ravagnani (2008) stresses the unrealistic character of the perfect foresight hypothesis used in theoretical models, for instance concerning the date of exhaustion of the resource. The historical evidence drawn from the US oil industry is that the bargaining between landowners and oil companies led to long-term contractual arrangements in which the lessee pays the landowner a fixed percentage of the crude oil produced on his tract of land, that percentage being subject to changes over time due to the evolution of the respective bargaining positions. The evidence of the role of socio-historical factors in the determination of distribution is in line with the Classical approach, as exemplified by the determination of the 'natural' wage in that construction. Ravagnani thus invites the reader to consider the royalty as another independent distribution variable, and claims that there is no analytical difficulty in adapting Sraffa's price equations "by taking the share of the resource price attributed to landowners as a 'given' coefficient reflecting the (persistent) share paid on average in actual economies" (Ravagnani, 2008: 91).

The model that Ravagnani has in mind for the determination of royalties is Marx's theory of absolute rent (Piccioni and Ravagnani, 2002), introduced in connection with the thorny 'problem of transformation of values into prices': according to Marx, prices of production differ from labour values and sustain a transfer of surplus values from sectors with a low organic composition of capital like agriculture towards capital-intensive sectors, in order to make the rates of profit uniform across all sectors. According to Marx, the monopoly of the class of landlords on land allows them to resist that transfer by requiring the payment of an 'absolute rent' from farmers, even in the absence of scarcity on lands. However, as underlined recently by Fratini (2018), the level of absolute rent is fully determined by the condition that it compensates exactly for the transfer of surplus value: it is not a monopoly price, the level of which would depend on the elasticity of demand.

Most theoreticians consider the theory of absolute rent as frail, if not incompatible with the notion of pure competition. Ravagnani's reference to class struggle for the distribution of national income finds its roots in Classical analysis. But if the royalty is considered as an independent distribution variable, the theory of absolute rent is not the adequate model.

6 CONCLUSION

In this paper we have examined different attempts to integrate exhaustible resources into Classical theory. The main problem is the reconciliation of two seemingly opposing logics: on the one hand, changing relative prices induced by the Hotelling rule; on the other, constant relative prices characteristic of the long-term approach. As soon as the possibility of changing relative prices is admitted, the choice of the standard of value and the measurement of the rate of profits are no longer trivial matters. We have tried to show that the corn-guano model constitutes a good starting point to analyse these points, even if the perfect foresight hypothesis is a strong limitation of the approach. The model is a simplifying device which brings conceptual problems rather than technical points to the fore and provides a yardstick for further generalisations.

The great diversity of the points of view expressed by Sraffian scholars is quite striking. The driving force of the debate on how exhaustible resources should be integrated is not in the first place a difference of opinion about what the relevant facts are. As we have pointed out, there is criticism of the lack of realism of some of the models which have been proposed; nevertheless, correspondence to empirical reality has not been the main preoccupation. The core of the debate is theoretical: it concerns the very understanding of basic economic concepts and the characterisation of the main features of Classical theory.

APPENDIX

Given a stock of scarce resources today ('endowments': capital goods, labour, lands, exhaustible resources, ...) and the technology, the set F of overall feasible products is compact and convex in R^n. Productive efficiency is reached when it is impossible to increase the production of some good without decreasing that of another, i.e. when the product belongs to the outer frontier E of feasible productions. A necessary condition for efficiency is that the allocation of inputs between industries is such that the relative marginal productivities of any two inputs be the same in all industries, otherwise an adequate cross transfer of these inputs between industries would sustain an increase of the overall product for an unchanged total amount of inputs. A well-known economic argument in favour of a competitive organisation of production is that such an efficient allocation of resources is reached in a decentralised way when firms maximise individual profits, because each firm then equalises its own marginal rate of substitution between any two inputs with their relative price. In the debates on the economic organisation of socialism, that argument was considered to be strong enough to suggest that socialist firms should mimic a competitive behaviour (Lange, 1936, 1937).

The precise relationship between a given price vector p and the corresponding overall product d is

$$d'p = \max_{y \in F} y'p$$

It means that d is efficient and that the price vector p is normal to E (or to F) at point d. Note that, when the notion of preference is introduced, the maximality property in physical terms is required for the Pareto optimality of a competitive equilibrium.

For many years, Kurz and Salvadori have used the same theoretical framework to deal with more and more complex technical points related to exhaustible resources (costs of extraction, capacity constraints on extraction, etc.; see also Huang (2018) for the introduction of research-seeking activities) in more and more complex models. The question they set is that of the existence of a competitive price vector for given 'nominal rates of profit'. Their formalisation may look like a convoluted version of Sraffian models, but two hypotheses should have attracted the readers' attention: (i) the initial amounts of exhaustible resources are given, and also those of commodities at date 0; Kurz and Salvadori (2000: 362) note that "[the] second initial condition [...] is perhaps less obvious" and later justify it since "the analysis is not a long-period one" (Kurz and Salvadori, 2015: 294); (ii) in standard Sraffian models, the consumption basket d is given; in the version they retain, the final demand in every period is proportional to d and amounts to γd, where γ is an *endogenously* determined scalar, defined as the maximum feasible level of constant consumption with direction d.

An arbitrary demand basket $d > 0$ need not be either efficient or even feasible, but the line of direction d cuts the efficiency frontier at some point γd $(\gamma > 0)$, and the price vector normal to the efficiency frontier at that point sustains the production γd. The almost equivalence between productive efficiency and competitiveness also holds, after minor adaptations, in an intertemporal framework. A gap is that, in an intertemporal economy, a part of the product is (re)invested, therefore the product at some date is no longer identified with the basket available for final demand. The maximality property now concerns intertemporal final demand. Then efficiency is obtained when the rates of substitution between inputs and those between outputs at the same date are identical across all industries: this is indeed the case at equilibrium with perfect foresight under competitive conditions, the price of a 'dated good' being its present price. The case of infinite horizon sets a problem because the number of dated goods becomes infinite with the horizon. In a remarkable contribution to capital theory, Malinvaud (1953, 1962) studied that point and derived an extension of the equivalence under some additional conditions. As a consequence, for a given basket $d > 0$, consider the maximal scalar $\gamma > 0$ such that the intertemporal demand $(d_0 = \gamma d, ..., d_t = \gamma d, ...)$ is feasible: by definition, the corresponding production is efficient, therefore it is sustained by an intertemporal price vector. Hence, the existence result.[8]

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[8] Incidentally, Kurz and Salvadori's proofs are wrong. They are based on the assumption that, for a given basket *d*, the number $\gamma = \gamma(y)$ of produced units of d is a linear function of the activity levels *y*. But let $d' = (1,1)$, and the net product for activity levels y_1 (respectively y_2) be (1,2) (respectively (2,1)). Then $\gamma(y_1) = \gamma(y_2) = 1$ but $\gamma(y_1 + y_2) = 3$: the fundamental theorem of marginalist theory on efficient production is grounded in convex analysis, not in linear programming.

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