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# ON SOME MISSING EQUATIONS IN CONTEMPORARY TREATMENTS OF INTERTEMPORAL GENERAL EQUILIBRIUM

Pierangelo Garegnani

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Dipartimento di Economia
via Silvio D'Amico
00154 Roma
telefono 06 57334662
e–mail: sraffa@uniroma3.it



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*Abstract of the paper*

*It has not been sufficiently noticed that in an Intertemporal general equilibrium system as many Walras's laws underlie the equations as there are dates in the system, with the implication that an equivalent number of equations are not in fact independent. This has been hidden from view by the fact that an equal number of equations which had as much title to be included as the remaining ones have been omitted. These missing equations are those concerning savings and investment in each period of the life of the economy. Thus, when the system is correctly written, such equations emerge as a part of the system as essential, or inessential, as e.g. the equations concerning the demand and supply of labour for the same dates — i.e. as much independent of, or dependent on, the remaining equations as those labour equations.*

*The two associated shortcomings — i.e. the oversight of the dated Walras's laws and that of the dated savings and investment equations — compensate each other with respect to a merely formal view of the determinacy of the system, and are in fact likely to have seriously affected the analysis of its properties by leading the interpreter to ignoring the capital goods yearly indirectly demanded by the individuals as savers through the firms — a demand which obeys principles very different from those governing their demand for consumptions goods.*

# ON SOME MISSING EQUATIONS IN CONTEMPORARY TREATMENTS OF INTERTEMPORAL GENERAL EQUILIBRIUM

Pierangelo Garegnani

1. It seems to be generally assumed that in an intertemporal equilibrium the demands and supplies for commodities and factors are linked by a *single* intertemporal 'Walras's law', often expressed as a "wealth constraint"[1] or "wealth equation".

This has probably come natural given the origin of intertemporal general equilibrium essentially in the 'static' general equilibrium[2] of Hicks' *Value and Capital*, believed to be extensible to cover intertemporality by 'dating' the commodities.[3] The single Walras's law of that 'static' equilibrium, became accordingly a single 'intertemporal' Walras's law. That, I submit, has been erroneous: the constraints, or Walras's laws, are as many as the dates of the intertemporal equilibrium envisaged.

I shall consider here the case of an economy with production. Distinguishing commodities according to the dates at which they become available, means supposing processes of production in well defined, corresponding periods of time. And since to the value of any gross production, there must correspond equivalent gross incomes earned because of that production, individual incomes also come to occur in well defined corresponding periods of time.[4] Now, to that earning side of each individual's income there will correspond an expenditure or disposal side, and therefore an individual budget equation for the same period. And the sum of these individual budget equations relating to each period, will originate as many Walras's laws for the economy as a whole as there are periods.

This of course is no denial of the fact that the expenditure *on consumption* of the incomes earned for each period needs not, and generally will not, occur only on goods of that period, and can instead be moved over time, backwards by borrowing (i.e. decreasing the borrower's capital) and forward by lending (increasing the lender's capital). But as in any traditional non-intertemporal analysis, that will only mean gross dissavings or savings for the periods to which the incomes refer, *i.e.* differences between the earned gross income and the consumption expenditure of the period in question (the only difference from the traditional static analysis being that the dates of the future consumptions cor-

[1] E.g. Debreu (1959: 62).

[2] Cp. Part II of Hicks (1939). That 'static' equilibrium has been often qualified as "atemporal" or as relating to a puzzling 'atemporal' economy (cp. e.g. Bliss, 1975: 16).

[3] Cp. Arrow-Debreu (1954: 266 ff).

[4] Cp. Garegnani (2000: 399).

responding to the savings, will now appear in the system). And when we include, as usual, gross savings on the disposal, or expenditure, side of the gross incomes accruing to the individuals, it becomes evident that budget equations will bind the expenditures to the incomes accruing to each individual for each period of time and, therefore a 'Walras's law' will underlie the community's disposals (savings included) of the period in question, which will necessarily be equal to the value of the gross output of the period.

**2.** Indeed the 'wealth constraint' which is generally admitted to underlie the intertemporal disposals of each individual, is but the sum of the 'yearly' budget equations of the individual in question. Similarly, the single intertemporal Walras's law or 'wealth constraint' for the economy in current treatments of intertemporal equilibrium, is but the sum of the 'Walras's laws' for the whole economy linking the income disposals to the gross earnings (value of the output) in any period. In those sums both for each individual, and for the economy, the savings of period $t$, appearing on the *disposal side* of the individual budget equations for 'year' $t$, will in fact cancel out with the (discounted) gross incomes from owned capital goods on the *earning side* of the following 'year', so that in the sum, the savings will disappear (except for the savings of the last 'year' of life of the economy, generally supposed to be pointless and therefore zero).[5]

The argument has been implicitly simplified here, by assuming: (i) 'yearly' production cycles with the product becoming 'available' at the beginning of the following 'year', when the incomes are also paid; (ii) circulating capital only, so that the entire gross savings of the 'year' reappear as the (discounted) value of the gross capital income of the following 'year'. Nothing of substance changes however, if fixed capital is considered, as is evident when we make the fixed capital goods aged one year more, appear in the outputs and therefore in the gross incomes of their owners for the following 'year'. Nor anything of substance changes if production were to be continuous, as it generally is in industry (rather than in cycles as in agriculture) with the corresponding inclusion among the capital goods in existence of the intermediate goods which must then be always in the pipelines to realize the (seeming) simultaneity between the application of primary factors and the emergence of the products.

**3.** This correspondence between the single 'intertemporal' Walras's law and the sum of the proper 'yearly' Walras's laws, does not however detract from the basic fact that the intertemporal one constitutes one *single* constraint, whereas the 'yearly' Walras's laws whose existence we contend constitute $n$ constraints, holding for each of the $n$ dates of the intertemporal equilibrium. This fact is basic because by considering only a single intertemporal Walras's law, $(n - 1)$

[5] Garegnani (2000: 400).

constraints have been erroneously ignored among the $n$ in fact underlying the equations of an intertemporal equilibrium. Those $(n-1)$ constraints, that is, are not used as they should have been for recognizing the non-independence of $(n-1)$ equations — adding up to $n$ with the generally admitted wealth constraint — one, that is, for each date of the intertemporal general equilibrium.[6]

4. But, then, it is inevitable to ask: how is it that the intertemporal general equilibrium system, as generally written, did not emerge as correspondingly *under*-determined because of including $(n-1)$ equations that are in fact dependent?

The answer is that some equations which should have been present in the intertemporal system in order to complete the total expenditures in each period have simply been left out. And the overlooked equations are nothing less than those establishing the equality between the 'yearly' demand and supply of gross savings or, in the terminology generally accepted after Keynes, those establishing the equality between, respectively, yearly gross savings and yearly gross investment.

These equations are in fact those governing the size of the *aggregate* 'yearly' output of capital goods, like raw materials and other circulating capitals, or tractors, lathes, industrial buildings and the other fixed capitals. They pertain to the disposal of the individual incomes no less than the equations regarding the demand and supply of potatoes or of shirts, though they pertain to a disposal which is not on *the consumption goods* of the given date, but is (directly or indirectly) on *the capital goods* of that date. If we wished to describe that disposal of income in a unitary way as the buying of a single commodity, we could describe that commodity as 'future income' — as Walras put it in describing his own equation establishing equality between demand and supply of gross savings as pertaining to the demand and supply of the one commodity he called *perpetual net income.*[7]

The result may be envisaged as two shortcomings in the procedure which have, so to speak, compensated each other with respect to the determinacy of the system viewed as number of equations and unknows. The first error has been to ignore the existence of as many constraints (Walras's laws) underlying the equations of an intertemporal system as there are dates in the system — thus potentially eliminating the independence of as many of the written equations. The second is to have ignored an equal number of equations which are an essential part of the system no less, and no more, than those pertaining to potatoes, or to shirts or, as we shall stress for a stricter analogy, those pertaining to the labour of the same dates. In other words as we shall see, the character of

[6] The generally admitted 'intertemporal Walras's law' or 'wealth equation', entails the $n^{th}$ such 'yearly' law, once the remaining (n-1) are satisfied.

[7] Cp. Walras (1954: par. 242; pp. 275-276). That commodity entails a verbal reversal of the terms demand and supply as generally used today for that market: *i.e.*, the *saving supply* would become *demand* for the commodity 'future income' and the *investment demand* would become the *supply* of the commodity.

dependent equations has been unawarely and arbitrarily imposed on those expressing the demand and supply of savings. Now, as we shall see, these missing equations are also those expressing some important specific aspects of the system that have thus been hidden from view by the way the system has been generally written. They are the aspects which pertain to the forces of profit maximization which specifically control the demand for the different kinds of capital goods and which are of course quite different from the consumer preferences which control the demand for the different kinds of consumption goods.

5. It might in fact be objected that, from a strictly formal point of view no error has been in fact committed since, as just said, what happens in the current treatments, is that *implicitly* the Walras's constraint of each 'year' has been used in order to treat as dependent, and accordingly omit, the savings demand and supply equation of the same year.

The emphasized adverb 'implicitly', which entails here another adverb, *i.e.* 'arbitrarily', largely answers the objection, once its full consequences are understood. The omission of the 'yearly' savings investment equation without awareness that that was made possible by merely taking that equation rather than any of the others of the same 'year' as dependent because of an underlying Walras's law, has entailed a misunderstanding of the nature and properties of the system. The awareness of that omission, once gained, would in fact have made it evident, that any of the other yearly equations, for example the labour demand and supply equation for the same 'year', could have been suppressed equally correctly and equally incorrectly. That would in turn have made it shockingly evident that just as the omission of the labour equations would certainly not have eliminated from the system the specific problems and properties of labour demand and supply — for example the possible multiplicity of equilibria because of backward rising labour supply — so the omission of the savings demand and supply equations had certainly not cleared the system from the problems and properties of the savings demand and supply conditions, as instead seems to have been widely held.[8] They are for example the problems and properties which emerged for the demand and supply of 'capital', the single magnitude of the traditional normal position as represented in neoclassical theory.[9] That awareness would have made it evident, that is, that the possibility to omit one of the 'yearly' equations chosen as dependent from the remaining ones of the same year, only means that the specific problems and properties of the omitted market will then work through the remaining markets (the remaining equations) when these are analyzed correctly.

Now, all this is particularly relevant in the case of the yearly savings demand and supply equations, entailing as they do the demand for capital goods. The demand for those products, coming ultimately though indirectly (*i.e.* through the

[8] Cp. n. 11 below.

[9] Cp. Garegnani (1970) and (2000: parr. 27-29; 429-35).

firms using them) from the individuals as savers, follows in fact principles altogether different from those underlying their demand for the consumption goods. The general capital goods demand is specified into particular capital goods by the principle of the highest (riskless) rate of return over their supply prices and not by any preference system of the individual (the latter acts with respect only to the total value amount of that demand and to the future consumption goods to be produced by those capital goods). Omitting the savings investment equations unawarely of what makes that omission possible (the same for which any other equation could have been equally omitted) has in effect led to ignoring the specific questions raised by the demand for capital goods. In a system where only consumption goods have been made visible as products, consumer demand has been generally considered alone in discussing the properties of the system (*e.g.* the weak axiom of revealed preferences pertaining to the individual consumer, needs of course not be satisfied when capital goods appear directly or indirectly in the excess demands of that consumer through his gross savings).[10]

**6.** But, by now another objection (or what is perhaps more exactly a specific application of the previous one) will almost certainly have occurred to many readers against the argument above. Is it not, perhaps, that the yearly savings demand-and-supply equations have been made unnecessary in the system by a reason different from the overlooked Walras's laws (or may be by a specific result of that oversight)? Is it not that the future intertemporal consumption equilibria, which are now included in the system unlike what used to happen in the traditional 'static' theory, are what ensures the absorption of the present savings? In other words, does not the equality between the demands and supplies for consumption goods in the 'years' $(t + 1)$, $(t+2)$, etc., into which there will enter the 'use' of the gross savings of 'year' $t$, take care also of the equality between savings and investment in $t$? It is an argument which appears to be often advanced, though with varying degrees of explicitness.[11]

[10] Indeed the overlooked savings demand and supply equations are the core of the equations system which, according to Walras, makes the entire difference between 'production with capital' and 'production without capital', *i.e.* between the "Theory of Production" of Part IV of the *Elements of Pure Economics* and the "Theory of Capital Formation" of Part V of the book. Without capital, production can be envisaged as essentially an intermediary turning a 'pure exchange' between consumptions goods into a pure exchange between 'factors of production' on one side and consumption goods on the other. Not surprisingly this intermediation does not essentially affect the properties of the system with respect *e.g.* to the multiplicity and stability of the equilibria. Production with capital however is an altogether different matter since it introduces, as we mentioned in the text, products like the capital goods whose demand follows principles entirely different from those governing that of consumption goods.

[11] Cp. Garegnani (2000: 438, n. 62), quoting Arrow (1989: 155) to the effect that within intertemporal equilibrium savings investment adjustments raise "a perfectly consistent story that does not look any different from the story about choosing commodities today ". At the same page I also quote (n. 63) Keynes passage (1936: 10-11) "If savings consisted [of] placing simultane-

Some reflection and one example, can provide a more specific answer to this second objection, besides the one already implied in our counting of equations and unknowns. To simplify, let us suppose two 'years' only, $t = 0$ and $t = 1$, for the life of the economy, so that the savings in $t = 0$ must become consumption demand in $t = 1$, thus representing the objection in its most simple and strong form. What the consumption goods equilibria of $t = 1$ do entail is the equality between, on the one hand, the demand for the several goods in $t = 1$ planned by the consumers, and, on the other hand, the outputs of those goods as planned by the firms. This of course also entails the equality between the aggregate *values* of the consumption outputs demanded and supplied at the prices of the situation envisaged. It is this equality, I believe, which is what is more or less clearly thought to imply an equality between the savings and investment decisions in $t = 0$.

Now, that implication is not there. Under our simple hypotheses, the total gross value of the output in $t=1$ planned by the firms, call it $Q_1$, will consist (i) of the wages for the labour which the firms *would intend to hire* in $t = 0$ under the assumed price conditions, and (ii) of the gross investment of $t = 0$, which the firms *would intend to make* in order to produce the outputs in question as profitably as possible under those price conditions. We accordingly have

$$Q_1 = L_0^D w_0 + I_0 \qquad [1]$$

where $L_0^D$ is the labour demanded by firms at the assumed wages $w_0$ and corresponding prices; and $I_0$ is on the other hand the gross investment (coming, in the present case, out of given initial stocks in existence at the instant $t = 0$, and not out of any production in the year $t = -1$, ignored in our model) which producers would desire to make under the same conditions. On the other hand the gross purchasing power $Y_1$ consumers have to dispose of, including the saving $S_0$ of $t = 0$, and motivating the above plans of the firms, is given by

$$Y_1 = L_0^S w_0 + S_0 \qquad [2]$$

where $L_0^S$ is the labour workers *intend to supply* at the going prices and $S_0$ are the savings of the period $t = 0$ (at discounted prices) which savers *intend to make* at those prices and therefore to spend on the consumption of year $t = 1$. The above mentioned necessary equality between the value $Q_1$ of planned gross total output in $t = 1$, and the value $Y_1$ of total gross expenditure in that year thus entailed in intertemporal general equilibrium, can therefore be expressed as

ously a specific order for future consumption, the effect might be indeed [that] the resources released from preparing for present consumption could be turned over to preparing future consumption".

$$L_0^D w_0 + I_0 = L_0^S w_0 + S_0 \qquad [3]$$

And this does *not* entail $I_0 = S_0$, any more than $L_0^D w_0 = L_0^S \ w_0$. Of course, *given* $L_0^D = L_0^S$ (and the other equations of the period, together with those of the other periods) equation [3] does entail

$$I_0 = S_0 \qquad [4]$$

so that we could leave aside [4] and concentrate the discussion of the equilibrium *e.g.* on the labour market, where we shall have to consider what allows satisfying

$$L_0^D = L_0^S \qquad [5]$$

But, analogously, given equation [4], equation [5] will always be satisfied, so that it would be equally sufficient to concentrate discussion on [4] and on what allows equalising the savings and investment, thus leaving aside the labour market instead.

Now, I believe, nobody would probably argue that the equality between the demands and supplies of labour in each period can be omitted from an intertemporal general equilibrium system of equations since the equality between demand and supply of future consumption goods is sufficient to ensure the equilibrium in the labour markets of previous periods. Exactly the same appears however to be implied when it is argued, as in the objection above, that those 'future' consumption equilibria entail those of savings and investment in the previous periods.

The substantial error associated with this second, more specific, objection is then analogous to that entailed when justifying the absence of the savings investment equation from current formulations in terms of its dependence on the other equations of the same period: it is the error of ignoring that even if we take the savings investment equation as the one dependent on the equilibrium of future consumption markets, the remaining independent equations are in fact bound to reflect the special forces acting on the demand of each specific capital goods — a demand *which will result not only from consumer preferences, as for the consumption goods*, but also and even mainly from the methods of production requiring capital, and the relative outputs, most profitable for the firms. Whether or not we include the yearly equations of savings demand and supply among the independent ones, those special forces underlie the system and alter its behaviour with respect to what it would be if in fact consumption goods only were produced in each period.

## Appendix: A simple illustration of the overlooked "Walras's laws"

1. The purpose of this Appendix is to illustrate (i) the 'yearly' Walras's laws emerging from the budget equations of the individuals in a simple example of intertemporal equilibrium, and then, on the basis of the same example, show (ii) the consequent dependence of one whichever of the equations relating to the 'year' in question upon the remaining ones of the period, and (iii) how the equation for the saving-investment market, *i.e.* the market for Walras's commodity 'future income', has been arbitrarily ignored.

The example of intertemporal equilibrium is the one used in Garegnani (2000) (also 2003) borrowed from the model used in Hahn (1982). Thus we have an economy lasting two years $t = 0$, $t = 1$, where two consumption goods $a$ and $b$ are produced in yearly cycles by labour and themselves as circulating capitals: $a$ and $b$ are therefore, each, both a consumption and a capital good.

The equilibrium equations when correctly written are therefore

1.1 $P_{a1} = l_a W + a_a P_{a0} + b_a P_{b0}$

1.2 $P_{b1} = l_b W + a_b P_{a0} + b_b P_{b0}$

2. $P_{b1} = 1$

3.1 $\bar{A}_0 = D_{a0} + (a_a D_{a1} + a_b D_{b1})$

3.2 $\bar{B}_0 = D_{b0} + (b_a D_{a1} + b_b D_{b1})$

3.3 $\bar{L} = l_a D_{a1} + l_b D_{b1}$

3.4 $S = (a_a D_{a1} + a_b D_{b1}) P_{a0} + (b_a D_{a1} + b_b D_{b1}) P_{b0}$

where $\bar{A}_0$, $\bar{B}_0$, $\bar{L}$ are the initial given endowments of commodity and labour, the $D$'s are the consumption demand functions for the four commodities, $S$ is the aggregate savings function, the $l$'s, $a$'s and $b$'s are the production coefficients and the $P$'s are the prices 'discounted' at $t = 1$, as implied by the choice of $b_1$ as the numeraire, and, finally, $W$ is the wage.

As for the equations, the two numbered [1] are the price equations for $a$ and $b$ available in period $t = 1$, with equation [2] indicating the numeraire; equations [3] are then the basic demand and supply equations — including, under [3.4], that for Walras's commodity 'future income', where the demand function is saving $S$ to be equalised with the supply given by investment, *i.e.* the quantities of the two capital goods (the carriers of the commodity 'future income') required to produce, with labour, the consumptions of $t = 1$. Thus equations 3.3 and 3.4 express the equilibrium in the market, respectively, for labour and for savings and investment (in which the investment in $t = 0$ coincides with a part of the total stock $\bar{A}_0$ and $\bar{B}_0$, and not as is generally the case, with a part of the social product). On the other hand, by definition the investment in $t = 1$ is zero.

We have 7 equations in all, of which, as we shall presently see, two are dependent, with five independent equation determining the 4 unknown prices and the wage.*

**2.** Once we consider the matter sufficiently closely, it will be evident that in the course of the transactions, even if all carried out at an initial single moment (according to the usual assumption), the following two yearly budget equations, *one for each of the two years* of the model will hold for each consumer $i = 1 \ldots n$, where we indicate by small letters the individual endowments and functions. In particular for $t = 0$ we shall have the equations

$$a_0^i P_{a0} + b_0^i P_{b0} = d_{a0}^i P_{a0} + d_{b0}^i P_{b0} + s^i$$

which summed through from individual 1 to $n$ will give Walras's law (I) below for $t = 0$ underlying, as we shall see, equations 3.1; 3.2 and 3.4, *i.e.*

$$\bar{A}_0 P_{a0} + \bar{B}_0 P_{b0} = D_{a0} P_{a0} + D_{b0} P_{b0} + S \qquad \text{(I)}$$

On the other hand, for each individual the following budget equation will hold for $t = 1$

$$l^i W + s^i = d_{a1}^i P_{a1} + d_{b1}^i P_{b1}$$

which will sum up through all individuals for Walras's law (II) below relating to $t = 1$:

$$\bar{L} W + S = D_{a1} P_{a1} + D_{b1} P_{b1} \qquad \text{(II)}$$

This 'law' will underlie equations 3.3; 1.1; 1.2; and, again 3.4,which by its nature pertains to both periods being expenditure in $t = 0$ and factor supply for the outputs of $t = 1$.

This implies that one equation in each of the two groups can be obtained from the remaining ones and is therefore not independent.

**3.** As for $t = 0$ and therefore for Walras's law (I) we shall illustrate that by showing how 3.1 can be obtained from 3.2 and 3.4; or 3.4 from 3.1 and 3.2; or 3.2 from 3.1 and 3.4.

* It should of course be noted that the $S$ and the $D$'s are functions of the five prices and therefore not unknowns.

In fact starting from (I) above and using the savings equation 3.4 we have

$$\bar{A}_0 P_{a0} + \bar{B}_0 P_{b0} = D_{a0} P_{a0} + D_{b0} P_{b0} + [(a_a D_{a1} + a_b D_{b1}) P_{a0} + \\ + (b_a D_{a1} + b_b D_{b1}) P_{b0}$$

*i.e.*

$$\bar{A}_0 P_{a0} + \bar{B}_0 P_{b0} = (D_{a0} + a_a D_{a1} + a_b D_{b1}) P_{a0} + (D_{b0} + b_a D_{a1} + b_b D_{b1}) P_{b0}$$

Using then equation 3.1, we obtain 3.2 which can accordingly be taken as dependent and excluded from the system; or, alternatively, by using (3.2) we obtain (3.1).

Similarly starting again from the identity (I) and substituting into the L.H.S. by equations 3.1, 3.2 and simplifying we have

$$(a_a D_{a1} + a_b D_{b1}) P_{a0} + (b_a D_{a1} + b_b D_{b1}) P_{b0} = S$$

*i.e.* equation 3.4.

4. Passing then to Walras's law (II), we can illustrate by showing how 3.4 can be obtained from 3.3 plus 1.1 and 1.2 — and how, in turn, 3.3 can be obtained from 3.4 plus 1.1 and 1.2. No problem also in obtaining 1.1 (or 1.2) from 3.3, 3.4 and 1.2 (or 1.1).

Starting therefore from (II)

$$\bar{L} W + S = D_{a1} P_{a1} + D_{b1} P_{b1}$$

and substituting on the R.H.S. for $P_{a1}$ $P_{b1}$, by 1.1 and 1.2, and on the left for $S$, by 3.4 we have:

$$\bar{L} W + (a_a D_{a1} + a_b D_{b1}) P_{a0} + (b_a D_{a1} + b_b D_{b1}) P_{b0} = \\ = D_{a1}(l_a W + a_a P_{a0} + b_a P_{b0}) + D_{b1}(l_b W + a_b P_{a0} + b_b P_{b0})$$

and then $\bar{L} W = D_{a1} l_a W + D_{b1} l_b W$ *i.e.* equation 3.3.

But, by again substituting for $P_{a1}$, $P_{b1}$ and then, on the left, for $\bar{L}$ using 3.3, we can similarly have

$$(l_a D_{a1} + l_b D_{b1}) W + S = D_{a1}(l_a W + a_a P_{a0} + b_a P_{b0}) + D_{b1}(l_b W + a_b P_{a0} + b_b P_{b0})$$

*i.e.* equation 3.4.

As for obtaining 1.1 from 3.3, 3.4 and 1.2, from Walras's Law (II)

$$\bar{L}W + S = D_{a1}P_{a1} + D_{b1}P_{b1}$$

using 3.3 and 3.4, we obtain

$$(l_a D_{a1} + l_b D_{b1}) W + (a_a D_{a1} + a_b D_{b1}) P_{a0} + (b_a D_{a1} + b_b D_{b1}) P_{b0} =$$
$$= D_{a1}P_{a1} + D_{b1}P_{b1}$$

Using then 1.2 on the R.H.S and simplifying we have

$$(l_a D_{a1} + l_b D_{b1}) W + (a_a D_{a1} + a_b D_{b1}) P_{a0} + (b_a D_{a1} + b_b D_{b1}) P_{b0} =$$
$$= D_{a1}P_{a1} + D_{b1}(l_b W + a_b P_{a0} + b_b P_{b0})$$

that is

$$l_a D_{a1} W + a_a D_{a1} P_{a0} + b_a D_{a1} P_{b0} = D_{a1} P_{a1}$$

*i.e.* equation 1.1. after simplifying by eliminating $D_{a1}$.

In a strictly analogous way we can obtain 1.2 by using 1.1 together with 3.3. and 3.4.

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