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Circular vs. one-way production: Two different views on production and income distribution

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Abstract

In economic theory, there are at least two alternative ways to represent production processes: circular production processes, where the same commodities appear among the products and the means of production, and one-way production processes, starting with the factors of production and ending with finished goods. This distinction brings about two views concerning income distribution and the conception of profits. In circular production, profits arise from a difference between quantities, the quantities of commodities produced and those required as inputs (surplus theory of profit). In one-way production, profits are viewed as a difference between the prices of final goods and the prices of the productive factors employed to produce them (profits as a price phenomenon). This difference reflects contrasting views on the forces shaping income distribution and assessing the relative merits of production agents in generating profits. Moreover, it also sheds light on two specific issues: the neo-Walrasian scholars' attempt to circumvent Walras' problem of over-determination of the equilibrium system and the possibilities that wage moderation has for increasing international competitiveness.

Keywords: Circular production, One-way production, Sraffa, (Maximum) rate of profit, theory of value.

JEL Codes: B12; B13; B51; C67; D33.

1. Introduction*

In Appendix D (Reference to the literature), Sraffa (1960) acknowledges that his work is connected with the theories of the old classical economists and their representation of the ‘system of production and consumption as a circular process’. Such a view, Sraffa comments, ‘stands in striking contrast to the view… of a one-way avenue that leads from “Factors of production” to “Consumption goods”’.

Moreover, in paragraph 3 ibidem, Sraffa offers some remarks concerning the notion of the Maximum rate of profits, which hint at the basic difference between these two alternative views of production:

> **3.** The notion of a Maximum rate of profits corresponding to a zero wage has been suggested by Marx, directly through an incidental allusion to the possibility of a fall in the rate of profits ‘even if the workers could live on air’;[3] but more generally owing to his emphatic rejection of the claim of Adam Smith and of others after him that the price of every commodity ‘either immediately or ultimately’ resolves itself entirely (that is to say without leaving any commodity residue) into wage, profit and rent[4]—a claim which necessarily presupposed the existence of ‘ultimate’ commodities produced by pure labour without means of production except land, and which therefore was incompatible with a fixed limit to the rise in the rate of profits.
>
> [3] *Capital*, vol. III, ch. 15, sec. ii, Kerr’s ed. p. 290.
>
> [4] *Capital*, vol. III, ch. 49, p. 979, 981 ff., referring to the *Wealth of Nations*, bk. I, ch. v; Cannan’s ed. I, 52.

From this, we can infer that Sraffa is referring to a one-way system when describing a system with ‘ultimate commodities’ produced by pure labour (and land).[2] It, thus, follows that one key difference is that a circular production process has a ‘fixed limit to the rise in the rate of profits’, while a one-way process does not.

In various passages of his unpublished manuscripts, Sraffa reflects on some further consequences of this primary difference. For example:

> {In} the St. Syst., given the wage, we can deduce <spot, identify> the rate of profit without need of knowing the prices. Indeed, we see that even if arbitrary <whatever mad prices> were given to all commodities, no prices

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[2] See Section 2 of this paper.

whatever mad could change the rate {...} No more tangible evidence could be had <expected> of the rate of profits as a non-price phenomenon – contrary to the recurrent <old-established & persistent (hard dying)> notion from Adam Smith onwards of its being due to an addition to the price of the products. [Malthus, Bohm,....] (Sraffa, D3/12/57:17r-18r, dated December 1957; words inside <> denote additions to the main text by Sraffa)

Another example:

{the *q*-System} gives a tangible <proof> demonstration that the rate of profit is not <fundamentally, essentially, as in its essence not a> price-phenomenon. <does not arise from an addition to the price of product over that of raw mat. etc.>. [This refutes the widespread opinion that profits arise from adding something on to the price of the end product. Malthus is perhaps the most explicit supporter of this view; but the picture of a linear <straight line> (as opposed to a circular) production process, which begins with 'factors of production' and ends in 'consumption goods' provides ideal conditions for a 'price' theory of profit.] (Sraffa, D3/12/68:20r, dated August 1955)

In both passages, Sraffa argues that the notion of the Standard system is critical to demonstrate that the rate of profits is not a price phenomenon. Since the Maximum rate of profits is equal to the uniform rate of surplus (that is found when building the Standard or *q*-System) (see Pasinetti 1977, chap. V, § 9), it is possible to conclude that the rate of profit is a non-price phenomenon in a circular system.

This notion of profits is contrasted by Sraffa to the views 'from Adam Smith onwards of its being due to an addition to the price of the products'. Moreover, Sraffa states that a one-way production process 'provides ideal conditions for a "price" theory of profit'.[3]

In conclusion, a sort of logical equivalence between three notions arises from these remarks:

a) Production is a *circular* process;
b) There exists a Maximum rate of profit, R;
c) Profits are a *non-price* phenomenon (originated in the production sphere).

Which can be mirrored by another set of three notions:

α) Production is a *one-way* process;
β) The Maximum rate of profit does not exist;

[3] In the previous quotations, as well as in other papers, Sraffa used the term 'linear production process' to identify what in 'Production of Commodities by Means of Commodities' is described as 'one-way avenue' process. To avoid possible misunderstandings about the meaning of the term 'linear', we will speak of 'one-way production process'.

γ) It is possible to conceive profits as a *price* phenomenon (originated in the trade sphere).

This article explores these connections in detail. In order to simplify the comparison, we will adopt a unified framework, namely input-output analysis which, while belonging to the realm of circular production,[4] can be adapted also to represent one-way production.

2. One-way production processes

The concept of one-way production is typical of the Austrian branch of neoclassical economics. In this approach, consumption goods are produced by capital goods, land and labour. Capital goods required to produce consumption goods are called 'first-order' means of production. These, in turn, are produced by another set of capital goods, which are referred to as 'second-order' means of production. This sequence may continue, in principle, *ad infinitum*. Means of production of order n are produced by means of production of order $n+1$, and so forth. To overcome this difficulty, Böhm-Bawerk assumed that the means of production of the highest order are produced by labour alone (see Bortis 1990).

In the simplest one-way production system, a generic consumption good 'c' is produced only by labour employed for one period of production. By representing time on a horizontal axis, this case is depicted in Figure 1.

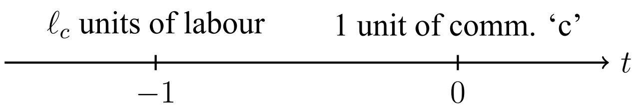

Figure 1: Production with 'direct' labour

In this case, the relation between the price of commodity c, p_c, the wage rate, w and the rate of profit, π is

$$p_c = w\ell_c(1+\pi). \tag{1}$$

It is apparent in this equation that the price of the commodity resolves itself into wages and profits. Furthermore, by setting $w = 1$ and rearranging the terms of the equation, we get

$$\pi = \frac{p_c - \ell_c}{\ell_c}.$$

As the expression shows, the rate of profits can be increased at will, apparently by suitably raising the commodity price.

[4]For example, in Kurz and Salvadori (1995, p. 58) a fundamental assumption is that 'the production of any commodity requires some material input(s)'; the very title of Sraffa's (1960) book underlines circularity of production as the essential element of the economic system analysed. Two fortuitous insights that go in the opposite direction are contained in Pasinetti (1973, § 15), Pasinetti (1986) and Fratini (2014, fn. 9).

Let us now analyse how we can extend this proposition to the general case of many commodities. Our point of departure is the price system developed by Sraffa (1960):

$$\mathbf{p}^{\mathrm{T}} = w\boldsymbol{\ell}^{\mathrm{T}} + (1+\pi)\mathbf{p}^{\mathrm{T}}\mathbf{A}. \tag{2}$$

As can be seen, the price vector can be found on both sides of the equation. In other words, it appears also on the cost side. Thus, a series of calculations are needed to 're-solve' the equation by transforming the cost of production into wages and profits. Such an operation consists in the 'reduction of prices to dated quantities of labour' (Sraffa 1960, ch. VI).

After replacing the expression for prices on the right-hand side of the equation with the price vector of the means of production, we get $\mathbf{p}^{\mathrm{T}} = w\boldsymbol{\ell}^{\mathrm{T}} + w(1+\pi)\boldsymbol{\ell}^{\mathrm{T}}\mathbf{A} + (1+\pi)^2\mathbf{p}^{\mathrm{T}}\mathbf{A}^2$. By iterating this operation $\bar{t}+1$ times, we get:

$$\begin{aligned}\mathbf{p}^{\mathrm{T}} = w\boldsymbol{\ell}^{\mathrm{T}} + w(1+\pi)\boldsymbol{\ell}^{\mathrm{T}}\mathbf{A} + w(1+\pi)^2\boldsymbol{\ell}^{\mathrm{T}}\mathbf{A}^2 + \cdots \\ + w(1+\pi)^{\bar{t}-1}\boldsymbol{\ell}^{\mathrm{T}}\mathbf{A}^{\bar{t}-1} + (1+\pi)^{\bar{t}}\mathbf{p}^{\mathrm{T}}\mathbf{A}^{\bar{t}}.\end{aligned} \tag{2'}$$

The operation of reduction, however far it may be pushed, always leads to a 'commodity residue', i.e. the term $(1+\pi)^{\bar{t}}\mathbf{p}^{\mathrm{T}}\mathbf{A}^{\bar{t}}$. In a one-way production system, the residue eventually becomes zero after a finite number of steps. Hence, there is a stage, let it be $\bar{t}-1$, where labour is the sole input of production. This can happen only if matrix $\mathbf{A}$ has the special property that $\mathbf{A}^{\bar{t}} = \mathbf{O}$. In matrix algebra, matrices that have this property are called 'nilpotent' (Meyer 2000, p. 391).

Definition (Nilpotent Matrixes). $\mathbf{N}_{n\times n}$ is said to be *nilpotent* whenever $\mathbf{N}^k = \mathbf{0}$ for positive integer k. $k = index(\mathbf{N})$ (referred to as the index of nilpotency) is the smallest positive integer such that $\mathbf{N}^k = \mathbf{0}$.

Since technical coefficient matrices are nonnegative, they can only be nilpotent if they are *strictly lower (or upper) triangular*,

$$\begin{bmatrix} 0 & 0 & 0 & \cdots & 0 \\ a_{21} & 0 & 0 & \cdots & 0 \\ a_{31} & a_{32} & 0 & \cdots & 0 \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ a_{n1} & a_{n2} & \cdots & a_{n,n-1} & 0 \end{bmatrix}.$$

As can be seen, this representation describes an Austrian production process for the general case. In this situation, all commodities are ultimately produced, that is, directly and/or indirectly, by labour only (we are disregarding land and other original inputs). Their price can be reduced to dated labour quantities—that is, wages and profits—in a *finite* number of steps 'without leaving any commodity residue' (Sraffa 1960, p. 94). Equation (2′) then becomes:

$$\mathbf{p}^{\mathrm{T}} = w\boldsymbol{\ell}^{\mathrm{T}} + w(1+\pi)\boldsymbol{\ell}^{\mathrm{T}}\mathbf{A} + w(1+\pi)^2\boldsymbol{\ell}^{\mathrm{T}}\mathbf{A}^2 + \cdots + w(1+\pi)^{\bar{t}-1}\boldsymbol{\ell}^{\mathrm{T}}\mathbf{A}^{\bar{t}-1}$$
$$= \sum_{i=1}^{\bar{t}} w\boldsymbol{\ell}^{\mathrm{T}}\mathbf{A}^{i-1}(1+\pi)^{i-1}. \quad (3)$$

In this equation, 'the price of commodities is arrived at by a process of *adding up* the wages, profit and rent' (Sraffa 1951, p. xxxv). Leaving aside rents, the terms of the right-hand side of Equation (3) not containing π, that is, $w\boldsymbol{\ell}^{\mathrm{T}}$, $w\boldsymbol{\ell}^{\mathrm{T}}\mathbf{A}$, $w\boldsymbol{\ell}^{\mathrm{T}}\mathbf{A}^2$, …, $w\boldsymbol{\ell}^{\mathrm{T}}\mathbf{A}^{\bar{t}-1}$, are wages; the remaining terms of (3) containing π or its powers, that is, $\pi w\boldsymbol{\ell}^{\mathrm{T}}\mathbf{A}$, $2\pi w\boldsymbol{\ell}^{\mathrm{T}}\mathbf{A}^2$, $\pi^2 w\boldsymbol{\ell}^{\mathrm{T}}\mathbf{A}^2$, …, $\pi^{\bar{t}-1}w\boldsymbol{\ell}^{\mathrm{T}}\mathbf{A}^{\bar{t}-1}$, are profits.

At this point, it is useful to recapitulate the logical sequence we have gone through. We began with solving the cost side of Sraffa's price system into wages and profits. As suggested by Sraffa, the operation required is the reduction to dated quantities of labour, which consists of replacing the prices of commodities forming the means of production with the prices of their own means of production and labour. After several replacements, we found that there is always the so-called 'commodity residue'. Yet, if the matrix of technical coefficients is nilpotent, the residue disappears, and the cost side consists exclusively of a sum of wages and profits. In that case, by looking at the price equations, profits arise 'from an addition to the price of product over that of raw mat{erial}' (Sraffa, D3/12/68:20r, dated August 1955).

Since a nilpotent matrix of technical coefficients provides a fair representation of a one-way production process, this allows for a logical connection between one-way production and the conception of profits as a price phenomenon to be established.

Moreover, since in such a case, the price vector does not appear on the cost side, it seems possible to raise profits by increasing sale prices. In other words, any non-negative predefined level of the rate of profit could, in principle, be reached if prices are varied at the suitable level.[5]

Now that the logical connections have been established, the following distinction

[5] Varying prices at will in the equations reproduced above, or in Sraffa's words: 'even if arbitrary whatever mad prices were given to all commodities', it is pretty certain that the rate of profit does not remain uniform among industries, flukes aside. It is, however, possible to derive a rigorous analytical expression of the relation between the prices of commodities and the rates of profit of the various industries. Starting from the price equations

$$\mathbf{p}^{\mathrm{T}} = w\boldsymbol{\ell}^{\mathrm{T}} + \mathbf{p}^{\mathrm{T}}\mathbf{A}(\mathbf{I}+\hat{\mathbf{\Pi}})$$

and substituting $\mathbf{p}^{\mathrm{T}}$ at the right-hand side, we have

$$\mathbf{p}^{\mathrm{T}} = w\boldsymbol{\ell}^{\mathrm{T}} + w\boldsymbol{\ell}^{\mathrm{T}}\mathbf{A}(\mathbf{I}+\hat{\mathbf{\Pi}}) + \mathbf{p}^{\mathrm{T}}[\mathbf{A}(\mathbf{I}+\hat{\mathbf{\Pi}})]^2$$
$$= w\boldsymbol{\ell}^{\mathrm{T}} + w\boldsymbol{\ell}^{\mathrm{T}}\mathbf{A}(\mathbf{I}+\hat{\mathbf{\Pi}}) + w\boldsymbol{\ell}^{\mathrm{T}}[\mathbf{A}(\mathbf{I}+\hat{\mathbf{\Pi}})]^2 + \mathbf{p}^{\mathrm{T}}\mathbf{A}[\mathbf{A}(\mathbf{I}+\hat{\mathbf{\Pi}})]^3.$$

needs to be addressed. Stating that in a one-way production process profits *arise* as an addition to the cost of production is not the same as stating that profits *are* effectively a price phenomenon.

To sustain the latter, it is required to clarify what is understood as a cost of production. If it means real costs, which include the reproduction of the present and future workforce, i.e., the real wage, it follows that profits are a price phenomenon if they can be increased or decreased while keeping real wages constant. Yet, it is readily apparent that this cannot be done.[6] Any price increase required to raise profits necessarily reduces the real wage. A simple example can be seen in 1. Letting y_c represent the net output, equation (1) can be restated as follows:

$$p_c y_c = w l_c y_c (1 + \pi),$$

where $p_c y_c$ is the value of the net output, and $l_c y_c$ is the quantity of labour required to produce it. Following Sraffa, we can set these terms equal to one. In that case, we get

$$1 = w(1 + \pi),$$

where w represents the wage share of output. Then, any increase in the rate of profit necessarily entails a reduction of the purchasing power of workers measured in terms of the net output. Since it is impossible to escape this constraint in any production system, we conclude that if production is a one-way process, profits cannot be regarded as a price phenomenon.

Garegnani (1984, §9) warned against this misunderstanding in a succinct way:

> [the] dependence of the value of the product upon distribution means that, when we look at the Social product and the Necessary consumption in value terms, the constraint by which one class cannot have more without the other class having less—so evident if we could look at the product in physical terms—is no longer apparent: might not the real wage rise without affecting the rate of profit, or vice-versa? Indeed Smith himself often lost sight of the constraint and envisaged the rate of profit and the wage as determined independently of each other. He wrote that 'the natural price varies with the natural rate of each of its component parts' [Smith (1776, Book I, ch. VII, § 33)] giving rise to what has been described as Smith's 'adding up theory of

Since $\mathbf{I} + \hat{\mathbf{\Pi}}$ is a diagonal matrix, if $\mathbf{A}$ is strictly lower triangular, $\mathbf{A}(\mathbf{I} + \hat{\mathbf{\Pi}})$ is strictly lower triangular as well: hence $\mathbf{A}(\mathbf{I} + \hat{\mathbf{\Pi}})$ is nilpotent. Consequently, after $\bar{t} - 1$ iterations, the price equations become

$$\mathbf{p}^{\mathrm{T}} = w\boldsymbol{\ell}^{\mathrm{T}} + w\boldsymbol{\ell}^{\mathrm{T}}\mathbf{A}(\mathbf{I} + \hat{\mathbf{\Pi}}) + w\boldsymbol{\ell}^{\mathrm{T}}[\mathbf{A}(\mathbf{I} + \hat{\mathbf{\Pi}})]^2 + \cdots + w\boldsymbol{\ell}^{\mathrm{T}}[\mathbf{A}(\mathbf{I} + \hat{\mathbf{\Pi}})]^{\bar{t}-1}.$$

[6] It remains impossible unless one admits that the physical output can vary, but even in this case, one may find some analytical difficulties to substantiate the claim. See, for example, Ricardo's critique of Smith and Malthus' theory of profits (Garegnani 1978, pp. 338-341).

> prices' Sraffa (1951, p. xxxv). And, after Smith Malthus could argue that a tariff on corn would raise both the rent of land and the rate of profits, without apparently seeing the consequences these rises would be bound to have upon the real wage [e.g. Malthus (1836, Book II, ch. I, Section IX)].

This was also mentioned earlier by Marx in quite ironic terms:

> [T]he outcome of this competition between land, capital and labour finally shows that, although they quarrel with one another over the division, their rivalry tends to increase the value of the product to such an extent that each receives a larger piece, so that their competition, which spurs them on, is merely the expression of their harmony (Marx (1863, p. 503); quoted in Garegnani (2018, p. 6)).

One-way systems are different in one aspect from circular production systems (to be discussed in the next section). The relation between the wage rate and the rate of profit is defined *for all* $\pi > 0$; it is *asymptotic* to the horizontal axis in a w-π plane (since $\lim_{\pi \to \infty} w(\pi) = 0$). As emphasised by Sraffa (1960, Appendix D, § 3), this entails that '[t]he notion of a Maximum rate of profits corresponding to a zero wage' can no longer be conceived for one-way systems. The same result can be regarded by observing that since the eigenvalues of a nilpotent are all equal to zero,[7] the Maximum rate of profit (as well as the uniform physical rate of surplus) *does not exist*: if $\lambda^* = 0$, the expression $(1 - \lambda^*)/\lambda^*$ is not well defined (approximately, we might say that Π (or R) tends to infinite when λ^* tends to 0). In parallel, the Standard system cannot be constructed. In this regard, Fratini and Ravagnani (2023) conjecture that because of this impossibility, Sraffa conceives one-way production processes as the 'ideal conditions for a "price" theory of profit'. The absence of a standard system 'precludes the conception of the rate of profit as the ratio of two quantities of the same commodity' (Fratini and Ravagnani 2023, §4). Our previous analysis enlightens another side of this same coin. A one-way system provides such an ideal setting because, since the matrix of technical coefficients is nilpotent in such a system, it is possible to resolve prices entirely into wages and profits without leaving a residue.

3. A comparison with circular production processes

To appreciate the different nature of profits in circular production processes, let us suppose that the technical coefficient matrix $\mathbf{A}$ is not nilpotent. Clearly, in such a situation, a Maximum rate of profit exists, $\Pi = (1 - \lambda^*)/\lambda^*$, and a Standard system exists, whose

[7] Let λ be any eigenvalue of a nilpotent matrix $\mathbf{A}$ of index k, and $\mathbf{x} \neq \mathbf{o}$ the correspondent eigenvector. Then $\mathbf{A}\mathbf{x} = \lambda\mathbf{x}$. Pre-multiplying both sides by $\mathbf{A}$ we get $\mathbf{A}^2\mathbf{x} = \lambda\mathbf{A}\mathbf{x}$, that is, $\mathbf{A}^2\mathbf{x} = \lambda^2\mathbf{x}$. By induction, we obtain $\mathbf{A}^k\mathbf{x} = \lambda^k\mathbf{x}$, but since $\mathbf{A}$ is nilpotent of index k, $\mathbf{A}^k\mathbf{x} = \mathbf{O}\mathbf{x} = \mathbf{o}$ and, consequently, $\lambda^k = 0$ and $\lambda = 0$.

gross product, $\mathbf{q}^*$, the vector of the means of its production, $\mathbf{Aq}^*$, and the net product, $\mathbf{y}^* = \mathbf{q}^* - \mathbf{Aq}^*$, are all constituted by the right-hand eigenvector of $\mathbf{A}$ associated to its dominant eigenvalue.

In order to better understand the nature of profits in this case, we take wages as given in *physical terms*. Therefore, let vector $\mathbf{d}$ be a given bundle, representing the real wage received by 1 unit of labour. Hence, $w = \mathbf{p}^{\mathrm{T}}\mathbf{d}$, and the price system $\mathbf{p}^{\mathrm{T}} = w\boldsymbol{\ell}^{\mathrm{T}} + (1+\pi)\mathbf{p}^{\mathrm{T}}\mathbf{A}$ becomes $\mathbf{p}^{\mathrm{T}} = \mathbf{p}^{\mathrm{T}}\mathbf{d}\boldsymbol{\ell}^{\mathrm{T}} + (1+\pi)\mathbf{p}^{\mathrm{T}}\mathbf{A}$, which can be written as follows:

$$\mathbf{p}^{\mathrm{T}} = (1+\pi)\mathbf{p}^{\mathrm{T}}\mathbf{A}(\mathbf{I} - \mathbf{d}\boldsymbol{\ell}^{\mathrm{T}})^{-1}. \tag{4}$$

Matrix $\mathbf{d}\boldsymbol{\ell}^{\mathrm{T}}$ is a semi-positive square matrix of rank 1.[8] Its dominant eigenvalue, $\lambda_{\mathbf{d}\boldsymbol{\ell}^{\mathrm{T}}}$, is $\sum_{c=1}^{C} \ell_c d_c$ (the other $C-1$ eigenvalues are all zero). Hence, $(\mathbf{I} - \mathbf{d}\boldsymbol{\ell}^{\mathrm{T}})^{-1} \geq \mathbf{O}$ if

$$\sum_{c=1}^{C} \ell_c d_c < 1. \tag{5}$$

That is, if the quantity of direct labour necessary to produce the subsistence of one unit of labour is lower than one. The price system can thus be written as

$$\mathbf{p}^{\mathrm{T}} = (1+\pi)\mathbf{p}^{\mathrm{T}}\mathbf{A}^{\oplus}, \tag{4$'$}$$

where

$$\mathbf{A}^{\oplus} = \mathbf{A}(\mathbf{I} - \mathbf{d}\boldsymbol{\ell}^{\mathrm{T}})^{-1} \geq \mathbf{O} \tag{6}$$

is the ‘socio-technical’ matrix. The solution of (4) is

$$\pi = \Pi^{\oplus} = \frac{1-\lambda^{\oplus}}{\lambda^{\oplus}} \quad \text{and} \quad \mathbf{p}^{\mathrm{T}} = \mathbf{p}^{\oplus\mathrm{T}} \tag{7}$$

where $\lambda^{\oplus}$ is the dominant eigenvalue of $\mathbf{A}^{\oplus}$ and $\mathbf{p}^{\oplus\mathrm{T}}$ the corresponding left-hand eigenvector. The viability condition, $\lambda^{\oplus} < 1$, ensures that $\Pi^{\oplus} > 0$.

We are now in a position to draw the main differences between one-way and circular production processes. Firstly, as already shown, the rate of profits has no upper limit in a one-way system. Instead, circular systems have a Maximum rate of profits, which coincides with the uniform rate of (physical) surplus.

Secondly, the price system that arises in a one-way system (Equation (3)) suggests a *sequencing* on the determination of the rate of profit and prices. Given any rate of profit, it is always possible to determine the price system which yields it. Conversely, in the price equations of a circular system (Equation (4′)), the determination of prices and the profit rate is *simultaneous*: it is not possible to freely vary the prices on the left-hand side in

[8] This descends from $r(\mathbf{d}\boldsymbol{\ell}^{\mathrm{T}}) \leq \min(r(\mathbf{d}), r(\boldsymbol{\ell}^{\mathrm{T}})) = 1$ since $\mathbf{d}$ and $\boldsymbol{\ell}^{\mathrm{T}}$ are vectors.

order to raise or lower the profit at will, because it also affects prices on the right-hand side.

It might be observed that since the price system could always be reduced to dated quantities of labour, the commodity residuum could be made as small as possible. Thus, one could argue that any price increase on the left-hand side will have little impact on the right-hand side, allowing an increase in the profit rate. Yet, since the system is circular, as the actual profit rate approaches the Maximum, the cost side consists entirely more and more of the commodity residue (like in Equation ($4'$)). There, any price increase is of the same magnitude on both sides, leaving the rate of profit unchanged.

Finally, when production is circular, profits depend first and foremost on the economic system's capacity to generate a surplus, that is, the possibility to produce commodities such as corn, cloth, wood, steel, and chips to a larger degree than required, and second from the capitalists' power to appropriate a part of it (if workers also get a share of the surplus) or all of it (if workers earn only the subsistence wage). Profits thus emerge in the *sphere of production* and in the relative bargaining strength of capitalists in fixing real wages. They are not the merit of a particular input, firm, or industry. The profits of an individual are a claim on the surplus, a claim which is independent of the risk taken, the ability to compete, or the sector in which it produces. These qualities may explain why one capitalist can get more than others, but not why there are profits in the economic system. In a circular system, the surplus precedes profits. No matter how cunning the firm's managers are—to compete or avoid competition—there will be no such profit if the system does not generate a surplus.

On the contrary, in one-way representations of the production system, profits emerge due to the ability to set a markup over production costs. Therefore, the conditions for establishing a markup, such as the firms' or their managers' entrepreneurial capabilities or the market conditions of the sector in which they operate, are relevant. Notably, the causality commented earlier is reversed: capitalists seem to be able to create a growing surplus by raising the markup over production costs. Yet, as discussed, this is illusory for the system as a whole, given the restriction that 'one class cannot have more without the other class having less' (Garegnani 1984, p. 301).

4. Real wages and circularity

It is now noteworthy to highlight a straightforward consequence that emerges when explicitly recognising that real wages are constituted by produced commodities.

Consider a one-way production system and imagine that the purchasing power of wages, in terms of any *numéraire*, is at a shallow near-zero value. Although setting the real wage at this level is feasible (i.e., we can raise prices to reach any desired level of profits), it may not be sustainable over time. That is, for a given monetary wage, workers cannot purchase the amount of the commodities required for their subsistence, and thus, sooner or later, the labour force would not be reproduced.

Commodities would not be produced, therefore not marketed, and the rate of profit would become zero as well. Thus, it could hardly be denied that there is a lower limit below which wages physically cannot fall, usually called subsistence wage. Marginalist authors like Wicksell (1901, p. 148) also recognised it.

Therefore, to ensure the repetition of the production process in the long run, the money wage should be at least sufficient to purchase a basket of necessary consumptions (subsistence). This acknowledgement is not trivial and has substantial analytical consequences.

1) Even if $\mathbf{A}$ is nilpotent, there will be a Maximum rate of profit. Suppose that the technical coefficients matrix is nilpotent of index 2, i.e. $\mathbf{A}^2 = \mathbf{O}$. In that case, equation (3) becomes:

$$\mathbf{p}^{\mathrm{T}} = w\boldsymbol{\ell}^{\mathrm{T}} + w\,(1+\pi)\,\boldsymbol{\ell}^{\mathrm{T}}\mathbf{A} \tag{8}$$

If we admit that money wages must be at least sufficient to purchase a bundle of commodities $\mathbf{d}$, then $\bar{w} = \mathbf{p}^{\mathrm{T}}\mathbf{d}$. Substituting $w = \bar{w}$ into equation (8) gives:

$$\mathbf{p}^{\mathrm{T}} = \mathbf{p}^{\mathrm{T}}\mathbf{d}\boldsymbol{\ell}^{\mathrm{T}} + (1+\pi)\,\mathbf{p}^{\mathrm{T}}\mathbf{d}\boldsymbol{\ell}^{\mathrm{T}}\mathbf{A} \tag{9}$$

Since by assumption $\mathbf{A}$ is nilpotent of index 2, we know that $(1+\pi)^2\mathbf{p}^{\mathrm{T}}\mathbf{A}^2 = \mathbf{o}^{\mathrm{T}}$. Thus, we can add it to the right-hand side of (9) and obtain:

$$\mathbf{p}^{\mathrm{T}} = \mathbf{p}^{\mathrm{T}}\mathbf{d}\boldsymbol{\ell}^{\mathrm{T}} + (1+\pi)\,\mathbf{p}^{\mathrm{T}}\mathbf{d}\boldsymbol{\ell}^{\mathrm{T}}\mathbf{A} + (1+\pi)\mathbf{p}^{\mathrm{T}}\mathbf{A}^2,$$

which may be restated as follows:

$$\mathbf{p}^{\mathrm{T}} = (1+\pi)\mathbf{p}^{\mathrm{T}}\mathbf{A}(\mathbf{I} - \mathbf{d}\boldsymbol{\ell}^{\mathrm{T}})^{-1} \tag{10}$$

It is quite easy to prove now that $\mathbf{A}^{\star} = \mathbf{A}(\mathbf{I} - \mathbf{d}\boldsymbol{\ell}^{\mathrm{T}})^{-1}$ is not nilpotent. For this, it suffices to prove that its dominant eigenvalue is greater than zero. Let us note that the matrix $\mathbf{I} - \mathbf{d}\boldsymbol{\ell}^{\mathrm{T}}$, with $\boldsymbol{\ell}^{\mathrm{T}}\mathbf{d} < 1$, is an elementary matrix (Meyer 2000, p. 131). This matrix is non-singular and:

$$(\mathbf{I} - \mathbf{d}\boldsymbol{\ell}^{\mathrm{T}})^{-1} = \mathbf{I} + \frac{\mathbf{d}\boldsymbol{\ell}^{\mathrm{T}}}{1 - \boldsymbol{\ell}^{\mathrm{T}}\mathbf{d}}.$$

Therefore,

$$\mathbf{A}^{\star} = \mathbf{A} + \frac{\mathbf{A}\mathbf{d}\boldsymbol{\ell}^{\mathrm{T}}}{1 - \boldsymbol{\ell}^{\mathrm{T}}\mathbf{d}}.$$

Since the second *addendum* is greater than zero, $\mathbf{A} < \mathbf{A}^{\star}$. Moreover, according to the Perron-Frobenius theorems, the dominant eigenvalue λ_m of $\mathbf{A}$ is a continuous, increasing function of the elements of $\mathbf{A}$ (Pasinetti 1977). Therefore, we can assert that:

$$\lambda_m(\mathbf{A}) < \lambda_m(\mathbf{A}^\star).$$

Finally, as stated previously, since $\mathbf{A}$ is nilpotent, all its eigenvalues are zero. Thus, $\lambda_m(\mathbf{A}) = 0$, and, consequently, $\lambda_m(\mathbf{A}^\star) > 0$; that is, $\mathbf{A}^\star$ is not nilpotent.

Thus, even if the technical matrix is nilpotent, that is, even if we start from a one-way production system, the explicit consideration that (subsistence) wages consist of produced commodities immediately brings us back to a circular system: equation (8) can be transformed into equation (10), which is identical to equation (4).

2) If $w > \bar{w}$, then wages also include a 'share of the surplus product' (Sraffa 1960, p. 9). Following Sraffa (1960, Ch. II, § 8), we can distinguish two components in the wage and establish $w = \bar{w} + \sigma$, where σ is the surplus part. Introducing this definition into equation (8), we get the following price system:

$$\mathbf{p}^{\mathrm{T}} = \mathbf{p}^{\mathrm{T}}\mathbf{d}\boldsymbol{\ell}^{\mathrm{T}} + \sigma\boldsymbol{\ell}^{\mathrm{T}} + (1+\pi)\mathbf{p}^{\mathrm{T}}\mathbf{d}\boldsymbol{\ell}^{\mathrm{T}}\mathbf{A} + \sigma(1+\pi)\boldsymbol{\ell}^{\mathrm{T}}\mathbf{A}.$$

As before, adding $(1+\pi)^2\mathbf{p}^{\mathrm{T}}\mathbf{A}^2$ to the right-hand side, and rearranging the terms, allows restating the previous equation as follows:

$$\mathbf{p}^{\mathrm{T}} = \sigma\boldsymbol{\ell}^{\star\mathrm{T}} + (1+\pi)\mathbf{p}^{\mathrm{T}}\mathbf{A}^\star,$$

where $\boldsymbol{\ell}^{\star\mathrm{T}} = \boldsymbol{\ell}^{\mathrm{T}}(\mathbf{I} - \mathbf{d}\boldsymbol{\ell}^{\mathrm{T}})^{-1}$. As we have shown, $\lambda_m(\mathbf{A}^\star) > 1$. In other words, this production system is also circular.

Instead, if we do not distinguish between the two components and consider 'the whole of the wage as variable', we return to equation (8), which suggests a one-way production system. In this case, we obtain an alternative interpretation of the one-way production system, according to which the 'necessaries of consumption' are relegated to non-basic products, which do not 'affect the rate of profits and the prices of other products' (Sraffa 1960, p. 10). In the end, this relegation (coupled with assuming a nilpotent technical coefficient matrix) allows conceiving the rate of profits as an addition to the cost of production of commodities.

However, this view does not seem particularly convincing. Since the products that workers consume are 'essentially basic', they influence prices and the rate of profits. Therefore, the production system can be one-way in appearance only. If production requires any human activity, it is necessarily a circular process.

5. Circular vs. one-way systems: some theoretical and 'practical' implications

The dichotomy between the circular and one-way representation of production also sheds light on more specific theoretical and applied aspects of production and distribution theory.

5.1. Circularity and 'one-wayness' in Walras and neo-Walrasian analysis

The different conceptions of production here emphasised help us to understand an essential juncture between the treatment of capital proposed by Walras and the subsequent transition to neo-Walrasian approach. Walras (1874) in Lesson 21, among the general equilibrium conditions of an economy with production, establishes the competitive condition of equality between the prices of newly produced capital goods, $\mathbf{p}$, and their expenses of production:

$$\mathbf{p}^{\mathrm{T}} = \mathbf{v}^{\mathrm{T}}\mathbf{A} + w\boldsymbol{\ell}^{\mathrm{T}}, \tag{11}$$

where vector $\mathbf{v}$ is the vector of rental prices of existing capital goods employed in producing new capital goods. Later on, in Lesson 23, Walras establishes a link between the prices of newly produced capital goods and the rental prices of existing capital goods:[9]

$$\mathbf{v}^{\mathrm{T}} = (1+\pi)\mathbf{p}^{\mathrm{T}} \tag{12}$$

Substitution of (12) into (11) yields a price equation coinciding with Sraffa's price equation (2). This element clearly places Walras' model among circular production systems.

However, the neo-Walrasian reformulation of the general equilibrium system drops condition (12) from the equilibrium conditions, decoupling thus the rental prices of capital goods from their production prices, and changing Walras' perspective. In fact, while in Walras' model, thanks to condition (12), production is circular, in neo-Walrasian reformulations this characteristic is deliberately removed. Although the representation of the production process is sufficiently general also to include the production of commodities by means of commodities (like, for example, in Malinvaud (1953)), the circularity is broken down by the device to consider the commodities entering as inputs as goods *different* from the commodities that are produced because they refer to different dates.

Indeed, many neoclassical economists have often regarded Walras' capitalisation equations as an element which is quite extraneous to the logic of neoclassical analysis.[10] In addition, as proved by Garegnani (1990, Section II), this set of equations make Walras' model over-determined. Evidently, its elimination prevents the model from being at-

[9]For the sake of simplicity we consider the case with circulating capital only and do not consider any insurance premium.

[10]Donzelli (1989), for example, maintains that this condition is to be interpreted regarding the demand price, instead of the supply price, of each capital good, but adding the condition that those capital goods whose demand prices is lower than their production cost will not be produced. Morishima (1960) gave formal proof of this statement.

tacked by this type of logical criticism. Yet its expunction makes the model *unable* to outline capitalistic production: one of its fundamental characteristics—the circularity of production—and one of the fundamental forces of capitalism—the tendency towards a uniform rate of profit—have both been sacrificed to the rigour of the analysis, in order to continue to apply the logic of supply and demand to explain the determination of wages and profits.

5.2. International trade theory

The distinction between one-way and circular production processes is also relevant for understanding some relevant issues concerning international trade in the presence of capital goods. Some quite recent contributions pointed out the possibility that under a regime of capital mobility, one country can be out-competed in all international markets (Brewer 1985, Parrinello 2010, Bellino and Fratini 2022). This result, which describes the emergence of the international trade pattern as a problem of choice of the methods of production among the technology of the entire world (or of the countries involved in trade relations), is based on the assumption of a *given* level of real wages in each country. Consequently, it might be maintained, that a suitable variation of wages (a reduction in countries that risk the exclusion and/or an increase in the others) could re-channel the excluded country within international trade relations. However, Crespo, Dvoskin, and Ianni (2021) showed that such a wage reduction may not be sufficient to do the job in a system where commodities are produced by means of commodities, that is, in a circular system. The reason for this impossibility may be easily grasped by the analysis here developed. While in one-way systems a suitable reduction of wages may drag the price of commodities down to any desired level, in circular production systems this cannot happen. In this case, in fact, for a small price-taking economy, there is a lower bound greater than zero of unit production costs, given by

$$\mathbf{c}^{\mathrm{T}} = (1 + \bar{\pi})\bar{\mathbf{p}}^{\mathrm{T}}\mathbf{A}, \tag{13}$$

where $\bar{\mathbf{p}}$ and $\bar{\pi}$ are the vector of international prices and the international rate of profits.

This lower bound reflects the fact that even if the wage rate is zero, firms still need to cover capital costs valued at international prices and earn the profit rate expected in international markets. Otherwise, they would invest their capital abroad. Under these conditions, nothing prevents these domestic unit costs from being higher than international prices, $\bar{\mathbf{p}} < \mathbf{c}$, making the economy unable to compete in any industry.

If commodities were ultimately produced only by labour—that is, if production is one-way—the lower bound equals zero. In this case, as the wage rate falls, the price of commodities produced with pure labour tends to zero. This price reduction would be translated to the production cost of commodities requiring these commodities and, together with falling direct labour costs, it would make the price of these commodities also tend to zero. The process proceeds until we arrive at consumption commodities.[11] In this case, any sufficient wage reduction would eventually make the economy competitive.[12]

In conclusion, the 'strong equilibrating forces that normally ensure that any country remains able to sell a range of goods in world markets' (Krugman 1996, p. 89) might be effective only if production could be represented as a one-way process. However, since production is circular, such equilibrating forces may not always work.

6. Concluding remarks

In this paper, we have compared two alternative ways to regard production: the 'circular' view, typical of classical political economy, where the same commodities appear both among the inputs and the outputs ('production of commodities by means of commodities') and the 'one-way' view, typical of neoclassical economics, where one starts from the factors of production and ends up with final goods.

The circular view of production, reappraised thanks to Piero Sraffa's book, and also adopted in input-output analysis, by bringing out the notion of social surplus—that is, the excess of produced quantities over the quantities that must be re-employed to repeat the production process on an unchanged scale, brings to regard profits in physical terms—as the difference between produced and employed quantities. This is a view that can be traced back to the Physiocrats, as well as to Ricardo and Marx. Evidently, such an emphasis on the notion of social surplus brings about the problems connected to the measurement of commodity aggregates with different physical compositions. But, beyond these analytical problems, which have been analysed in-depth and solved by Piero Sraffa and the extensive related literature, the substance of profits remains something which ultimately pertains to the quantity (material, physical) side. The amplitude of the social surplus cannot be attributable to a particular input, firm or industry. Rather, its magnitude depends on the entire system's technological capacities. Profits are merely a claim over the surplus due to ownership of the means of production and reflect the bargaining power that capitalists have over workers.

On the other hand, the one-way view, starting from production factors and ending with consumption goods, involves quantities of resources that cannot be compared in any way. The only magnitudes that can be compared are the prices attributed to these magnitudes so that profits emerge as a difference of prices: the prices of goods and the prices of the factors of production. Profits thus arise from a markup to the costs of production. This

[11] Another way to circumvent the problem—the preferred way in neoclassical analysis—is to assume that all the national income accrues to labour. That is, the profit rate is zero. In that case, it can be shown that there is at least one competitive industry in the economy (see Deardorff 2005).

[12] This different result is the flip side of the fact that circular production systems have a Maximum rate of profit, whereas one-way systems do not.

view can be recognised more or less explicitly both by some classical authors like Smith or Malthus and by some neoclassical authors, like Böhm-Bawerk and the Austrians.

However, a 'price' theory of profit can be established only if real wages remain unaffected when prices change, i.e., if the constraint, that one class cannot have more without the other having less, is overcome. As observed, this is not possible. Even in one-way production systems, there is an inverse relationship between the rate of profit and real wages. Hence, the theory can only be established in a 'weak' formulation (price increases raise profits at given nominal wages). This may explain Sraffa's caution in claiming that non-circularity 'provides *ideal* conditions for a "price" theory of profit' (Sraffa, D3/12/68:20r, dated August 1955; authors' emphasis).

Furthermore, we have also shown that a one-way system is automatically transformed into a circular system if it is recognised that wages cannot descend below a value that ensures the purchase of a commodity basket required for the subsistence of the workforce. In other words, production processes are one-way only in appearance once it is acknowledged that workers consume 'basic' commodities.

In Section 5, the distinction between circular and one-way production systems has been exploited to shed light on two additional specific issues: i) the choice of neo-Walrasians to distinguish goods not only by their physical qualities but also by the date in which they become available; this apparent dynamisation of the analysis rids the requirement to obtain a uniform rate of return on the supply price of capital goods, and *de facto* abolishes the circularity elements connected with the presence of capital goods, returning to depict production as a one-way process; and ii) the effectiveness of wage moderation to pursue competitiveness in international trade relations. If production requires produced means of production, there is a positive lower bound to the prices of commodities, which does not disappear, even if wages fall to zero. This entails that wage reduction may not be able to re-establish a country's competitiveness if it is excluded from international trade.

Editorial note

Some of the points addressed in this paper were anticipated in a handwritten note by the late Frederic Lee, which Andrew Trigg gave to one of us (G. B.) some years ago. Nonetheless, our paper was conceived independently; only later did we discover the similarity in the arguments. Moreover, it must be said that our paper emphasises the case of production as a circular process. In contrast, Lee was more attracted by the case of one-way production processes since this view allowed him to regard profits as a markup.

References

Bellino, E., and S. M. Fratini (2022): "Absolute advantages and capital mobility in international trade theory," *The European Journal of the History of Economic Thought*, 29, 271–293.

Bortis, H. (1990): "Structure and change within the circular theory of production," in *The economic theory of structure and change*, ed. by M. Baranzini, and R. Scazzieri, pp. 64–94. Cambridge University Press, Cambridge.

Brewer, A. (1985): "Trade with fixed real wages and mobile capital," *Journal of International Economics*, 18, 177–186.

Crespo, E., A. Dvoskin, and G. Ianni (2021): "Exclusion in 'Ricardian' Trade Models," *Review of Political Economy*, 33, 194–211.

Deardorff, A. V. (2005): "Ricardian comparative advantage with intermediate inputs," *The North American Journal of Economics and Finance*, 16, 11–34.

Donzelli, F. (1989): "Teorie classiche e neoclassiche del valore. Alcune riflessioni critiche," in *Aspetti controversi della teoria del valore*, ed. by L. Pasinetti, pp. 69–95. Il Mulino, Bologna.

Fratini, S. M. (2014): "The Hicks-Malinvaud average period of production and 'marginal productivity': A critical assessment," *The European Journal of the History of Economic Thought*, 21(1), 142–157.

Fratini, S. M., and F. Ravagnani (2023): "Sraffa and the 'slogans not used'," MPRA Paper No. 117815.

Garegnani, P. (1978): "Notes on consumption, investment and effective demand: I," *Cambridge Journal of Economics*, 2(4), 335–353.

——— (1984): "Value and Distribution in the Classical Economists and in Marx," *Oxford Economic Papers*, 36(2), 291–325.

——— (1990): "Quantity of capital," in *Capital Theory – The New Palgrave*, ed. by J. Eatwell, M. Milgate, and P. Newman, pp. 1–78. Macmillan Press, London.

——— (2018): "On the Labour Theory of Value in Marx and in the Marxist Tradition," *Review of Political Economy*, 30(4), 618–642.

Krugman, P. (1996): *Pop Internationalism*. MIT Press.

Kurz, H. D., and N. Salvadori (1995): *Theory of Production – A Long-Period Approach*. Cambridge University Press, Cambridge.

Malinvaud, E. (1953): "Capital Accumulation and Efficient Allocation of Resources," *Econometrica*, 21(2), 233–268.

Malthus, T. R. (1836): *Principles of Political Economy – Considered with a View to Their Practical Applications*. Blackwell, Kelley, London and New York, 2nd edition, published in 1951.

Marx, K. (1863): *Theories of Surplus Value*. Progress Publisher, Moscow, Part III.

Meyer, C. D. (2000): *Matrix Analysis and Applied Linear Algebra*. SIAM.

Morishima, M. (1960): "Existence of Solutions to the Walrasian System of Capital Formation and Credit," *Zeitschrift für Nationalökonomie – Journal of Economics*, 20, 238–243.

Parrinello, S. (2010): "The notion of national competitiveness in a global economy," in *Economic Theory and Economic Thought. Essays in honour of Ian Steedman*, ed. by J. Vint, J. S. Metcalfe, H. D. Kurz, N. Salvadori, and P. A. Samuelson, chap. 4, pp. 49–68. Routledge, London and New York.

Pasinetti, L. L. (1973): "The Notion of Vertical Integration in Economic Analysis," *Metroeconomica*, 25(1), 1–29.

——— (1977): *Lectures in the Theory of Production*. Macmillan, London.

——— (1986): "Sraffa's Circular Process and the Concept of Vertical Integration," *Political Economy – Studies in the Surplus Approach*, 2(1), 3–16.

Smith, A. (1776): *An Inquiry into the Nature and Causes of the Wealth of Nations*. W. Strahan and T. Cadell, London, 2 vols.; edition used ed. by R. H. Campbell, A. S. Skinner and W. B. Todd (Glasgow Edition of the Works and Correspondence of Adam Smith), Clarendon Press, Oxford, 1979.

Sraffa, P. (1951): "Introduction," in *The Works and Correspondence of David Ricardo*, ed. by Piero Sraffa with the collaboration of Maurice H. Dobb, Vol. I, xiii–lxii.

——— (1960): *Production of Commodities by Means of Commodities – Prelude to a Critique of Economic Theory*. Cambridge University Press, Cambridge.

Walras, L. (1874): *Éléments d'économie politique pure, ou théorie de la richesse sociale*. Corbaz, Lausanne, CH, Engl. trans., *Elements of Pure Economics, or the Theory of Social Wealth*, translated by W. Jaffé, Richard D. Irvin, Inc., Homewood, Illinois, 1954.

Wicksell, K. (1901): *Föreläsningar i Nationalekonomi*, vol. I. Första delen: Teoretisk Nationalekonomi, Lund, Engl. transl., *Lectures on Political Economy*, Vol. I, Routledge, London, 1934.

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