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On the Origins of Piero Sraffa's Equations. New Evidence Following Pierangelo Garegnani's Lead

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Abstract

The paper investigates the origins of the equations which form the structure of Piero Sraffa's *Production of Commodities by means of Commodities*. Following an interpretation first developed by Pierangelo Garegnani in a paper that highlighted the importance of a manuscript headed 'Notes London, Summer 1927 (Physical Real Costs etc.)', we single out new evidence relevant to the reconstruction of the path which led Sraffa to conceive his equations. In particular, we stress how Sraffa came to pay special attention to the case of a subsistence economy ('a community that produces just what is sufficient to keep it going') and how this led him to shift his attention from the idea of reducing heterogeneous physical costs to an 'absolutely necessary commodity' to the determination of exchange ratios by the solution of systems of simultaneous equations.

Keywords: Sraffa; Piero Sraffa Papers; Production of Commodities; costs; relative prices

JEL codes: B24; B31; B51

1. Introduction

This paper investigates the origins of the equations which form the structure of Piero Sraffa's 1960 book *Production of Commodities by means of Commodities*. The question

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has already been approached during the two decades that followed the opening of the papers of Piero Sraffa to the public of scholars and, even though every interpreter has acknowledged a central role to Sraffa's critical stance against subjectivism and marginalist economics and to his attention to alternative approaches, the answers put forward have been different and sometimes contrasting. A first interpretation indicated Marx's *Theories of Surplus Value* and the reproduction schemes contained in the second volume of *Capital* as Sraffa's main source of inspiration (de Vivo 2000, 2003 and 2016 and Gilibert 2001 and 2003).[2] A subsequent interpretation focussed on Sraffa's reading of contemporary economists, on his interest in objective descriptions of economic processes and on his attention to natural sciences, notably physics and chemistry (Kurz and Salvadori 2004 and 2005; Gehrke and Kurz 2006 and 2018; Kurz 2006 and 2012). Thirdly, it has been argued that Sraffa's equations were an offspring of an endogenous evolution of his analysis of Marshallian economics which took place in summer 1927 (Garegnani 2004 and 2005).

The former two interpretations are crucially based on recognition of similarities between the equations Sraffa started to develop in late 1927 and specific passages in books or articles he certainly knew. Indeed, there may be no doubt that Sraffa's interest in Classical political economy and in Marx's contributions, his study of contemporary economists, his search for an approach to the theory of value and prices based on objective magnitudes, and also his interest in natural sciences provided a broad background to the development of his own equations. But when we try to gather information on the specific process which led Sraffa to write the earliest formulations of his equations, we see that none of those interpretations point to a list of manuscripts which may be read as part of a sequence – a paper trail – providing us testimony of the path actually followed by Sraffa.[3]

On the contrary, the interpretation originally put forward by Pierangelo Garegnani explicitly indicates some crucial steps and some manuscripts as parts of the process which led Sraffa to write his equations. Indeed, even though we have argued that the list of documents suggested by Garegnani is largely unsatisfactory,[4] we believe that he rightly pointed to a manuscript which, if considered with greater attention, may allow us

[2] This view is criticized in Kurz (2012 and 2015) and in Kurz and Salvadori (2015); see also de Vivo and Gilibert (2013).

[3] More recent interpretations also share the same general approach. Marchionatti stressed the importance of the debates on Marx's theory of value which followed the publication of volumes I-III of *Das Kapital* (Marchionatti 2018). Sinha pointed at an analogy between Sraffa's reading of Pareto and the relationship between his own equations and Classical political economy (Sinha 2016 pp. 48-9). The potential importance of Pareto's analysis had already been stressed by Gehrke and Kurz (2006 pp. 99-101), but, if the documents they referred to may reveal how some of Sraffa's early reflections on the equations he had conceived were to a certain extent guided by his knowledge of general equilibrium analysis, they do not illuminate the spring of the process which had led him to conceive those very equations: as we shall see in Sections 4 and 5 below, manuscripts kept in the Sraffa Papers suggest that that process had not been originated by a line of thought focussed on systems of simultaneous equations.

[4] See note 26 below.

to attain a deeper understanding of the origins of Sraffa's equations. This is what we propose to do in this paper: follow Garegnani's footsteps and single out new evidence directly relevant to the reconstruction of the path which led Sraffa to conceive his equations.

Our aim will be pursued from two opposite directions. On the one hand, we will take advantage of our knowledge of the shape that Sraffa's equations assumed in *Production of Commodities* and move backward from 1960 towards the late 1920s – swimming, so to speak, against the current of documents produced by Sraffa while preparing his book, up to the point where we may find information on the shape of the equations he originally wrote. On the other hand, we will select a moment in time remote enough to be able to assume that Sraffa's equations were still out of his own sight. We will then follow the string of Sraffa's extant manuscripts up to the point where we may see how those equations emerged. Luckily enough, we may have a rather precise idea of the point where movements from both directions should meet: few days before the 26th of November 1927.

2. The 26th of November 1927

In a letter to his wife dated Monday, 28 November 1927, Keynes wrote:

> On Sunday I had a long talk with Sraffa about his work. It is very interesting and original […] Sraffa is in so much intellectual ferment and excitement about his ideas since I said that I thought there was something in them that he walks very fast up and down his room all day thinking about them. It is impossible for him to write them down, because as soon as he thinks about them, he has to start walking again. He is now inclined to give up his Christmas visit to Italy so that he can be able to continue in these courses for several weeks more (King's College, Modern Archive Centre, JMK/PP/45/190/3/268-9, J. M. Keynes to L. Lopokowa; emphasis as in the original).

Most likely, Keynes' sentences were somewhat imprecise. It is true that Sraffa did not leave England during the 1927 Christmas vacations, but we may presume that the reason was that (because of a letter on Gramsci's detention he had sent to the *Manchester Guardian* at the end of October) he was in danger of being arrested by the Italian police. Furthermore, in that very end of November, he certainly paused from what Keynes described as a sort of compulsive walking and drafted many notes, including a long one, presently to be considered, relating at least in part to the content of a conversation with Keynes he explicitly mentioned as having taken place on Saturday 26 November 1927 – not unlikely the conversation that Keynes wrote to his wife they had had on "Sunday". Finally, even though Keynes attributed Sraffa's excitement exclusively to his own approval of Sraffa's ideas, we may assume that it had been produced, at least in part, by those very ideas and by the results Sraffa perceived they promised to bring.

This is confirmed by the manuscript we have just mentioned:

> I foresee that the ultimate result will be a restatement of Marx, by substituting to his hegelian metaphysics and terminology our own modern metaphysics and terminology: by metaphysics here I mean, I suppose, the emotions that are associated with our terminology and frames (schemi mentali) – that is, what is absolutely necessary to make the theory living (lebendig), capable of assimilation and at all intelligible.
>
> If this is true, it is an exceptional example of how far a difference in metaphysics can make to us absolutely unintelligible an otherwise perfectly sound theory. This would be simply a translation of Marx into English, from the forms of Hegelian metaphysics to the forms of Hume's metaphysics (Keynes to-day, 26 XI. 27, has clearly outlined the divorce between English and Continental thought: the first descending from Descartes and Hobbes, the two original geniuses, to Locke, Hutcheson and ultimately Hume; the second from Spinosa (did he say that of S.?) from Kant to Hegel: they always remained foreign to one another).
>
> If this is true it also shows (or is it an exceptional case? in physics it doesnt seem to be indifferent) how little our metaphysics affect the truth of our conclusions, and how the same truths can be expressed in two widely divergent forms. Our metaphysics is in fact embodied in our technique; the danger lies in this, that when we have succeeded in thoroughly mastering a technique, we are very liable to be mastered by her.
>
> The typical case of Marx's metaphysics is his statement that «only human labour produces (causes) values», «values are embodied human energy (crystallized)»: there is no doubt that he attached to it some metaphysical meaning (D3/12/4: 15-16).

Sraffa had just found something which was allowing him to see how Marx, which we may guess he felt he knew quite well,[5] could be "restated" or "translated" and, presumably, placed on the forefront of contemporary economic science. But this perception must have been based on something else beside a discussion of the main strands of European philosophy. Indeed, we may presume that that basis rested on what Sraffa marked in his pocket diary under the very date of November 26 1927: 'K. approves 1st eq.' (Sraffa Papers E1) – which we may read as *Keynes approves first equations*.

Sraffa at that time was certainly familiar with many instances of equations used by modern economists – we find an inkling of this in the very manuscript quoted above:

> All the inquiry about value has always been (and still is and probably always will be) a purely metaphysical quest. When the old economists asked for the "causes" or the "measure" of value, they really were looking – as in fact we are, under the illusion of our equations "determining" value – for the «nature» of value (it is not an accident, as Cannan, elsewhere, says that the word is in A. Smith's title) in the same metaphysical sense in which we look for the nature of «matter» or of «mind» (D3/12/4: 16).

[5] Parenthetically, we would like to mention that knowing that Sraffa and Antonio Gramsci met quite often between 1924 and 1926 and had long and engaging conversations (Naldi 2000 pp. 88-92, Steve 2004 p. 17) allows us to strengthen the twofold presumption that Sraffa in 1927 knew Marx's *Theories of Surplus Value* and all the three volumes of Marx's *Capital*. Gramsci, before his arrest, already owned a copy of the French translation of Marx's *Theories of Surplus Value* – the *Histoire des doctrines économiques*, published in 1924-25 (letter from A. Gramsci to T. Schucht, 24 March 1929; Gramsci and Schucht 1997 p. 332), and in his Prison Notebooks, discussing how a compendium of Marx's *Capital* should be prepared, he stressed the importance of basing it on all the three volumes and also on the *Theories of Surplus Value* (see Quaderni, 10 XXXIII § 37 II, in Gramsci 1995 pp. 176-9). Obviously, this is no proof of Sraffa's reading of Marx's *Capital* and *Theories of Surplus Value* in the 1920s, but it would be strange if he had not shared at least Gramsci's degree of acquaintance with those books.

From his papers, however, we know that what Sraffa meant by "first equations" was something very precise: a set of equations describing production processes in an economy producing no surplus. Therefore, the entry in Sraffa's pocket diary may be interpreted in the sense that he had received Keynes' approval after showing him equations he had recently conceived, that those equations described a no surplus economy, and that by that time he had already conceived also a second set of equations describing the case of an economy producing a positive surplus ("first equations" are likely to be so named only together with "second equations").[6] We may then presume that by the 26th of November 1927 Sraffa had already drawn an early sketch of the equations he was eventually to display in Chapters I and II of his 1960 book, that he attributed great importance to those equations, and that, for some reasons, he felt he could show to Keynes only the first set (or that Keynes felt he could approve only that set).

3. Earliest drafts of Sraffa's equations

Having argued that by the 26th of November 1927 Sraffa had already written an early draft of his equations, that he believed that they were relevant to a restatement of Marx's contributions, and that they put him in a state of considerable excitement, we may start our search for their origins by moving backward from the vantage point of the schemes that we find in *Production of Commodities* and look for their primitive formulations.

Given our aim, we may skip much of the thirty-three years separating 1960 from 1927 and direct attention towards some notes written between late 1927 and early 1928 which reveal that at that time Sraffa introduced price variables into equations where only material inputs and outputs had explicitly appeared. This information, already highlighted by Garegnani (2004 pp. 176-8; 2005 pp. 468-70), emerges from a manuscript kept in a folder titled 'Winter 1927-28'.[7] Here Sraffa, considering a case of positive net product and including a distributive variable r, came to discuss this system (D3/12/6: 17):[8]

$$\begin{aligned} A + A_S &= (5a_1 + 6b_1 + 3c_1)\, r \\ B + B_S &= (4a_2 + 2b_2 + 6c_2)\, r \\ C + C_S &= (7a_3 + 2b_3 + 3c_3)\, r \end{aligned} \qquad [1]$$

[6] For instances of Sraffa's use of the phrases "first equations" and "second equations" in late 1927 and early 1928 see D3/12/9: 9, D3/12/10: 33 and D3/12/11: 17, 35 (see also D3/12/23: 1, f.1, dating to 1942).

[7] The manuscript could be part of the lecture notes mentioned by Sraffa in a draft letter to the Secretary of the General Board of the University of Cambridge dated 11 January 1928: 'now that I have prepared a certain number of lectures I am convinced that the subject I had chosen is really quite unsuitable' (B9/1: 16; see also B9/1: 11).

[8] The two components of each commodity's output distinguish input replacement (A, B, C) and positive surplus product (A_S, B_S, C_S).

Pursuing its solution, Sraffa stressed the importance of replacing 'apparent unknowns' with 'real unknowns', or 'values'. What he meant by 'apparent' and 'real' unknowns can be understood by considering how he modified the previous equations (D3/12/6: 18):

$$\begin{aligned} V_{a/b}\,(A + A_S) &= (V_{a/b}a_1 + b_1 + V_{c/b}c_1)\,r \\ (B + B_S) &= (V_{a/b}a_2 + b_2 + V_{c/b}c_2)\,r \\ V_{c/b}\,(C + C_S) &= (V_{a/b}a_3 + b_3 + V_{c/b}c_3)\,r \end{aligned} \qquad [2]$$

$V_{a/b}$ and $V_{c/b}$ were the 'real unknowns': the value of A in terms of B and the value of C in terms of B. The 'apparent unknowns' previously employed (a_i, b_i and c_i, and also A, B and C)[9] were 'only «one unit of measure of each commodity» (1 bushel of wheat, 1 ton of coal etc)' (D3/12/6: 17).

Our search for the earliest formulations of Sraffa's equations must then look for notes where no price variable is explicitly introduced and production processes are described by straightforward combinations of material inputs and outputs. Such systems of equations must approximate to the following two instances, which can be found more than once in the Sraffa Papers (see D3/12/6: 6 and D3/12/10: 95 for the former, and D3/12/11: 77, 87, D3/12/5: 2, and D3/12/8: 8 for the latter):

$$\begin{aligned} 13A &= 4A + 5B \\ 7B &= 9A + 2B \end{aligned} \qquad [3]$$

and

$$\begin{aligned} A &= a_1 + b_1 + c_1 \\ B &= a_2 + b_2 + c_2 \\ C &= a_3 + b_3 + c_3 \end{aligned} \qquad [4]$$

A system with these characteristics can be found in manuscripts D3/12/2: 32 and D3/12/2: 34, and, in our view, that is the earliest extant formulation of Sraffa's equations: possibly the earliest formulation. Manuscripts D3/12/2: 32 and 34 analyze a no-surplus case and may be associated to two other documents (manuscripts D3/12/2: 33 and D3/12/2: 35) which are likely to have been written immediately afterwards and discuss two cases of positive surplus.

These documents are kept among others dating to the 1940s and 1950s in a folder headed 'USEFUL (All dates) ex black cover, 1955', but on manuscript D3/12/2: 32 Sraffa annotated: '(From folder headed: 'End of Nov. 1927')' – a folder with such heading actually exists and is classed D3/12/4: it is the folder which contains the manuscript

[9] As already noted by Garegnani, a_i, b_i and c_i play different roles in the two sets of equations (units of commodities in the first case; quantities of commodities in the second case). Close inspection of the manuscript reveals that Sraffa's original formulation had no figures and inputs were simply described as $a_i + b_i + c_i$. Figures were inserted as an afterthought, which also gave a different status to a_i, b_i, and c_i on one side and to A, B, C, A_S, B_S, and C_S on the other. They disappeared, and a_i, b_i, and c_i were returned to their original role, when Sraffa substituted 'real unknowns' for 'apparent unknowns' (see Garegnani 2004 p. 177 fn. 30 and 2005 pp. 469, 487 fn. 26). See also manuscripts D3/12/8: 26-29, most likely written in early 1928.

on metaphysics quoted above.[10] Manuscript D3/12/2: 32 was written on the back of the second page of a letter that presumably reached Sraffa between the 23rd and the 25th of November.[11] These pieces of information imply that item D3/12/2: 32 could not have been written more than a couple of days before the 26th of November, when Sraffa showed his "first equations" to Keynes, and – having been part of a folder headed 'End of November 1927' – it should not have been written more than a couple of days after that meeting; the same presumption we may also extend to document D3/12/2: 34, where, as we shall see, the same system appears, and to documents D3/12/2: 33 and D3/12/2: 35.

As we have just mentioned, manuscripts D3/12/2: 32 and D3/12/2: 34 elaborate upon the same system of equations. The system is formed by three equations; three commodities may be identified and outputs and inputs may be recognized as described by letters and figures to be associated to each commodity and its quantity; the net product is clearly nil:[12]

$$\begin{aligned} 10A &= 3A + 7B + 4C \\ 20B &= 6A + 5B + 1C \\ 15C &= 1A + 8B + 10C \end{aligned} \qquad [5]$$

In sheet D3/12/2: 32, the equations are arranged exactly as we have just written them and are followed by calculations strewn in a rather casual and haphazard way throughout the page. We reproduce them in a single column:

$$\begin{aligned} &\frac{4}{5}A + \frac{32}{5}B \\ 7A - \frac{4}{5}A &= 7B + \frac{32}{5}B \\ \frac{31}{5}A &= \frac{67}{5}B \\ A &= \frac{67}{31}B \\ B &= \frac{31}{67}A \\ C &= \frac{63}{67}A \\ 7A &= 4C + \left[\frac{6A + 1C}{15}\right]7 \end{aligned} \qquad [6]$$

[10] The actual heading of the folder is 'End of November 1927 (large sheets)' – with the latter two words Sraffa probably meant to distinguish it from the set of notes now classed D3/12/11.

[11] The first page of the letter was also used by Sraffa as writing sheet: it is kept in another folder and marked D3/12/5: 32 (manuscript D3/12/5: 33 is dated '2.12.27'). The letter, which is dated 19 November 1927, was sent from Britain to Sraffa's address in Milan and forwarded from Milan to Cambridge. Evidence from the correspondence between Piero Sraffa and Tatiana Schucht suggests that a letter sent from England to Milan and readdressed to Cambridge could have completed the whole travel in five to seven days.

[12] On the top of the sheet, next to '(From folder headed: 'End of Nov. 1927')', Sraffa also wrote: 'Without surplus' – the two phrases seem to have been written at different times.

$$7A - \frac{42}{15}A = \frac{63}{15}A = \frac{67}{15}C$$
$$A = \frac{67}{63}C$$
$$7B = \frac{217}{67}A$$
$$4C = \frac{252}{67}A$$
$$4C + 7B = \frac{469}{67}A = 7A$$

It is clear that Sraffa, who placed the solutions for *A* in terms of *B* and for *A* in terms of *C* within circles and also stressed the last line, focussed his attention on solving the system.[13]

In sheet D3/12/2: 34 the same equations are arranged in a row.[14] Under the equations we find approximately the same calculations as in document D3/12/2: 32, but they are accomplished as if to put in good order what in the other manuscript had been jotted down quite quickly. The calculations lead to solutions for *B* and *C* in terms of *A* (placed by Sraffa, also in this case, within circles):[15]

$10A = 3A + 7B + 4C$
$7A = 7B + 4C$
$7A = 7B + \frac{4}{5}A + \frac{32}{5}B$
$7A - \frac{4}{5}A = 7B + \frac{32}{5}B$
$\frac{31}{5}A = \frac{67}{5}B$
$31A = 67B$
$B = \frac{31}{67}A$

$15C = 1A + 8B + 10C$
$5C = 1A + 8B$
$C = \frac{1}{5}A + \frac{8}{5}B$
$C = \frac{1}{5}A + \frac{8}{5}\left(\frac{6}{15}A + \frac{1}{15}C\right)$
$C = \frac{1}{5}A + \frac{48}{75}A + \frac{8}{75}C$
$C - \frac{8}{75}C = \frac{1}{5}A + \frac{48}{75}A$
$\frac{(75-8)}{75}C = \left(\frac{15}{75} + \frac{48}{75}A\right)$
$\frac{67}{75}C = \frac{63}{75}A$
$67C = 63A$
$C = \frac{63}{67}A$

$20B = 6A + 5B + 1C$
$15B = 6A + 1C$
$B = \frac{6}{15}A + \frac{1}{15}C$

[7]

Repeating in good order previously scattered calculations and arranging equations in a row are quite unusual features in the Sraffa Papers. In this case, we take the liberty of interpreting them as signs of Sraffa's surprise in the face of a result he had reached somewhat unexpectedly, and of his desire to show the system to another person (possibly, Keynes). Be that as it may, it does not seem out of place to assume that these manuscripts contain the earliest formulation of Sraffa's equations. This view is consistent with both the peculiarities of manuscript D3/12/2: 32 which allow to date it to the days

[13] Other calculations, mainly located on the bottom of the page, were crossed out by Sraffa and are not reproduced here.

[14] On the top of the sheet Sraffa wrote: 'No surplus (stesse equazioni)' (No surplus (same equations)).

[15] Other calculations, located on the bottom of the page, were crossed out by Sraffa and are not reproduced here. On the back of the sheet Sraffa wrote and crossed out another no-surplus system.

around the 26th of November 1927 and with the reconstruction of how and when Sraffa introduced price variables into his systems.

But we may also presume that, having the system in D3/12/2: 32 and 34 been written and solved, a question concerning what would happen if there were a positive surplus would have immediately been considered: could the same sort of solutions be determined? Indeed, attempts to tackle such a question may be recognized in manuscripts D3/12/2: 33 and D3/12/2: 35.

Manuscript D3/12/2: 33 shows the following system and calculations:

$$\begin{gathered} 10A + 4A = 3A + 9B \\ 12B = 7A + 3B \end{gathered}$$

$$\begin{gathered} 14A = 3A + 9B \\ 12B = 7A + 3B \end{gathered} \qquad [8]$$

$$\begin{aligned} 11A &= 9B & A &= \frac{9}{11}B \\ 9B &= 7A & A &= \frac{9}{7}B \end{aligned}$$

While on the top of the sheet Sraffa annotated 'Try negative surplus (loss)' and 'Surplus only in A with two unknowns there are two solutions Why?',[16] the two results for *A* in terms of *B* are accompanied by a big question mark and by the following comments: 'V. Chini p 41 (le equazioni sono contraddittorie quindi non esiste alcuna soluzione) Le equazioni devono essere non contraddittorie indipendenti'.[17] These annotations are consistent with the presumption that the document was written at a very early stage of Sraf-

[16] These sentences seem to have been written at different times. We may also stress that the phrase 'Surplus only in *A* with two unknowns there are two solutions Why?' may imply that, consistently with what have been observed above, *A* and *B* were simultaneously conceived as units of commodities and unknowns. Several manuscripts in the Sraffa Papers show that the solutions of such early systems had been immediately interpreted by Sraffa as values and exchange ratios (see for instance D3/12/11: 54, 89, 101, D3/12/6: 4 and D3/12/5: 2).

[17] 'See Chini p 41 (equations are contradictory therefore no solution exists) Equations must be non contradictory independent'. 'Chini page 41' certainly refers to the book by Mineo Chini *Corso speciale di matematiche. Con numerose applicazioni. Ad uso principalmente dei chimici e dei naturalisti* (the book is still kept in Sraffa's library – see de Vivo 2014, p. 89). Chini's *Corso speciale* was an introductory handbook based, as Chini himself stated in its preface, on university-level teaching experience of the mathematics needed by students who were to engage in experimental sciences. The latter characteristic, together with examples and exercises distributed throughout the text mainly relating to chemistry and to natural sciences in general, justified the subtitle of the book. We may presume that Sraffa bought it between 1923 and 1926, i.e. in the earliest years of his academic career. Indeed, Sraffa, who began university teaching in the academic year 1923-24, owned the sixth edition of the book, issued in 1923 (a subsequent edition was issued in 1926). It would not be surprising if that book had been recommended to Sraffa by Ettore Molinari, professor of chemistry at Bocconi University (Sraffa's father was the Vice-Chancellor of that university) and prominent figure of the Anarchist movement, or by his son, Alessandro Molinari, who had graduated from Bocconi University in 1920 with a thesis on Russian Soviets (when, in June 1922, Piero Sraffa was appointed director of the Labor Office of the Province of Milan, Alessandro Molinari was director of the Labor Office of the Municipality of Milan, and they certainly were acquainted with each other). For other references to Chini's book in the Sraffa Papers, see Kurz (2012, pp. 1545-47).

fa's work on his equations. It may certainly be taken to have been produced immediately after documents D3/12/2: 32 and 34, and, not unlikely, it could be the earliest formulation of Sraffa's equations for the case of a positive surplus.

Document D3/12/2: 35 contains another attempt to tackle the same point. Here we find the following system:[18]

$$\begin{gathered} 11A = 3A + 9B \\ 13B = 7A + 3B \\ S = 1A + 1B \end{gathered}$$

$$\begin{gathered} S = 1A + \frac{7}{10}A = \frac{17}{10}A \\ A = \frac{9}{8}B \\ B = \frac{7}{10}A \end{gathered} \qquad [9]$$

The results were similar to those reached in the previous manuscript, and, next to the three equations, Sraffa wrote: 'These are contradictory, whether *S* equal or not to zero.'

4. The spring of the equations

If moving backward from 1960 towards November 1927 we have been able to locate four manuscripts and list them as the earliest extant drafts (not unlikely, the very earliest drafts) of Sraffa's equations, we may now consider Sraffa's research work in the early 1920s, look for traces of the path which led him to write his equations – i.e. a paper trail illuminating their origins – and see if the results of this search are consistent with those achieved moving from the opposite perspective.

As far as we know, Sraffa began to devote most of his time to studying economic theory in 1923, when he decided to turn to academic teaching as his main occupation (see Naldi 2005 pp. 382-4). Even though many documents suggest that he focussed his attention on Marshall's *Principles*, we may presume that his interests had a larger scope and encompassed contemporary economic theory in general, Classical political economy and Marx. As a matter of fact, when we read his 1925 and 1926 articles we may appreciate both his acute penetration of Marshall's theory and his references to Classical economists. As is well known, in those articles Sraffa put forward a critique of Marshall's theory of prices, suggested that within the compass of full competition an assumption of constant costs could provide a better solution than Marshall's approach, and proposed to develop the analysis of price determination in the direction of a more realistic case laying between full competition and monopoly.

[18] On the top of the sheet Sraffa had annotated: 'Surplus «a separate industry».'

Thanks to the interest raised by his 1925 and 1926 articles, Sraffa was invited to teach in Cambridge and the subject of his main course was to be *Advanced Theory of Value* (Marcuzzo 2005 pp. 426-8, Naldi 2005 pp. 386). The course was due to start in October 1927 and some months before Sraffa set to prepare his lectures placing them – quite obviously – on a canvas broader than that of his articles. The result of that preparation can be recognized in a document kept in a folder headed 'Notes London, Summer 1927 (Physical Real Costs etc.)' (D3/12/3).

In those notes Sraffa meant to put recent theories of value in historical perspective, to illustrate the evolution of Classical and later analysis of value and prices and – above all – to highlight characteristics, weaknesses and limitations of each theory. The approach to value of Ricardo and the Classics – spanning from a search for ultimate causes and standards of value to an interest for the 'relation of commodities as a whole to mankind' (Sraffa Papers D3/12/3: 8-9) – was put into a nutshell and interpreted through a conception of 'physical real costs' and through the idea of reducing any cost of production to an 'absolutely necessary commodity' (actually, a wage basket). Sraffa described this structure as logically defective, but he argued that it could provide a better approximation to an exact explanation of value and prices than any theory based on subjective magnitudes. The parts of the lectures devoted to discuss modern theory, on the other hand, could not be restricted to Marshall's *scissors*, as Sraffa had done in the 1925 and 1926 articles. They had to consider also schools emphasizing utility as sole determinant of value, opportunity costs and general equilibrium. With regard to each of these strands, Sraffa set to develop detailed criticism and to show that they were not satisfactory.

Most likely, after having shown the limitations of any available conception of price determination in a context of perfect competition, Sraffa would have introduced a case laying between full competition and monopoly, but this discussion is not part of the pre-lectures. What we may take for granted is that Sraffa – as it emerges from letters dating to late September and mid-October 1927 (see Naldi 2018 p. 139) – was not happy with the draft he had prepared. Indeed, that draft is far from polished, and, if Sraffa had meant to lecture by reading it to his students, he had many reasons to be unhappy. But quite apart from its being polished or unpolished, we may see that the content of those notes suggests that from the point of view of the theory of prices in a context of full competition Sraffa had reached a stalemate. He had indicated fundamental differences between Classics and modern theorists, he had highlighted limitations of both approaches, but he had been unable to put forward a general solution to the problem of price determination – and we may presume that, even introducing a case of semi-monopoly and describing it as more realistic than perfect competition, he could not have avoided thinking that discussion of the latter remained crucial.[19]

[19] Some points are worth mentioning with regard to the manuscript of the pre-lectures. On the one hand, we may wonder to what extent the content of folder D3/12/3 actually reflects the whole set of notes

In this state of things, Sraffa asked and obtained his course to be delayed by some months (on the 18th of October the *Cambridge University Reporter* announced the course was due to start in Lent term – see Marcuzzo 2005 p. 446 fn. 8).[20] Having been relieved of the pressure of a close start of his lectures, we may guess that he approached with more ease the most tedious tasks involved by their preparation and (not unlikely after the earliest phase of the crisis originated by his letter to the *Manchester Guardian* on Gramsci's detention had been overcome and after having read a paper on the 'Revalorization of the Lira' at The Economics Society of Emmanuel College, on the 3rd of November, and another one on 'The Corporative State' at Keynes' Club, on the 14th of the same month)[21] allowed himself to engage in some of the parts of the work he had done up to that point which he felt more lively and in tune with his own emotions (we are freely borrowing words from Sraffa's note on metaphysics, quoted in Section 2 above). And we may be sure that the parts dealing with Classical economists and with Marx, and, among them, those dealing with the conception of physical real costs, ranked high in such a hypothetical list.

Be it as it may, the latter conception and its keystone (the "absolutely necessary commodity") deserve to be considered with some attention, and the text of the relevant part of the notes prepared by Sraffa in summer 1927 must be quoted in full:

> Physical real costs.This conception would be tenable only if all the commodities considered (or at least one of them) had, each of them, no possible substitute (and therefore were absolute necessaries, since luxuries are naturally substitutes among themselves). But if commodities have substitutes, there is no more "one" real cost composed of a series of various quantities of commodities, which don't require a common measure: so soon as there are substitutes, there is an infinite number of combinations of the different commodities, which satisfy the condition of maintaining life and efficiency of the producers. How are we to choose between these combinations? [But in a community that produces just what is sufficient to keep it going would there not be only one combina-

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prepared by Sraffa for the lectures due to start in October 1927. Indeed, the order of the sheets (originally unnumbered) may have been altered in time, and some sheets may have been dispersed, or moved to other folders by Sraffa himself. On the other hand, we may stress that discussion of physical real costs occupies less than ten percent of the notes in D3/12/3, and the results of that discussion are presented with no emphasis. This suggests to interpret the prominence that the subject is given in the heading of the folder by supposing that Sraffa wrote the latter after November 1927, when, as we shall argue, he had already come to see that physical real costs could play a crucial role in the analysis of exchange value – something he had been far from recognizing in summer 1927.

[20] In the end, Sraffa started his course only a year later, in October 1928, and he used another set of notes (now kept in the Sraffa Papers under the archive number D2/4); for this reason Garegnani dubbed the former document "pre-lectures" (Garegnani 2005 p. 453; Naldi 2018, p. 146 fn. 5) – we will follow the same convention.

[21] The letter to the *Manchester Guardian* had been published on October the 24th and on that very day Sraffa sent a second letter, intended to conceal the authorship of the former (see Naldi 2005 pp. 380-81 and 388-89; Naldi 2008 p. 17; Lattanzi and Naldi 2018 pp. 77-8). For the two papers see D2/3, D2/2, entries on the 3rd and 14th of November 1927 in Sraffa's diary E1, and Emmanuel College Magazine, 1927-28, vol. XXVI, n.1, p. 35.

tion which satisfies the above condition? it would be «the cheapest»][22] it is of course impossible to choose between 1 kg of bread + 1/2 kg of meat and 1/4 kg of bread + 1 kg of meat, unless we introduce the common measure of their value—and that would beg the question.

It should be remarked that if this difficulty (of no substitutes) were overcome and an absolutely necessary commodity found, the difficulty of reducing [to a common measure] the various ~~factors~~ [things] entering into real cost would solve by itself. In effect, it would be easy to find the cost of all the other things in terms of the necessary one, and thus by going back enough in the genealogy of production (and stopping along each branch so soon as we have resolved it into our necessary commodity) we might find exactly the total amount of ~~wheat~~ [corn] (if this were the ideal necessary commodity, which it is not) that has actually entered into the production of, say, this book, and covers entirely its cost of production, at the exclusion of any other commodity. (This is true: it is just as true as saying that a man has not a drop of blood that does not come from a man called A…: in fact if we followed each branch of his genealogy up to when we find an A… and stopped there in each case, this would happen. In the case of corn the process would be different, because at each step backwards we would find a part of cost being wheat and the other not, and setting aside the first, while going on analysing the latter, this non-wheat residue would ultimately be reduced to practically nothing – would have zero as limit.)

There is however something to be said for this conception of real cost. It is true that there is an infinite number of combinations of commodities which would be «the minimum» necessary to support permanently a labourer working 8 hours a day at a given standard of efficiency. But this difficulty arises only in so far as we abstain from using a unit of measure for the different commodities, and simply say that the real cost of producing a given article is a given set of diverse commodities—and this would be an «ultimate» conception if there were no possible substitutes for those commodities.

This not being the case, we must find a unit of measure for cost: the necessity for this unit arises, not from a desire of actually measuring, – it is prior to it, and is required even for thinking of cost. The best measure available is the amount of various commodities that is required to support during an hour, ~~or day or year~~ a [average] common labourer: if there are many of such sets of commodities, we can choose the one that can be produced with a minimum of labour (this is ambiguous!). Of course, not all individuals in one trade require the same amount of necessaries, and persons in different trades require different amounts – and to this extent our measurement is inexact, and ~~cost~~ real cost is slightly different (in excess or deficiency) from number of hours of labour. I contend however that the amount of necessaries varies much less ~~in the case of di~~ between different workers, than vary a) their disutilities, b) their wages.

Thus to Ricardo's T. V.,[23] based on amount of labour, two interpretations can be given: 1) the subjective [psycholog.], disutility one, 2) the objective [physical], necessaries of existence one. He probably had not always clear in mind the distinction, but I believe that the latter is the one that underlies his T. V.

(Sraffa Papers D3/12/3: 44-47, spelling and words underlined or crossed out are as in the original manuscript; smaller case indicates words inserted above the line).

In the two previous sheets of the pre-lectures (D3/12/3: 42-3) Sraffa had already hinted that physical real costs are opposed to psychological standards, but no proper

[22] The sentences within square brackets were written by Sraffa, within square brackets (see D3/12/3: 44), on the left hand margin of the sheet.

[23] Read: theory of value.

presentation of the concept can be found in the manuscript – most likely the sheets which contained it have been lost, or had never been written. The text, in any case, reveals that this approach describes real costs as sets of commodity inputs employed in production processes, and conceives also labour inputs as sets of commodities. If the difficulties implied by the existence of substitutes could be overcome or if a unique set of commodities expressing labour inputs could be identified, this conception would allow us to go back through the genealogy of production and find the total amount of a hypothetical absolutely necessary commodity directly and indirectly employed in the production of any other commodity. This would allow to express the cost of any commodity by a definite magnitude and would answer the question of the existence of an ultimate standard or of a common measure of value (something which is 'required even for thinking of cost'). But Sraffa stressed that the existence of substitutes would stand as an obstacle against the possibility of reaching a general solution by this route – and he made equally clear that he was not prepared to assume that difficulty away: only an empirically approximate solution could be reached. However, in the note appended on the left-hand margin of sheet D3/12/3: 44, he also hinted that the difficulties generated by the existence of substitutes could be sidestepped by confining the analysis within the boundaries of 'a community that produces just what is sufficient to keep it going'. He foresaw that in that special case 'only one combination' would satisfy the condition of 'maintaining life and efficiency of the producers'. Accordingly, a description of real costs as sets of commodities – physical real costs – could be seen as the ultimate foundation of cost and could be expected to lead to an exact determination of the latter by reducing inputs of any individual commodity to amounts of an absolutely necessary commodity.

This approach shows clear similarities with fundamental features of *Production of Commodities*. The description of production processes as "sets" of physical quantities of diverse commodities representing the physical real cost of producing a given article is very close to the way methods of production are tabulated in the opening chapters of Sraffa's book. The note in the margin of sheet D3/12/3: 44, hinting at a subsistence economy (an economy so poor that the wage basket could only be 'the cheapest'), obviously leads us towards the 'extremely simple society which produces just enough to maintain itself' we meet in the first paragraph of the book (Sraffa 1960 p. 3),[24] and towards the distinction between production for subsistence and production with a surplus of Chapters I and II (Sraffa 1960 pp. 3, 6). The very description of 'production and consumption as a circular process' emphasized by Sraffa both in the Preface and in Appen-

[24] We may mention that, if the opening sentence of the Italian edition of *Production of Commodities* ('Consideriamo una società primitiva che produce appena il necessario per continuare a sussistere') does not translate word-for-word the English edition, it is because of a suggestion by Raffaele Mattioli, which led Sraffa, in Spring 1960, to replace the original phrase 'società elementare' with 'società primitiva' (Fondazione Mattioli, Carte Piero Sraffa, Box n. 4).

dix D (Sraffa 1960 pp. v, 93) is implicit in the idea of 'a community that produces just what is sufficient to keep it going' that we find in D3/12/3: 44.

But these elements allow us to do more than record similarities: they allow us to formulate a detailed hypothesis on the path followed by Sraffa up to the point of writing the earliest sketch of his equations. We may presume that Sraffa, in late November 1927, reconsidering the lecture notes he had prepared during the summer, decided to explore the possibility of reducing production processes to a hypothetical absolutely necessary commodity in the special case of an economy that barely "keeps going". The latter specification is crucial: if he had not identified a case within whose boundaries the problems posed by the existence of substitutes could have been sidestepped, he would have had little or no incentive to attempt an analytical reduction of production processes of individual commodities to a hypothetical absolutely necessary commodity: that was a direction that he had otherwise described as untenable.[25]

These results have been reached following a route traced by Pierangelo Garegnani in the paper he read at the 2003 Lincei conference on Piero Sraffa (Garegnani 2004 pp. 159-83 and 2005 pp. 453-75). However, of the three elements outlined above, Garegnani stressed only Sraffa's interest in the reduction of inputs to a hypothetical absolutely necessary commodity, most likely taking for granted the description of economic activity by means of sets of commodities employed in production and as a circular process. But Garegnani paid no attention to the note on a community that barely "keeps going", which, in our view, was crucial. This led him to direct his search for early drafts of Sraffa's equations looking for manuscripts which could resemble attempts to reduce production processes to a hypothetical absolutely necessary commodity. This search, however, did not prove fruitful: it led Garegnani to single out documents dating to a stage of Sraffa's research more advanced than the early approach he was looking for, and we may also doubt that those manuscripts actually contain any reduction at all.[26]

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[25] This point deserves to be considered more closely: why did Sraffa describe the existence of substitutes as an obstacle which prevented any general step forward within the physical real cost approach, while he seems to have presented the subsistence community case as an interesting one? Why did he not treat in the same way the hypothetical cases of an economy with no substitutes and of a subsistence community? In our view, Sraffa could have seen a difference between the two cases in the fact that assuming away the existence of substitutes would have meant assuming away an aspect of real life; on the contrary, a case of strict subsistence as the one he had envisaged in his note could have been taken as a case – however extreme – of real economic life. Such a distinction elicits an analogy with another one we find in a note most likely relating to a conversation Sraffa had with Wittgenstein in the early 1930s. In that note Sraffa distinguished between two different 'conditional propositions' and argued that '«If I were the king» is nonsense. For either I, or the job, would have to be entirely different', while '«If I were a lecturer» has sense. For I was last year, and I don't think I have changed much since, nor has the job' (D3/12/7: 174; see also D3/12/7: 42-3).

[26] In our view, as argued in greater detail in Naldi (2018 pp. 135-9), contrary to what had been suggested by Garegnani (2004 p. 173, 2005 pp. 465, 487 fn. 21), manuscripts D3/12/6: 1(f.1-f.6) do not stem directly from the pre-lectures and do not contain an attempt to develop a reduction to an absolutely necessary commodity. Most probably, they reflect an attempt to generalize results concerning the solution of systems of equations. Indeed, items D3/12/6: 1(f.1-f.6), just like items D3/12/6: 17-18, already dis-

Our reconstruction, on the contrary, while recognizing the importance of Sraffa's interest in the reduction of inputs to an absolutely necessary commodity, directs our research towards manuscripts where the barely "keep going" aspect of the system and the simple description of production processes, possibly in the form of simple tabulations of numerical magnitudes, in the "1 kg of bread + 1/2 kg of meat" style, are prominent.

Looking for manuscripts with these characteristics, we are brought once again to document D3/12/2: 32, already considered in Section 3 above. Even though no reduction of inputs to an absolutely necessary commodity may be seen in that manuscript, the tabulation it contains, clearly describing production processes as lists of quantities of commodities, may be interpreted as groundwork laid down in view of an attempt to calculate such a reduction within the boundaries of a subsistence economy. The fact that nothing actually amounting to that reduction may be found in the manuscript does not necessarily weigh against this interpretation. The physical real costs description of a no-surplus economy's production processes would have been a necessary step towards it; but, once Sraffa had written down that description, he may have immediately seen it as a simple system of equations, whose straightforward algebraic solution would have determined exchange ratios: physical real costs provided a basis to exchange ratios determination with no need to reduce inputs to an absolutely necessary commodity. We may then expect that a reduction to an absolutely necessary commodity immediately fell outside of Sraffa's analytical horizon. At the same time, the question of how a positive surplus case could have been dealt with by means of the same equations would have emerged. Indeed, while in the Sraffa Papers we find no document that may be straightforwardly associated with that reduction, an attempt to answer the question concerning an economy producing a positive surplus may be immediately recognized, as we have already seen, in manuscripts D3/12/2: 33 and D3/12/2: 35.

5. The folder 'Physical Real Costs'

Having put forward an explanation of the origins of Sraffa's equations which emphasizes the role played by physical real costs and by their description as sets of commodities, by the idea of reducing production processes to an absolutely necessary commodity and by the identification of the case of a subsistence economy as of special importance, it may be appropriate to complete our discussion by gathering some details on how Sraffa came to develop the crucial elements of this structure. To this end we may consider manuscripts kept in a folder headed 'Physical Real Costs' (see D3/12/42: 32-56 and also D3/12/2: 24-25). Most likely, the items it contains were written (all but one, as we shall see) before the relevant sections of the pre-lectures, and, being mainly focussed on the

cussed in Section 3 above, are kept in a folder headed 'Winter 1927-28' and most likely date to that period.

distinction between cost and income,[27] they may be seen as part of an attempt aimed at providing a rigorous foundation to a theory of cost. As we know that in the pre-lectures the existence of substitutes is said to invalidate any attempt to construct a general physical real costs theory and to reduce production inputs to an absolutely necessary commodity, we may also assume that the manuscripts in the 'Physical Real Costs' folder were not seen by Sraffa as providing a basis for a successful elaborations. Yet, they may give us some indications on the closest background of Sraffa's analysis.

Some of those manuscripts show that Sraffa's idea of founding cost on physical inputs and on a reduction of production processes to quantities of an absolutely necessary commodity was crucially influenced by his critical approach to Marshall and marginalism in general (physical real costs are an obvious counterpart to disutility and to Marshall's real cost) and by his attention to Classical political economy in general, and to Ricardo in particular, as an alternative to marginalism itself. In one of them we read that the '[Physical Real Cost] theory coincides with labour theory of cost' (D3/12/42: 56).[28] This conclusion is reached considering that different inputs cannot be reduced to a common unit and directing attention towards reduction of production processes to quantities of labour employed in production and to the 'quantity of goods necessary to support a labourer for one day'.[29]

Other manuscripts kept in the same folder allow us to illuminate an influence which may have contributed to lead Sraffa to introduce into his analysis the entwined ideas of describing economic systems as circular processes and the notion of a subsistence community. The relevant points can be found in manuscripts where Sraffa's inquiry into the distinction between cost and income is pursued by referring to Hobson's 1910 description of costs as 'economically necessary to support and keep in operation the existing structure of industry' (D3/12/42: 52; see also D3/12/42: 53) and by summarizing Jannaccone's 1901 analysis of cost as pivoting on a level of production which is maintained constant 'as a continuous flow' and where 'restoration [...] of productive energy [is] opposed to remuneration' (D3/12/42: 54 recto).[30] These passages mainly relate to the activity of individual firms or industries producing typical surplus income (interest, profits, rents and above-subsistence wages), but we may nevertheless presume that they had some importance as part of the background against which Sraffa came to think of a "community" which barely "keeps going" – i.e. which constantly reproduces itself and

[27] In Sraffa's words: distinguishing 'in the total money cost of a thing what is real cost and what is surplus' (D3/12/42: 35), 'physical costs' from 'moral incentives' (D3/12/42: 39), 'bare necessaries of life' from 'necessaries for efficiency' (D3/12/42: 48), 'restoration' from 'remuneration' (D3/12/42: 54).

[28] We may note that this conclusion is very close to the sentences on Ricardo's theory of value (D3/12/3: 47) at the end of the excerpt from the pre-lectures quoted in Section 4 above.

[29] Sraffa wrote this phrase between inverted commas, but he added no reference to identify it as a quotation. He could have used inverted commas, as he did also on other occasions (see for instance D3/12/42: 33), as a way to stress a concept.

[30] See Hobson (1910 pp. vii, xii, 109) and Jannaccone (1901 pp. 333-9) (the same passages are also referred to in item D3/12/2: 24).

whose net product is nil. Indeed, a somewhat similar idea had been explicitly proposed by Jannaccone with regard to the analysis of cost at the individual firm level: in the very pages referred to by Sraffa, Jannaccone outlined a hypothetical firm, called 'intrapresa-limite', whose distinctive feature was that it reproduced the conditions of its own economic existence and that it produced no surplus.[31]

Quite interestingly, the only manuscript in this folder which, in our view, was written by Sraffa after having already drafted his earliest equations (D3/12/42: 33) shows that he had come to attach crucial importance to the ideas of a circular process of production and of a subsistence economy, and that he did not associate them to Classical economists and Marx. On the contrary, he used those concepts in an attempt to highlight the difference between his own views and that approach. However, the manuscript, which opens suggesting that treatment of natural resources which can neither be reproduced nor substituted may mark 'the difference between «Physical real costs» and the Ricardo-Marxian theory of «labour costs»',[32] ends by reversing that proposition: those natural resources

> cannot find a place in a theory of <u>continuous</u> production and consumption: they are dynamical facts, i.e. a stock that is being gradually exhausted and cannot be renewed, and must ultimately lead to destruction of the society. But this case does not satisfy our conditions of a society that just manages to keep continuously alive (D3/12/42: 33 verso; emphasis as in the original).[33]

This conclusion implies that by the time he wrote that note Sraffa had already developed the analysis of a circular process and of a subsistence economy – his "first equations" – and that in no way he saw the latter as a case of limited validity. But the emphasis on 'our conditions' may also be taken to imply that Sraffa felt that the results he had reached should not be read as a close development of the work of earlier authors. Indeed, he was the first to give to the analysis of a subsistence economy the importance of a founding stone of price determination. Before 1927 no other author had even approached the question of exchange ratios determination in such a context as a case which deserved to be examined on its own. To stress this originality, we may mention two schemes which, even though not encompassing the peculiar feature at the root of Sraffa's own approach, show interesting similarities with his analysis.

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[31] Jannaccone's *Il costo di produzione* reads: 'nell'intrapresa-limite, non essendovi prodotto netto, non v'è rendita, né profitto, né interesse, né salario, ma solo quote di reintegrazione della produttività' ('in the limit-firm, there being no net product, there is no rent, no profit, no interest, no wages, except for amounts set to reintegrate productive powers' – Jannaccone 1901, p. 338).

[32] This sentence allows us to see that Sraffa had come to consider the conception of physical real costs as a fully fledged theory of costs, which could be compared with those developed by Ricardo and Marx. On the contrary, in the lecture notes he later prepared for the course on Advanced Theory of Value he taught for three years from October 1928 we find several references to that theory, but in that case it is associated to William Petty, to the Physiocrats and to Classical economists in general (see SP D2/4).

[33] We may note the analogy between the structure of this argument and the way Sraffa had argued against Marshall's theory of costs in his 1925 and 1926 articles.

Isnard, in his *Traité,* discussed the relationships between production, prices and distribution employing a system of simultaneous equations and a description of production processes based on physical inputs and outputs strikingly close to Sraffa's positive surplus schemes. But he did not use his equations to determine relative prices. His focus on a critique of Quesnay's thesis of exclusive productivity of agriculture led him to assume different sets of exchange ratios and study how they affected the distribution of the value of the net product among sectors (Isnard 1781 pp. 35-7).[34] Few pages before, he had determined exchange ratios by solving a similar system (Isnard 1781 pp. 18-21), but that he had done for a case of pure exchange. A no surplus production system may be placed behind the latter construction to generate the quantities of the goods exchanged, but there is no reason to presume that it should be a subsistence system (just like Marx's simple reproduction schemes, it could include a surplus product consumed by a proprietary class). Similarly, we may find notable analogies between Sraffa's earliest tabulations and a system which appeared in Leontief's December 1927 doctoral dissertation (*Die Wirtschaft als Kreislauf* – see Leontief 1928, 1991a, 1991b). That system, subservient to discuss how changes in various parameters would divert the economy from following a stable path, clearly shows a physical input-output description of an economy constantly reproducing itself, which, at first glance, may appear perfectly identical to the schemes in Chapter I of *Production of Commodities*. But Leontief did not qualify his system as a subsistence economy. Indeed, his interest in the concept of circular flow in no way implied that a surplus beyond subsistence could not be part of the system. No hint suggests that either Isnard's book or Leontief's research were known to Sraffa.

6. Conclusions

As generally acknowledged, the broad background of the developments which led Sraffa to draft his equations may certainly be identified in his study of contemporary economic theory, in his critical stance against subjective foundations of economics, in his interest in Classical political economy and in Marx's contributions, in his study of the epistemological bases of natural sciences, and, in particular, in his search for an approach to the theory of value and prices based on objective magnitudes. But the focus of this paper has not been directed to delve into such a broad background; we have tried to identify the earliest drafts of Sraffa's equations and to single out the closest spring which led Sraffa to write them. While the former have been recognized in manuscripts D3/12/2: 32-35, the latter has been found in the pages of the summer 1927 pre-lectures headed 'Physical Real Costs', and, in particular, in the note appended on the left hand margin of sheet D3/12/3: 44, which pointed to the case of 'a community that produces

[34] For a reprint and translation of Isnard's *Traité,* see van den Berg (2006).

just what is sufficient to keep it going'[35] as of special importance to the reduction of a commodity's production process to an absolutely necessary commodity. As we have argued, there is no contradiction between the role we attribute to Sraffa's interest in that reduction and the absence in the Sraffa Papers of any calculation actually pursuing it. We may presume that that aim immediately lost importance when Sraffa's first step towards it – the tabulation of the production processes of a subsistence economy – led him to realize that exchange ratios could be determined by solving what his tabulation turned out to be: a system of simultaneous equations entirely based on objective data (quantities of physical inputs and outputs). At the same time his attention to a subsistence economy had also brought to the centre of the stage the description of the economic system as a circular process and the distinction between no-surplus and positive surplus cases. These two elements had played no significant role in Sraffa's earlier manuscripts, while later developments of Sraffa's analysis came to hinge upon them.

The striking resemblance between the descriptions of 'a community that produces just what is sufficient to keep it going' and of the 'extremely simple society which produces just enough to maintain itself' that we find in the opening Chapter of *Production of Commodities* (Sraffa 1960 p. 3) is also worth noting. In Sraffa's book, that chapter certainly emphasizes how material conditions of production lay at the bottom of the whole analysis of prices and how the existence of a physical net product is a necessary condition to the existence of any distributive variable, but the note in document D3/12/3: 44 allows us to see that that chapter is not only an analytical and didactical device: it also directs the reader to adopt the author's original starting point. Furthermore the originality of Sraffa's research path may be also appreciated by considering that we know of no other author who, before 1927, approached the question of exchange ratio determination in a subsistence economy. Actually no one paid much attention to such a case, and certainly no one gave to the analysis of a subsistence economy the importance of a founding stone of the analysis of price determination.

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