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Centro Sraffa Working Papers

n. 11

April 2015

ISSN: 2284 -2845

Centro Sraffa working papers

[online]

Garegnani on a way to avoid the value capital endowment in Wicksell (1898)

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Abstract

Pierangelo Garegnani has argued that the value capital endowment could have been avoided in Knut Wicksell's long-period general equilibrium in *Value, Capital and Rent* (1898); the capital endowment might have been specified as an amount of labour embodied in the economy's stock of capital goods, which would have avoided the vicious circle of a factor endowment dependent on the prices the equilibrium must determine. The paper argues that this thesis cannot be accepted.

JEL classification: B13, D50

§1. This short note discusses the late Pierangelo Garegnani's thesis that the value capital endowment could have been avoided in Knut Wicksell's *Über Wert, Kapital und Rente* (1898). Its purpose is to clarify the reason – succinctly expressed in Petri (2004, pp. 112-13, fn. 40) – why I, a great admirer of Garegnani's contributions and in full agreement with his views on most issues, found it impossible to agree with him on this point.

In the 1898 book Wicksell avoids the mistake one finds in J. B. Clark and even sometimes in Marshall, of formulating production functions with value capital as one of the inputs; the single consumption good (the net product) is produced by a production function with labour and the average period of production as independent variables; but capital is still conceived as a single factor of variable 'form', and the economy's given total capital endowment is a quantity of exchange value (in terms of the consumption good). In *Il capitale nelle teorie della distribuzione* (1960) Garegnani, after presenting

the system of equations that determine the long-period equilibrium in Wicksell's book, points out

how the measurement in terms of value of the existing capital is not *essential* to the theory presented above. In fact as long as one deems it possible to use the average period of production in the production function, the same measurement can also be adopted to express the economy's available capital stock.¹

So, according to Garegnani, the *inevitable* deficiencies of the analysis of the early Wicksell are rather the need for simple interest, absence of durable capital, and absence of scarce natural resources: the three assumptions needed for the use of the average period of production. When the admission of the excessively restrictive nature of these assumptions brings Wicksell in the *Lectures* to abandon the average period of production, the measurement of capital in value becomes indispensable and "with it Wicksell has fallen back into the same circular reasoning avoided for the production function"². Were the three assumptions needed for the use of the average period legitimate, then according to Garegnani a circular reasoning in the measurement of the capital endowment would be avoidable.

The proposed modification capable of avoiding the value capital endowment is the following. Let A be the given labour supply, L labour employment, T the average period of production, w the real wage in terms of the consumption good, K the economy's capital endowment measured as an amount of exchange value (in terms of the consumption good). It seems unnecessary to reproduce here all four equations of the basic model of *Value, Capital and Rent*. (Cf. Garegnani, 1990, pp. 26-28, or 1960, pp. 125-7, for succinct presentations.) It will suffice to indicate the modification, suggested by Garegnani, of the fourth equation, that imposes the equality between supply and demand for capital. In this equation Wicksell assumes a given endowment of value capital K and writes the equality between supply and demand for capital as³

$$K = AwT.$$

¹ "...come la misurazione in termini di valore del capitale esistente non sia *essenziale* alla teoria esposta sopra. Infatti finché si ritenga possibile il periodo medio di produzione nella funzione della produzione, la stessa misurazione può essere adottata anche per esprimere il capitale disponibile nell'economia." (Garegnani 1960, p. 127, italics in the original). In this paper all translations from Italian are mine. I thank Ian Steedman and Saverio Fratini for useful comments.

² "...per essa Wicksell è ricaduto nello stesso ragionamento circolare evitato per la funzione della produzione" (1960, pp. 147-8).

³ This is because with simple interest, once T is given, the optimal value of capital *per unit of labour* is wT, a function only of T because w is a function of T derivable from the production function of the consumption good through the condition that the marginal-product payments to factors must exhaust the product. It follows that when labour supply A is fully employed the optimal value of capital desired by firms is AwT.

Garegnani argues that, since the quantity of net product per unit of labour is a function of T and only of T , the average period indicates the quantity of capital per unit of labour whose variations determine the variations of the net product per unit of labour; but T “is also the number of labour years embodied in the capital goods that on average assist one labourer engaged in the direct and indirect production of the good we are discussing”⁴. Therefore AT is the labour embodied in the vector of capital goods present in the economy at full employment when the production methods are the optimal ones associated with an average period T . Garegnani argues that one can take this amount of labour embodied – indicated as K' – as the measure of the given capital endowment: “The labour embodied in the aggregate of capital goods needed to allow the use of the technique defined by T must therefore be $K' = AT$.”⁵ If one replaces Wicksell’s equation $K=AwT$ with this equation, where K' is given, the number of equations and of unknowns is not altered and thus the four unknowns net product, real wage, T , and rate of interest

can be determined without recourse to a given K measured as a sum of values. The given capital available in the economy would be measured in terms of the labour embodied in the capital goods, a quantity which, differently from value, is independent of variations of distribution ... The existing capital, measured in terms of average period, or of labour embodied, satisfies instead the requirements illustrated in chapter I: it is a magnitude definable in terms independent of the system of relative values and, when one uses simple interest, it has a known relationship with the value of capital.⁶

This thesis is repeated by Garegnani in “*Quantity of capital*” (1990, p. 28):

if social capital could in fact be measured by the average period of production T , then the value measurement of the quantities demanded and supplied of capital in equation (III:10) would not be essential. That measurement could be replaced by the measurement of capital used in the production function, where

⁴ “è anche il numero di anni di lavoro incorporato nei beni-capitali che assistono in media un lavoratore impegnato nella produzione diretta e indiretta del bene in questione” (1960 p. 127). For a simple proof cf. e.g. Petri (2004, pp. 108-9). As the observations in footnote 10 below make clear, to describe these capital goods as ‘assisting’ labour may be misleading. To speak of capital goods *present* in a subsystem employing one unit of labour is more precise.

⁵ “Il lavoro incorporato nell’aggregato dei beni capitali necessari per permettere l’uso della tecnica definita da T dovrà perciò essere $K' = AT$.” (1960, pp. 127-8)

⁶ “potranno essere determinate senza ricorrere a un dato K misurato quale somma di valori. Il capitale dato disponibile nell’economia sarebbe misurato in termini del lavoro incorporato nei beni-capitali, quantità questa che è, a differenza del valore, indipendente da variazioni nella distribuzione. ... Il capitale esistente, misurato in termini di periodo medio, o lavoro incorporato, soddisfa invece i requisiti visti nel capitolo I: esso è una grandezza definibile in termini indipendenti dal sistema dei valori relativi e sta, quando si usi il saggio semplice dell’interesse, in un rapporto noto col valore del capitale” (1960, pp. 128-130).

the ratio of capital to labour is expressed by T : the number of labour years embodied in the capital stock assisting, on the average, one worker in the economy. If we then take as given the capital K' existing in the economy, measured in terms of the labour years which have been necessary for its production, we shall have the following equation:

$$K' = AT.$$

... The quantity of capital appearing there would be independent of distribution.
... This discussion has shown how the average period of production could be used as a consistent measure of capital in marginal theory.

§2. What Garegnani seems to be suggesting in these passages is this: if labour employment L is given and equal to A , the capital endowment of the economy is such as to allow employing an average capital-labour ratio not greater than a certain maximum T^* , which is given once the capital endowment is given. If the capital-labour ratio is measured by the average period of production T , then the condition of full employment of capital simultaneously with the full employment of labour can be written $AT^*=AT$. Garegnani's given K' is AT^* .

But what is the capital endowment made of, that prevents an average period of production greater than T^* ? Capital is not *made of* a period of production, the maximum period of production must *result* from something that poses a constraint to the possibility of lengthening it further. In Böhm-Bawerk the constraint is the subsistence fund; the idea is that the maximum period of production results from the availability of anticipated wages relative to the future output of the consumption good, that is, it results from the cumulated abstinence that permitted allocating wages to, on average, more and more indirect production.

My perplexities concerning Garegnani's proposal arise from the fact that Garegnani does not explain how the labour embodied in the capital goods of an economy can be considered given *before* determining the equilibrium vector of capital goods, and can therefore act as a constraint. He speaks of this quantity of labour embodied as "a magnitude definable in terms independent of the system of relative values"; but it is not enough that it be so *definable*, it must also be possible to consider it as *given*, that is, as unchanging during the tendency toward equilibrium, in spite of the changes that quantities produced and productive methods are undergoing⁷.

Some admission of problems in this respect does appear in the 1960 book, in a footnote (attached to the passage quoted here in fn. 5) that deserves quotation in full. Footnote 17, p. 130, reads as follows:

⁷ The "requirements illustrated in chapter I" that Garegnani mentions in (1960, p. 130) are those Garegnani traces as necessary to determine the rate of profits without circular reasoning *in the classical approach*, and therefore on the basis of given produced quantities, given productive methods, and given real wage; thus only relative values and rate of profits remain to be determined. The marginal approach on the contrary must determine all these magnitudes simultaneously.

It is not possible to discuss the further problem of the limits within which it would be possible, in the face of variations in the techniques adopted for production, to postulate the invariance (or the magnitude of the variation) of the capital available in the economy, measured as a quantity of labour embodied, as required by marginal productivity theories. This is, because we have no experience of an economy where the three hypotheses necessary for the use of the average period of production hold true.⁸

This footnote raises three perplexities. First, it is unclear why the problem is seen as a *further* problem, not worth discussing in the main text, and that can be left without a clear answer: clearly Wicksell would not have considered his approach defensible if only applicable to economies without choice of techniques, so it is strange that a different specification of the capital endowment be proposed as an *improvement* upon Wicksell, when it is admitted that the proposal might be indefensible, or defensible only under conditions of insufficient generality, when choice of techniques is allowed.

Second, one wonders why in 1990 no similar need for caution is mentioned and Garegnani speaks assuredly of ‘consistent measure’.

Third, the reason adduced in this footnote, why it should be *impossible* to discuss the legitimacy of the given quantity of capital so measured when techniques are variable, is unconvincing. If, in order to understand the logic of Wicksell’s approach, it is possible to analyze the determination of the interest rate and of the average period of production (admitting, therefore, variability of techniques) in an economy with simple interest, no fixed capital, and no scarce natural resources (therefore definitely an economy of which “we have no experience”), why should it be impossible to study in the same economy the legitimacy of a given quantity of labour embodied in the capital goods?

The considerations below suggest that in fact it is not impossible.

§3. Let us examine whether one can consider the labour embodied in the capital goods as given before one has determined the production methods adopted, in a case where there is no problem with considering the capital stock as given in physical terms. Assume the simple neoclassical economy that produces corn with labour and corn-capital as inputs in yearly production cycles according to a differentiable production function with constant returns to scale $G=F(K,L)$, where G is gross output, L labour

⁸ “Non è possibile discutere l’ulteriore problema dei limiti entro cui sarebbe possibile, di fronte a variazioni nelle tecniche adottate per la produzione, postulare la costanza (o grandezza della variazione) del capitale disponibile nell’economia, misurato come quantità di lavoro incorporato, così come si richiede per le teorie della produttività marginale. Ciò perché non si ha alcuna esperienza di un’economia dove le tre ipotesi necessarie per l’uso del periodo medio di produzione siano verificate.” (1960 p. 130 nota 17).

employment (paid at the end of the production period), K corn-capital. Suppose the corn-capital stock to be given and fully employed, and that income distribution changes: the production method will change, and labour employment with it. Will the labour embodied in the given capital stock remain constant? In general no, as I proceed to show.

In this economy each production method is characterized by two technical coefficients: k , corn-capital per unit of gross output, and λ , direct labour per unit of gross output. For each production method (k, λ) , the labour h embodied in 1 unit of corn is determined by $h = hk + \lambda$, that is, $h = \lambda + \lambda k + \lambda k^2 + \dots = \lambda / (1 - k)$.

The average period of production is the labour embodied in the capital goods present in a subsystem that employs 1 unit of direct labour, and therefore that produces $x = 1/\lambda$ units of gross output and uses k/λ units of corn-capital; the labour embodied in k/λ units of corn-capital is $T = hk/\lambda = k/(1 - k)$. It coincides numerically with the physical capital per unit of *net* product: if $x - kx = 1$, then $x = 1/(1 - k)$ so the corn-capital employed to produce it is $k/(1 - k)$.⁹

Garegnani's given capital endowment amounts to taking LT as given. An example will show that in this economy $LT = Lk/(1 - k)$ can vary with the production method adopted when physically the stock of corn capital does not change.

Suppose the gross-output production function is Cobb-Douglas with labour exponent α , that is, $G = L^\alpha K^{1-\alpha}$. Suppose $K = 1$, so $G = L^\alpha$, which means that the production method changes if L changes. Then $k = K/G = 1/G = 1/L^\alpha$. (Note that viability, in the sense of capacity to produce a positive net product, that is, $k < 1$, is not guaranteed if factor proportions are given: if labour employment is sufficiently small, gross output is less than the corn-capital employed; then $k > 1$ and labour embodied is negative; therefore for labour embodied to make sense we must assume levels of labour employment such that $k < 1$, that is, $L > 1$.) Then LT varies with L , because

$$LT = Lk/(1 - k) = L \frac{\frac{1}{L^\alpha}}{1 - \frac{1}{L^\alpha}} = \frac{L}{L^\alpha - 1}.$$

For example if $\alpha = 1/2$, then $L = 4$ implies $LT = 4$, while $L = 9$ implies $LT = 4.5$.

More generally, an increased labour employment with an unchanged corn-capital increases production, so k decreases, hence $T = k/(1 - k)$ decreases, but there is no reason why the increase in L and the decrease in T should leave LT unchanged; the way LT changes will depend on the production function.

⁹ In this economy the long-period price of corn, which with compound interest satisfies $p = 1 = w\lambda + (1 + r)k$, with simple interest must satisfy $p = 1 = wh + rwhT = wh(1 + rT) = w\lambda/(1 - k) + rw\lambda k/(1 - k)^2 = [w\lambda/(1 - k)] \cdot [1 + rk/(1 - k)]$. So the relationship between r and w is different from the correct one. For example, let $\lambda = 1$, $k = 1/4$, $r = 1$; with compound interest this implies $w = 1/2$; with simple interest it implies $(4w/3) \cdot 4/3 = 1$, that is, $w = 9/16$.

To see the relevance of this result, let us consider an economy where there is a given stock of corn-capital and there is full labour employment and there results a certain $T=k/(1-k)$. Now immigration raises labour supply from A to A' . Would a neoclassical economist have the right to determine the new equilibrium taking K' as given at its initial value AT ? The usual comparative statics assumes no net savings; in this economy the meaning of no net savings is clear: the corn-capital endowment does not change; then, as shown, K' changes as the decrease in the real wage induces firms to adopt a lower capital-labour ratio and $A'T'$ will be generally different from the initial $K'=AT$. Differently – in this example – from the case with the capital endowment specified as an amount of value, the capital endowment specified *à la* Garegnani does not stay unchanged as the economy tends to the new equilibrium, so it cannot be taken as given.

The example also shows that there is no reason why the labour embodied in capital goods should be a *constraint* on the possibility to vary production methods; it shows that it is the physical endowment of corn-capital that acts as a constraint, not the labour it embodies, which is *determined by* the production methods.

Here a point emerges, that will acquire greater evidence as one considers more complex economies (§5): it seems impossible to find reasons why agents may want to keep the labour embodied in the capital goods unchanged. Neither the saver nor the entrepreneur *cares* about this magnitude, which (unlike net savings, or methods of production) is not directly observable nor directly affecting income or profits, and therefore is not an object of choice.

§4. But problems arise even in the absence of choice of techniques. Let us consider a different economy, the simplest case of wage fund theory: the economy produces corn with unassisted labour as input, production takes one period, and wages are paid at the beginning of the period, so advanced capital consists of advanced wages. If t indicates the *date* (the day or moment) at the end of year t , then output of date t requires payment of wages at date $t-1$. Suppose only one production method is known, that requires λ units of labour to produce one unit of corn. Clearly $T=1$, and the labour embodied in the capital goods utilized in the economy is measured by labour employment L ¹⁰. The wage fund is given, so an increased labour employment requires a decrease in the real wage w by the same percentage. If K measures the given wage fund, the demand for labour is

¹⁰ That $T=1$ is clear, because it is the moment when wages are paid that determines labour advancement, so all labour is advanced one period; but the other interpretation of T , as labour embodied in the capital goods present in the subsystem that employs 1 unit of labour, might seem difficult to sustain in this case, since it may seem that there is no capital good *assisting* labour. But the average-period approach assumes labour is paid the moment its output comes out. That is, for this economy it is implicitly assumed that it is as if labour were employed from date $t-2$ to date $t-1$, at that date wages are paid, and a product comes out which needs one period of ripening (unassisted by labour) before it can be sold at date t . The product coming out at date $t-1$ and then ripening is an intermediate product, it is the capital goods *present* in the subsystem and embodying the amount of labour indicated by T .

K/w ; if labour supply is A then equilibrium requires $K=Aw$, which, remembering that in this economy $T=1$, coincides with Wicksell's equilibrium condition $K=AwT$. So in this economy Wicksell's given capital endowment is the given wage fund, and to take it as given before the equilibrium is determined is *logically* legitimate (although criticisable on other grounds as even J. S. Mill eventually admitted). On the contrary the capital stock measured *à la* Garegnani as the labour embodied in the capital goods, $K'=L$, is not given before equilibrium labour employment is determined, because it coincides with it. And this, in spite of the absence of choice of technique: in this economy the potential problem, admitted by Garegnani in the 1960 footnote quoted in §2, does not arise, but the impossibility to take as given the capital stock measured *à la* Garegnani seems clear.

§5. With heterogeneous capital, and technical choice or several consumption goods demanded in variable proportions, there arises the additional issue of the need for an endogenous determination of the 'form' of capital; that is, since the equilibrium proportions among capital goods will generally be different from the ones in the economy whose equilibrium one wants to determine, the need arises to indicate a choice process of economic agents, at least to some extent plausible, that aims at changing the 'form' of capital when it is not the most convenient one, while leaving its quantity unchanged (in conformity with the traditional neglect of net savings during the adjustment process). Differently from the case of a value capital endowment, it is not easy to find a choice process that might aim at adapting the 'form' of capital to a changed income distribution or composition of demand while maintaining the labour embodied in the capital goods unchanged. That individuals may choose to change the 'form' of their capital without altering its *value* (by utilizing depreciation funds to buy capital goods different from the used-up ones) is perfectly conceivable (although such choices are unable to legitimize a given aggregate value endowment of capital, and are destined to fail if relative prices are changing); on the contrary not only is it unclear why a capital owner should *care* about maintaining the labour embodied in her capital goods unchanged; it is also unclear how this individual could possibly contemplate such a decision, since the labour embodied in her capital goods is a quantity she does not know and that anyway depends on technical choices taken by other people.

§6. I conclude that replacing in Wicksell (1898) the given value capital endowment with a given quantity of labour embodied in the capital goods would make the theory not more but less consistent, because the quantity of capital so measured would not be independent of the variables the equilibrium must determine even in the very special cases in which such an independence would hold for a value specification of the capital stock, and in addition it seems impossible to find reasons why economic decisions should aim at keeping that quantity unchanged in the face of changes of its 'form'. Therefore the measurement of the capital endowment as a quantity of exchange value remains inevitable even in the versions of the marginal approach based on the average period; the logical circularity implicit in such a measurement cannot be avoided.

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