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The Dissolution of Two Stability Problems of General Equilibrium

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1. The problem of intertemporal equilibrium (Garegnani 2000,2003, 2009, Petri 2004, Parrinello 2005-2009-2010, Tosato 2006, Schefold 2008)

2. The numeraire problem (raised by many economists and resumed in Parrinello 2010)

CLAIMS

The problem of intertemporal equilibrium and the numeraire problem can be clarified within a pure **exchange economy**, without the intrusion of the complexities of the theory of capital (reswitching of techniques and reverse capital deepening).

The problem of intertemporal equilibrium is at the same time more confined and more general than some economists (Garegnani and others, included myself to some extent) have argued.
It is confined to the stability and does not impinge upon the existence and uniqueness of an equilibrium.
It is not confined to a production economy with productive capital, although it has been suggested by the capital controversies of the SIXTIES and formalized by the Hahn-Garegnani model.

The problem of intertemporal equilibrium dissolves jointly with the numeraire problem.

A conjecture. The existence of a threat for the stability of an equilibrium remains whenever a market for an aggregate value intervenes and is combined with the choice of a dated standard of value for each period . This feature seems to be especially relevant for a monetary production economy

AN EXCHANGE ECONOMY WITH 4 GOODS

Given endowments $X_1^0, X_2^0, X_1^I, X_2^I$ Nominal prices $P = \left(P_1^0, P_2^0, P_1^I, P_2^I\right)$

Market clearing equilibrium in terms of demand *D* and excess demand *Z*

$$\left.\begin{array}{l} Z_1^0 \equiv D_1^0(P) - X_1^0 = 0 \\ Z_2^0 \equiv D_2^0(P) - X_2^0 = 0 \\ Z_1^I \equiv D_1^I(P) - X_1^I = 0 \\ Z_2^I \equiv D_2^I(P) - X_2^I = 0 \end{array}\right)$$

Walras Law $P_1^0 Z_1^0(P) + P_2^0 Z_2^0(P) + P_1^I Z_1^I(P) + P_2^I Z_2^I(P) \equiv 0$

The standard ADJUSTMENTS of “non normalized” or “normalized” prices. Of course “normalized” prices are not “normal” prices

$$\left.\begin{array}{l} \text{Non normalized } \Delta P_1^0 \Leftarrow Z_1^0 \quad \text{or normalized } P_1^0 = 1 \rightarrow \Delta P_1^0 = 0 \\ \Delta P_2^0 \Leftarrow Z_2^0 \\ \Delta P_1^I \Leftarrow Z_1^I \\ \Delta P_2^I \Leftarrow Z_2^I \\ \text{where} \Leftarrow \text{means a causal relation from Z to } \Delta \text{P, that preserves the sign of Z.} \end{array}\right)$$

Assume that the **four goods** are available in **2 periods t = 0,1**.

Goods X_1^0, X_2^0 in period 0; goods X_1^I, X_2^I in period 1.

The goods are **NON STORABLE** and in each period they fall like **MANNA** from heaven with their own property rights. The form of the equilibrium model of the exchange economy does not change. The existence and uniqueness of an equilibrium do not depend on the fact that the goods are dated or undated

The problem of Intertemporal equilibrium: Can the previous mechanism of price adjustment be preserved in the analysis of the stability in the context of such intertemporal interpretation/transformation ?

- A preliminary criticism

- An argument in favour of the preservation

- An argument in favour of an alternative adjustment mechanism

A preliminary criticism of the non normalized standard mechanism:

The mechanism is not consistent with the rational behaviour expressed by the homogeneity of the excess demand functions.

The problem

$$\left.\begin{aligned} \Delta P_1^0 &\Leftarrow Z_1^0(2P) = Z_1^0(P) \\ \Delta P_2^0 &\Leftarrow Z_2^0(2P) = Z_2^0(P) \end{aligned}\right)$$

If we start from given nominal prices *P* and we double all *P*,

the excess demands (positively homogeneous) of degree 0 in P), do not change,

$\Delta P_1^0, \Delta P_2^0$ does not change, but P_1^0 / P_2^0 may change.

Correction. (Parrinello 2010). The change in price can be assumed to depend on the value of the physical excess demand; in other words let assume that the physical excess demand affect a proportional change (a pure number) in price.

$$\left.\begin{aligned} \Delta P_1^0 &\Leftarrow P_1^0 Z_1^0 \\ \Delta P_2^0 &\Leftarrow P_2^0 Z_2^0 \\ \Delta P_1^I &\Leftarrow P_1^I Z_1^I \\ \Delta P_2^I &\Leftarrow P_2^I Z_2^I \end{aligned}\right)$$

Even after this correction , the economic meaning of the non normalized adjustment remains questionable and it will not be pursued here.

Argument in favour of the standard normalized adjustment mechanism

In the exchange model with all commodities available in the same period, abstract "MARKETS" translate a signal of an intended exchange between 2 goods (barter) into distinct signals of demand and supply. Each signal must be *COVERED, MATCHED, BACKED, VALID*ATED by available income. Otherwise the signal is a bluff and remains ineffective.

In the exchange model with dated goods, the adjustment mechanism requires that each demand or supply be **covered by the TOTAL WEALTH**, the value calculated by adding dated incomes measured in terms of an arbitrary dated commodity chosen as a numraire ("discounted" values) .

Then the standard mechanism of price adjustment can be preserved without any special problem except for the numeraire problem to be discussed later

Argument in favour of an alternative adjustment mechanism

The argument is based on Garegnani 2000,2003, 2009 , his unpublished paper, and my own writings.

I will not stress the role of capital goods as perfect substitutes for savings , but what seems to be a correct interpretation of the market signals of intended exchanges of goods against goods (instead of money).

In disequilibrium the market splits a signal of an intended intertemporal exchange of physical goods into

- A demand for a good available for example in t = 0 covered by income available in t = 0
- A supply of another good in t =1 covered by income available in t = 1

Even in the presence of a one-shot negotiation and absence of renegotiation, not only the aggregate wealth, but **a dated income must guarantee the availability and delivery of dated goods in each period** . An income constraint must be fulfilled in each period in order to guarantee that the promises of delivery will be carried out at the due time .

The excess demand for income available in period 1, that is covered by income in period 0, corresponds to the market of aggregate savings with its own rate of interest.

A n alternative MODEL OF ADJUSTMENT

A Walras Law holds in each period (Garegnani).
In our exchange model with t = 0, 1

$$P_1^0 Z_1^0(P) + P_2^0 Z_2^0(P) + S \equiv 0$$

$$P_1^I Z_1^I(P) + P_2^I Z_2^I(P) - S \equiv 0$$

S is the total amount of income "moved " between the two periods.
S can be called intended SAVINGS.
S **is null in equilibrium because all goods are not storable. But the intended savings S can be positive or negative in disequilibrium**

The Walras law in terms of total wealth follows from the two dated Walras laws.

$$\text{REPLACE} \quad \left. \begin{array}{l} \Delta \dfrac{P_2^0}{P_1^0} \Leftarrow Z_2^0 \\ \Delta \dfrac{P_1^I}{P_1^0} \Leftarrow Z_2^I \\ \Delta \dfrac{P_2^1}{P_1^0} \Leftarrow Z_2^I \end{array} \right)$$

$$\text{WITH} \quad \left. \begin{array}{l} \Delta \dfrac{P_2^0}{P_1^0} \Leftarrow Z_2^0 \\ \Delta \dfrac{P_2^I}{P_1^{'}} \Leftarrow Z_2^I \end{array} \right)$$

$$\text{and} \quad \left. \Delta \dfrac{P_1^I}{P_1^0} \Leftarrow \dfrac{S}{P_1^o} \right)$$

$$\frac{P_1^0}{P_1^I} \equiv 1 + r$$

r is the own rate of interest on the numeraire and adjusts itself in response of *real* S

Now not all relative prices can adjust each independently from the others in response of their respective physical excess demand: this occurs for the relative prices of goods available in different periods.

Suppose that S is positive. This means an excess supply of income available in period t = 0 and not spent for goods available in the same period (a demand for coupons with promises of income in period 1) and also means an excess demand for income in the next period t = 1 to be spent for goods in that period.
From the equation

$$\Delta \frac{P_1^I}{P_1^0} \Leftarrow \frac{S}{P_1^o}$$

a positive S "causes" an increase in the relative price of income available in t = 1 over the price of income available in t = 0 , where the numeraire is good 1.

Combined with the definition

$$\frac{P_1^0}{P_1^I} \equiv 1 + r$$

It means that a positive S causes a decrease in the rate of interest on the numeraire or an increase in the price of a Walrasian "coupon"

- It can be proved (Parrinello 2010) that the received stability conditions (Hicks 1939, Arrow-Block-Hurwicz 1959 et al.) cannot be extended to the alternative adjustment mechanism. In fact the static stability and the dynamic (tàtonnement) local stability of an intertemporal equilibrium in general become dependent not only on the slopes (partial derivatives) of the excess demand functions at the equilibrium point, but also on the very equilibrium prices.

The numeraire problem:
Does the stability depend on the choice of the numeraire?

Arrow (1980, p. 775): "Suppose, for example, that at some point in time, there is disequilibrium. In particular, **suppose that both gold and silver have excess demand**, that is, demands exceed supplies for both. If gold is the numeraire, then its price is fixed. If the price of silver responds only to the difference between supply and demand on the silver market, then its price will rise. Hence the price of gold relative to that of silver falls. But, if silver were the numeraire, then exactly the same reasoning shows that the price of silver falls relatively to that of gold. Equivalently, the price of gold is now rising relative to that of silver.

This problem concerns both the standard normalized adjustment and the alternative adjustment mechanism. Is the theory of gravitation towards normal prices and quantities fully exempted from such problems, in particular the numraire problem ?

Arrow's argument points out a PSEUDO numeraire problem. Parrinello (Metroeconomica 2010) suggests that the adoption of a standard commodity as distinct from the choice of the numeraire can solve the "numeraire problem".

In the normalized adjustment, let us choose a composite commodity **which cannot be in excess demand**, instead of an arbitrary numeraire (e.g. gold or silver), and put the nominal value of this commodity at the denominator of each nominal price. This composite commodity is made of all 4 goods taken in the same proportions in which they enter the given endowments.

$$M \equiv P_1^0 X_1^0 + P_2^0 X_2^0 + P_1^I X_1^I + P_2^I X_2^I \text{ price deflator}$$

The aggregate value of the excess demand of such composite commodity is zero not only in equilibrium but also in disequilibrium in force of the Walras law.

$$M \equiv P_1^0 X_1^0 + P_2^0 X_2^0 + P_1^I X_1^I + P_2^I X_2^I \text{ price deflator}$$

$$\left.\begin{array}{l} \mathbf{\Delta} P_1^0 / M \Leftarrow Z_1^0 \\ \mathbf{\Delta} P_2^0 / M \Leftarrow Z_2^0 \\ \mathbf{\Delta} P_1^1 / M \Leftarrow Z_1^1 \\ \mathbf{\Delta} P_2^1 / M \Leftarrow Z_2^1 \end{array}\right)$$

The revised normalized adjustment

$$\text{Now } \Delta \frac{P_1^I}{P_1^0} \Leftarrow \frac{S}{P_1^o} \text{ must be dropped}$$

It would be cumbersome to assign any other special role to the market of aggregate savings *S*, although *S* is implicit and our (Garegnani) interpretation of S in terms of dated Walras Laws is not undermined

We might write $P_1^0 X_1^0 + P_2^0 X_2^0 + P_1^I X_1^I + P_2^I X_2^I = 1$ and say that the implicit composite standard commodity is the numeraire. Yet, we are also entitled to choose a different good as numeraire and fix its price = 1 , e.g. $P_1^0 = 1$ still maintaning $M \equiv P_1^0 X_1^0 + P_2^0 X_2^0 + P_1^I X_1^I + P_2^I X_2^I$ in the adjustment equations written above.

Arrow's puzzle disappears and the received stability theorems can be extended to the case of intertemporal equilibrium. The two problems are dissolved from this point of view.

With hindsight it is easy to say that such a (dis)solution of problems was already there in many received mathematical demonstrations of the existence, uniqueness and stability of a general equilibrium, where we find the following equation of price normalization

$$P_1^0 + P_2^0 + P_1^I + P_2^I = 1$$

This is a convenient way to constraint the prices within a simplex of a given dimension. Many non mathematical economists may wonder what is the economic meaning of this equation and whether there is some advantage for adopting this equation instead, for example, the simpler equation

$$P_1^0 = 1$$

I was inclined to mention to my students the need for not pre-committng the strict positivity of one particular price, which instead might be zero in equilibrium and this is an advantage in the choice of the former equation inastead of the latter. However another remarkable advantage should be stressed.

The equation $P_1^0 + P_2^0 + P_1^I + P_2^I = 1$ can be interpreted as a transformation of the equation $P_1^0 X_1^0 + P_2^0 X_2^0 + P_1^I X_1^I + P_2^I X_2^I = 1$ that is obtained by means of an appropriate choice of the units of measure of $X_1^0, X_2^0, X_1^I, X_2^I$

Therefore the solution to the intertemproal problem was already offered by the familiar equation $P_1^0 + P_2^0 + P_1^I + P_2^I = 1$ ……*with hindsight*

GENERAL PROBLEMS OF STABILITY: open questions

The problem of the intertemporal equilibrium amounts to ascertain whether there are theoretical reasons to adopt a unique standard of value (deflator M) or a different standard for each period.

No definite answer: a fully fledged micro-foundation of the adjustment process is missing and this is a serious shortcoming for the typical neoclassical methodology. Still the alternative mechanism characterized by multiple standards of value seems more “plausible” and capable of extension to a production monetary economy in real time

Should we (they) surrender and retreat to the conclusion that we (they) cannot prove or disprove the stability of a general equilibrium, but only the stability of many different disequilibrium processes which may or may not have in common the same equilibrium state?

A Conjecture

The existence of a threat for the stability of an equilibrium remains whenever a market for an aggregate value intervenes and is combined with the choice of a distinct standard of value for each period .

This feature seems to be relevant for a monetary economy where the standard in each period is a dated money and the market of an aggregate value is the market of real money (nominal money deflated by a price index of heterogeneous goods)

An anticipation of this issue, related to foreign currencies, can be found in Mundell R. (1968):"*Hicksian Stability, Currency Markets, and the Pure Theory of Economic Policy*".

Relative price-effects on aggregate values make not sufficient the stability conditions imposed on the preferences (utility functions) and the technology (production functions).

In particular the stability of an equilibrium may depend on the equilibrium prices

Path dependency is not at issue here

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