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# A Search for Distinctive Features of Demand-led Growth Models<sup>1</sup>

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## Abstract

This paper aims at a critical and constructive assessment of some extensions of Keynes's analysis of effective demand to the long period and growth. A criticism is addressed to a single-cause interpretation of the demand-led growth models and to the notion of normal capacity utilization adopted in such models. A positive argument tries to find a distinctive characterization of those extensions in the productive and financial conditions that make *effective* the autonomous changes in aggregate demand. It suggests a notion of normal capacity utilization as a range of *distributions* of normal utilization, related to the cost minimizing choice of techniques and to the persistence of the long term expectations of normal prices underlying the investment decisions. In such a context a long-period analysis with normal prices should avoid a dual steady growth where constant relative prices of capital goods correspond to constant proportions among their *total* stocks.

**Keywords:** Growth, Keynesian long-period analysis, capacity utilization, productive-financial capacity

**JEL Codes:** B50, E22, O40

## 1. Introduction

This paper aims at a critical and constructive assessment of some past and recent efforts to extend Keynes's principle of effective demand to the long period and to the theory of economic growth. A criticism is addressed to a certain causal interpretation of the demand-led growth models and to the notion of normal capacity utilization adopted in such models. A positive argument tries to achieve a consistent characterization of the same type of models by stressing certain conditions that allow the autonomous changes

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<sup>1</sup> A first draft of this paper was presented in the keynote lecture given by the author at the Edward J. Nell's Retirement Conference, The New School, NYC, May 3<sup>rd</sup> 2014.

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in aggregate demand to become *effective*. The paper is divided in two parts. The first part deals with two main arguments as an extended introduction to the rest of the paper: 1) a basic property of the models of compound (geometric) accumulation and growth, 2) a pseudo demand-led growth model derived by opening Harrod's model through the inclusion of an autonomous demand. The second part presents 1) a generalized notion of normal capacity utilization and the idea of normal prices without normal quantities; 2) an investment-constrained growth model and 3) suggests an extension of normal capacity utilization to the financial sector. The paper is articulated through the following sections.

Section 2 deals with a property of the compound rate of growth already noticed in the 1950s by Samuelson and Solow in their stability analysis of linear disaggregated models and resumed in the recent debate. Section 3 emphasizes some basic non-Keynesian features of Harrod's model when it is augmented by an autonomous aggregate demand. In such a context, an increase of the foreign demand for exports brings about a negative impact on the equilibrium rate of growth, despite the assumption of an unlimited supply of labour. A numerical example (section 4) serves to describe the impact of the autonomous demand on the moving equilibrium and the distinction between pseudo, hypothetical and effective multipliers related to virtual and effective changes in demand (sections 5, 6). The rest of the paper is concerned with some difficulties to integrate a leading role of demand with normal values and distribution without falling into the trap of a steady growth where such integration seems to fade out. A sequence of related arguments deals with a simple investment-constrained growth model with variable capacity utilization (sections 7, 8), the balance-of-payments constrained growth model (section 9), the distinction between demand-led and closed growth models (section 10), a generalized notion of normal capacity utilization (sections 11, 12). The different threads of reasoning mentioned above lead to the conclusion (section 13) that a model of growth confined to the *real* economy is not adequate for an integration between a theory of demand-led growth and the theory of normal values.<sup>3</sup> The extension of the *effectiveness* of demand from the short term to the long term should cover two possible sources of flexibility: a range of utilization of productive capacity and a range of access to financial capacity, that cannot be reduced to the mere assumption of endogenous money.

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<sup>3</sup> This conclusion shares similar persuasions held by Foley (1986), Foley-Michl (1990), Nell (1998), Duménil and Lévy (2013).

## Part I

### 2. A property of compound growth

Samuelson and Solow (1953) in their analysis of stability, applied to disaggregated models, have stressed the following property of compound accumulation:

In the case where the system is expanding geometrically, one unit of output lost in the distant past could have been the progenitor of an ever-growing quantity of output lost since that time. This capacity is clear if, as suggested earlier, we think of  $X_i(t)$  as being the size of a population in the  $i^{\text{th}}$  age group. Now suppose one mother had been subtracted from the population many generations ago. If the population is expanding geometrically in the Malthusian manner, the number of potential descendants would also be increasing approximately geometrically. The loss in potential population attributable to the past disturbance would be increasing, not decreasing. The steady growth solution cannot be stable in the absolute sense that changes in initial conditions have effects ultimately damping to zero. What we might expect, however, is that the equilibrium *relative* age distribution might tend to reestablish itself; that the population might tend to resume its geometrical expansion at the rate  $\lambda$ . (Solow –Samuelson 1953, pp. 418-420)

This property of compound accumulation suggests the following remarks.

Firstly, a small change in the capacity utilization in one period is a progenitor of cumulated effects in the next periods both in terms of output and capacity expansion or contraction. Garegnani (1992) has put forward this argument to argue that the potential multiplier effect of a change in demand in the long run can be much higher than in the short run and therefore the independence of normal prices and income distribution from the changes in the autonomous demand *a fortiori* can hold in the long period, contrary to the early Kaldor's theory of growth with his assumption of full employment. This argument will be reconsidered in section 12.

Secondly, the property of compound accumulation sheds a doubt on the relevance of the asymptotic,  $t \rightarrow \infty$ , stability of an equilibrium proved by means of difference/differential equations. It is not only the lack of persistence of the structure of the model at issue, but the fact that a recursive solution, starting from given initial conditions and moving ahead over an indefinite time horizon, may amplify any small error of specification or the omission of an interfering factor at a point of time, through a process of compound accumulation of the same error or omission. This possibility suggests that, at least for the purpose of a policy-oriented general theory, the explanatory scope of a dynamic growth model should be confined to the direction of the movement of the economy over a finite number of periods.

Thirdly, it has been invoked, in particular by Kaldor in his early writings, that a positive excess of the natural rate of growth over the warranted rate of growth, and a corresponding compound growth of unemployment, cannot persist indefinitely *and therefore* we should rely on full employment growth models. Similarly, Thirlwall (1979) has claimed that a persistent geometric growth of the deficit of the balance of

payments is not sustainable *and therefore* balance-of-payments constrained models are better descriptions of a growing economy than those which admit such unlimited imbalance. The property of compound accumulation, combined with the second remark, suggests that the choice of models, based on the *real world* evidence of the impossibility of unlimited unbalances, should be reconsidered if the growth models are bounded in their time-range.

### 3. Harrod's equation with autonomous demand: a pseudo demand-led growth model

Let us start from Harrod's equation

$$g = \frac{s}{v} \quad [1]$$

where

$s$ : the proportion of savings out of total income

$g$ : the *rate* of growth<sup>4</sup>

$v$ : the capital/output ratio

Equation [1] derives from the equilibrium equation of the product market  $sY_t = v(Y_t - Y_{t-1})$  where the propensity to save is subject to  $0 < s < 1$  and  $Y_t$  is the total output of period  $t$ . Let us interpret the equation  $I_t = K_t - K_{t-1} = v(Y_t - Y_{t-1})$  as a simple accelerator mechanism, where  $v$  is the *desired* capital/output ratio,  $(Y_t - Y_{t-1})$  is the expected *and* realized change in national income; and the capital depreciation is null<sup>5</sup>. [1] corresponds to the steady growth conditions  $g = g_k = g_s = g_y$ , where  $g_k$ ,  $g_s$ ,  $g_y$  denote the rates of growth of capital, saving and output, and  $g$  is, in Harrod's words, the *warranted* rate of growth.

This model seems to be a natural extension of Keynes' short period analysis to the long period. In the short period the *level* of national income is the adjustment variable between savings and investment; instead in the long period this role is played by the *rate of change* of national income. In both cases the supply of labour is assumed unlimited at a given conventional wage rate and the saving function *appears* distinct from the investment function; a departure from Say's Law. However, the attribution of a driving role to demand is unsustainable if the coefficient  $v$  in the investment equation is interpreted as a *fixed desired and actual* capital/output ratio, associated with the assumption of perfect foresight<sup>6</sup>. This assertion becomes more transparent if we open the model by the introduction of an autonomous demand.

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<sup>4</sup> *Change* and *rate of change* in the text mean *rate of change* and *proportional rate of change* respectively.

<sup>5</sup> It is assumed a one-commodity model where the same commodity can be used as a means of production not subject to depreciation or for consumption. The model of an open economy will deal with a composite commodity made of domestic and imported goods in fixed proportions.

<sup>6</sup> The assumption of perfect foresight in a deterministic model is equivalent to the assumption of rational expectations.

The emphasis of Harrod and subsequent commentators has been centred on the problem of the relative<sup>7</sup> instability of a moving equilibrium path determined by the equation [1] in the *closed* model, where no room is left to the autonomous components of aggregate demand. This instability is attributed to the rigidity of the accelerator mechanism, such that any initial deviation of the capital/output ratio from the equilibrium ratio  $\frac{K_0}{Y_0} = v$  brings about a cumulative deviation of the rate of growth from its equilibrium (warranted) value. The focus on the relative stability has obscured a more simple property of the open model. The autonomous components of aggregate demand typically consist in private or public expenditure for consumption or investment or in exports. We shall confine the analysis to a model in which the autonomous demand does not directly affect the productive capacity of the economy: the model of an open economy with exogenous exports  $X_t$  and endogenous imports  $M_t$ . The usual assumptions and conventions are adopted.  $X_t, M_t$  are measured in physical units. Imports are proportional to the national product:  $M_t = mY_t$  where  $0 < m < 1$ ; the terms of trade are fixed and the units of measure of the traded quantities are chosen so that the price of imports in terms of exports (the terms of trade) is equal to one. In the absence of international capital flows, the difference  $X_t - M_t$  is the balance of payments in real terms. We neglect the public sector and therefore  $m$  is the only leakage coefficient out of the national income circuit, beside the propensity to save,  $s$ . The macroeconomic equilibrium condition is

$$sY_t + mY_t = v(Y_t - Y_{t-1}) + X_t \quad [2]$$

Equation [1] is generalized accordingly:

$$g_t = \frac{s+m-h_t}{v} \quad \text{with } h_t \equiv \frac{X_t}{Y_t}, \quad [3]$$

This formalization resumes an old generalization<sup>8</sup> of Harrod's equation, in which the role of government expenditure and taxation is similar to that of exports and imports in the present model.<sup>9</sup> If we set  $h_t = m$  in equation [3], the balance of payments  $X_t - M_t = 0$  is satisfied in each period and the equilibrium rate of growth is determined like in a closed economy without autonomous demand by equation [1]. Instead, exports in the present model are exogenous and the export to income ratio,  $h_t$ , endogenous. In both cases an increase of  $h_t$  is associated with a *decrease in the equilibrium rate of growth*,

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<sup>7</sup> Relative stability means the instability of the warranted *rate* of growth as distinct from the stability of the *absolute* growth of income (cfr. Samuelson and Solow 1953). See section 2.

<sup>8</sup> Cfr. Baumol (1963, ch.4).

<sup>9</sup> The same equation has been recently reconsidered in (Commendatore, D'Acanto, Panico and Pinto 2013) who have tried a reconstruction of Harrod's ideas spread over different writings (Harrod's 1939, 1948, 1964, 1973). However, in their exegesis the adoption of a special unit of measure of quantities seems to imply that  $h_t$ , instead of  $X$ , is an exogenous variable.

$g_t$ . A crucial condition adopted in this open Harrodian model is the equality between the desired and the actual capital/output ratio in each period:  $v = \frac{K_t}{Y_t}$ ,  $t = 0, 1, 2, \dots$ . Some elementary tools of dynamic analysis are recalled in the Appendix and will be applied to the following numerical examples.

#### 4. A numerical example of moving equilibrium

Even without resuming the instability of the warranted rate of growth in the closed model, the moving *equilibrium* may go out of the track in the presence of an autonomous component of aggregate demand, exports in our case. Figure 1 describes the model with the parameters  $s+m = 0.3$ ;  $v = 5.0$  and  $X_t = 1$ . The corresponding general solution is  $Y(t) = (Y_0 - 1/0.3) \left(\frac{50}{47}\right)^t + \frac{1}{0.3}$  where  $Y_0$  is the initial level of national income. The figure shows that any initial slight deviation of  $Y_0$  from the stationary equilibrium  $Y^* = \frac{1}{s+m} = 10/3$  (the horizontal red line) brings about a cumulative decay or growth of  $Y_t$ .

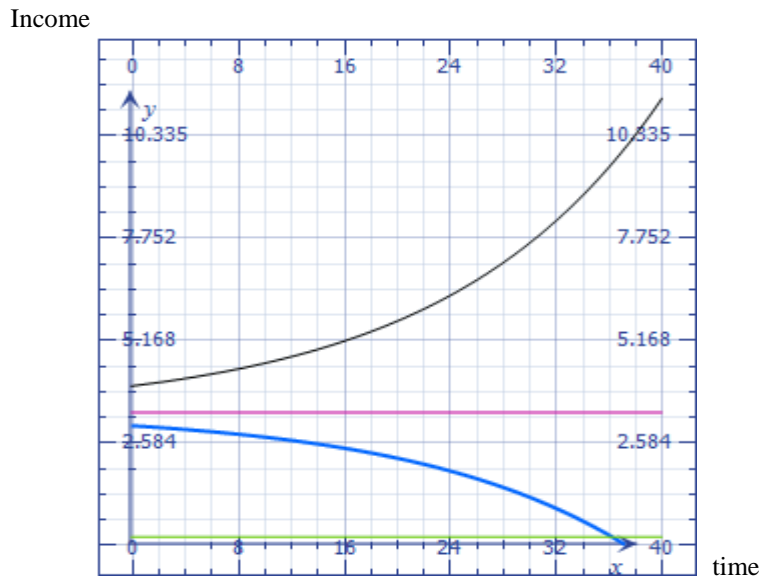
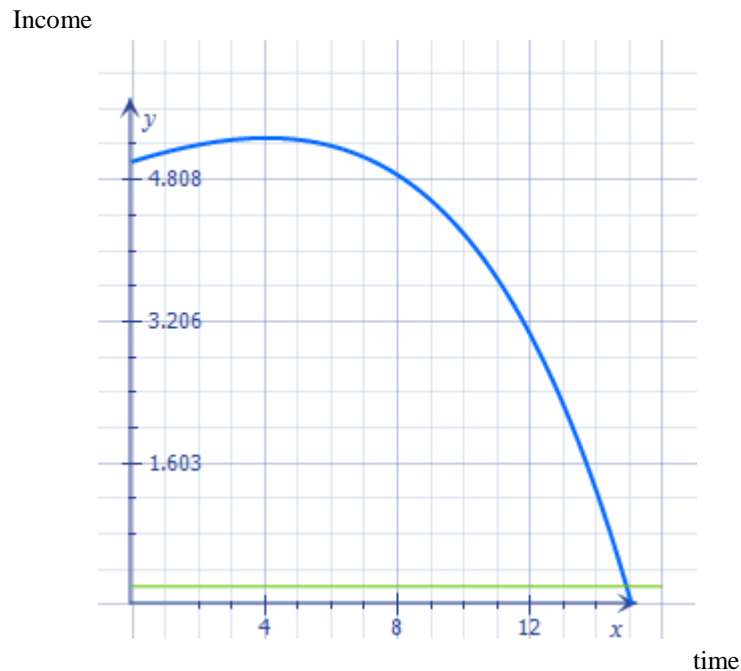


Figure 1

A deviation below, with  $Y_0 = 3$ , will lead to the blue line down to the green floor where  $Y_t = X/v = 1/5$ . Instead a deviation above, with  $Y_0 = 4$ , leads to the gray line, that describes a growth path at a variable positive rate which *tends* to a steady warranted rate of growth, because the influence of the constant foreign demand  $X$  becomes more and more negligible relatively to the endogenous growth mechanism.

This conclusion can be generalized assuming that the volume of exports, instead of being constant, follows an exponential trend  $X_t = X_0 b^t$ . The curve in Figure 2 describes the case in which the parameters are those of the previous example with the new parameters  $b = 5/4$  and the initial condition  $Y_0 = 5$ . The curve shows that the negative

influence of the exogenous trend of exports is initially dominated by the endogenous growth and then such influence is reversed leading to the decay of the national product down to the limit green line.



**Figure 2**

The examples suggest that the equilibrium dynamics is the combined result of two components: the endogenous and the exogenous or autonomous dynamics. Depending on the values of the parameters, one of the two components can dominate the variable rate of growth, relatively to the other. As argued in section 2, a special focus should be addressed to the initial periods, instead of the asymptotic tendency to either an exogenous or endogenous constant rate of growth.

The examples show that exports in the model play either a negative or a negligible role for the growth performance of the economy, as if the benefits highlighted by Keynes through his parable of digging holes into the ground would become damaging in the equilibrium dynamics of Harrod's model with autonomous demand. Although the model assumes an unlimited supply of labour, an initial equilibrium path can change only through a *trade-off* between the autonomous demand and the endogenous investment. This result is not surprising: it depends on the lack of excess capacity of the capital stock and on the assumption of perfect foresight (the accelerator coefficient  $v$  is assumed identical to the actual capital-output ratio) that make the investment function  $I_t = v (Y_t - Y_{t-1})$  implicit in the other functions, instead of being the result of the independent decisions of the entrepreneurs. This trade-off contrasts with the interpretation of Keynes's short period equilibrium when the activity level of the economy is supposed to reach the maximum capacity: in this case an increase of the autonomous demand brings about an inflationary pressure without a necessary substitution effect.



## 5. Hypothetical and effective multipliers

In figure 1 the comparison between the two colored curves is a comparison of *different* economies, each in its own dynamic equilibrium. They do not represent what will happen to an economy if at time  $t = 0$  its exports change, *ceteris paribus*. The *ceteris paribus* clause cannot be applied because the initial difference  $\Delta X$  has to be associated with the simultaneous differences  $\Delta Y = \frac{1}{s+m} \Delta X$  and  $\Delta K = \frac{v}{s+m} \Delta X$ . Let us write the macro-equilibrium condition [2] in the form:

$$\hat{Y}_t = \frac{1}{s-v+m} (X_t - vY_{t-1}) \quad [4]$$

and derive

$$\frac{\Delta \hat{Y}}{\Delta X} = \frac{1}{s-v+m} \quad [5]$$

The fraction  $\frac{1}{s-v+m}$  looks like a short period foreign trade multiplier, but it is a *pseudo multiplier*, independently of the fact that it may turn out less than 1. In fact it represents a one-period impact of the change in exports on the change in income. Next, subject to the stationary equilibrium condition  $Y_t = Y_{t-1}$ , (cfr. the appendix) the equation [5] becomes

$$\frac{\Delta \hat{Y}}{\Delta X} = \frac{1}{s+m} \quad [6]$$

The fraction  $\frac{1}{s+m}$  looks like the long period foreign trade multiplier. If the changes are attributed to the same economy and the change of exports is said to *cause* a multiplicative change in the national income of the same economy, this is a counterfactual assertion where the initial  $\Delta$ -occurrence would be like a *miracle* in the sense adopted by the philosopher David Lewis (1973) and [6] might be called a *miracle multiplier*. The expression [6] differs from the short period multiplier in Kahn and Keynes. The difference does not rest only on a different autonomous demand and the distinction between comparative dynamics and comparative static. In Kahn and Keynes the autonomous change  $\Delta X$  is supposed to be *effective* and the induced change in  $Y$  becomes actual through a feasible change in the rate of utilization of the existing capacity and an implicit soft budget constraint attributed to the investors. Instead the changes in equations [6] are *purely hypothetical*.

## 6. A digression: the effective demand and the “effectual” supply of exhaustible natural resources

Let us exploit once more the simple model of compound accumulation of an open economy. Imagine that figure 1 is extended to the left so that also a backward sequence of periods  $t = -1, -2, \dots$  is represented on the time axis. Suppose that the economy is in equilibrium before period  $t = 0$  with the actual capital/output ratio equal to the desired ratio. Normal and abnormal fluctuations, normal and abnormal accidents impinge both on the demand and the supply side of the economy. Suppose that something *new* happens in  $t = 0$  : an autonomous change either in the demand or in the supply. For example, the opening of a foreign market (a case of Schumpeterian innovation) is an autonomous increase in the foreign demand for exports  $X$ . Instead the closure of a foreign market is an autonomous change in the foreign supply of imports, that is transformed from an unlimited amount to a sort of rationing. In both cases we can calculate the effect of the hypothetical change by a *counterfactual reasoning*, but we wonder what are the conditions under which the hypothetical changes becomes *effective* for the economy instead of being only potential.

We have a ready answer in the case of an increase of exports. Suppose that before  $t = 0$  the producers had foreseen that some change might happen in the future, for example a normal seasonal fluctuation or an accident and some margins of productive capacity exist in period  $t = 0$  to satisfy a profitable change in demand. If the autonomous increase of exports is not too large it can become an actual change  $\Delta X_0$  without displacing other demand components and therefore it becomes a net increase in national income in period  $t = 0$ ,  $\Delta Y_0$ , followed by a multiplier effect. The actual autonomous change can be either an impulse that occurs once-for-all and with negligible effects on investment; or can be persistent and affect the investment plans.

Instead we have not a ready answer in the case of the sudden limitation of the foreign supply.

The structure of the model has to be changed. To avoid that this limitation bring about a collapse of the economy for the lack of a necessary imported commodity, let us suppose that the economy possesses a backstop technique that in case of need can be activated to provide the imported commodity, although at a higher cost. In this case the autonomous change in the foreign market becomes effective through such a substitution effect. However, we cannot at the same time *deny* that the growth performance of the economy is *led* (constrained) by the supply of imports and *admit* that it is demand-led in the case of the expansion of exports. There is no decisive reason for assuming a priori that of the two potential autonomous changes, in exports and in imports, only the former - on the demand side - is effective whereas the latter - on the supply side - remains potential. Another example, even more challenging for the claim of generality attributed to the demand-led models, is offered by what elsewhere I called *the effectual supply* of exhaustible natural resources in the context of Sraffa's price equations (Parrinello 2004). Instead of the total *stock* of the resource, say oil in the ground, it is its *flow* of

supply to be one of the determinants of the normal prices. This flow depends on the long term expectations of the owners of oil deposits whose decisions of extraction may conform to the Hotelling rule, like the investors in Keynes are supposed to conform to the rule of the marginal efficiency of investment. Once more there is no *logical* reason to deny that an abrupt change in the expectations of the owners of the resource is less effective for a decrease in the supply of oil than a change in the expectations of the investors.

We are still looking for some consistent characterization of demand-led growth models, instead of agreeing with the provocative title of Duménil-Lévy (1999) article “Being Keynesian in the short term and classical in the long term”

## Part II

### 7. An investment constrained growth model with variable capacity utilization

Not surprisingly we have not found a satisfactory notion of *demand-led growth* in Harrod’s model. Let us cover another route to escape from the ‘*cage*’ of the steady growth models that satisfy the dual condition: 1) a *constant* and *uniform* rate of growth of quantities and 2) *constant* relative prices with a *uniform and constant* rate of profit. A step in this direction consists in the recognition of different kinds of flexibility of an economy that undergoes foreseen and random deviations from a steady growth. A flexibility in a model with fixed prices is provided by inventories of storable goods and money, incomplete contracts, bank overdraft and lines of credit and margins of excess productive capacity. In the recent debate an overwhelming emphasis has been put on the latter, perhaps with an undue understatement of the other elements of flexibility. This flexibility can be operative *within certain limits* without relaxing the assumption of fixed prices and, in particular, a fixed rate of interest underlying the desired capital/output ratio. For the sake of argument we shall change the initial assumptions and move to a growth model of a closed economy with variable capacity utilization.

Let us assume a closed economy and the classical saving function: a positive proportion of saving out of profits,  $s_\pi$ , is given and the propensity out of wages is zero. The overall propensity to save  $s$  is equal to  $s_\pi$  multiplied by the ratio of total profits  $\Pi$  to the national income  $Y$ . The initial Harrod’s equation [1] becomes:  $g = \frac{s}{v} = \frac{s_\pi \Pi}{Y} \cdot \frac{Y}{K} = s_\pi r$  where  $r$  is the *rate of profit*<sup>1011</sup>. Let  $\lambda$ ,  $v$  denote *normal* technical coefficients: the amount of labour and capital per unit of output when the productive capacity is utilized at a *normal* rate.<sup>12</sup> Let us assume that a change in capacity utilization affects the

<sup>10</sup> Since the depreciation rate of capital is null  $r$  denotes both the gross and net rate of profits.

<sup>11</sup> The equation  $g = s_\pi r$  is the received Cambridge equation, that can be differently interpreted, but not in a certain way. In particular, both  $g$  and  $r$  can be either *actual/ex-post* or *normal/theoretical* rates of growth and profits, respectively, but not a mixture of such attributes. Cfr. Garegnani (1992).

<sup>12</sup> The notion of normal capacity will be critically reconsidered in sections 11, 12.

productivity of labour and capital in the same proportion<sup>13</sup>,  $u$ , and define the critical rates of capacity utilization by

$u = 1$ : normal capacity,

$0 < u < 1$ : excess capacity,

$1 < u < u_m$ : capacity overutilization, where  $u_m$  is a technical maximum.

$v$  is the normal capital/output ratio corresponding to  $u = 1$  and

$Y = \frac{K}{v}$  the normal output at a given  $K$

$Y_a = uY$  the actual output

$Y_m = u_m Y$  the full capacity output.

Let  $w$ ,  $r$  denote the real wage and the (net and gross) rate of profit. The general price equation is

$$w\lambda + rv = u \quad [7]$$

$w\lambda + rv = 1$  in the normal state.

The desired rate of capital accumulation depends on long term expectations. Instead of the dependence on the expected growth rate of output related to the idea of the accelerator, let us assume, according to the intuitive idea of the *animal spirits* of the entrepreneurs suggested by Joan Robinson, that  $g_k$  is an exogenous variable

$$g_k = \bar{g}_k$$

The equilibrium condition between desired investment and saving is the Cambridge equation:

$$\bar{g}_k = s_\pi r. \quad [8]$$

Suppose that the wage rate is fixed at a conventional level:

$$w = \bar{w}. \quad [9]$$

The solution to equations [14], [15], [16] is:

$$r = \frac{\bar{g}_k}{s_\pi} \quad [10]$$

$$u = \bar{w}\lambda + \frac{v}{s_\pi} \bar{g}_k. \quad [11]$$

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<sup>13</sup> The same assumption is adopted in Foley and Michl (2000). This section has been inspired by chapter 10 of their volume. An important difference remains and it is due to the fact that the classical saving function in the present work is formulated in the usual way, i.e., savings are a given proportion of the total profit flow. Instead Foley and Michl assume that savings are a given proportion of the total wealth of the capitalists. Such a stock/flow relation seems to be the correct formulation to be adopted in the model extended to the financial sector of the economy (see Foley 1986).

By substitution, the actual national income associated with a given K:

$$Y_a = K \left( \bar{w} \frac{\lambda}{v} + \frac{\bar{g}_k}{s\pi} \right) \quad [12]$$

and the actual consumption

$$C = Y_a - \bar{g}_k K = K \left[ \bar{w} \frac{\lambda}{v} + \bar{g}_k \left( \frac{1-s\pi}{s\pi} \right) \right]. \quad [13]$$

The solution [10],[11],[12],[13] recovers the Keynesian features that have been lost in the initial model of moving equilibrium. In fact, there is no trade off between the *actual* rate of profit and the *actual* real wage, on the side of prices, and no trade off between consumption and investment, on the side of quantities. In particular, if  $\bar{g}_k$  increases, the actual rate of profit and consumption per capita increase whereas the conventional wage rate remains fixed. However, how general is such a Keynesian result for a growth theory combined with a long period theory of distribution? As a matter of fact the *Keynesian features* mentioned above belong to the short period and do not subvert the inverse relation between the normal rate of profit and the real wage, that in this simple model is imposed by the price equation [7] subject to the norm  $u = 1$ .

## 8. The reduction of the model to a special case

We cannot pretend to have achieved a dynamic growth theory by means of a sequence of self-contained short period equilibria. The previous investment-constrained growth model has indeed such shortcoming, despite the possibility of determining a sequence of capital stocks K through a recursive solution. Depending on the specification of the investment function, instead of the assumption of an exogenous rate of accumulation  $g_k = \bar{g}_k$ , the Keynesian properties mentioned above may or may not be preserved. We shall stress two difficulties behind the choice of the investment function.

Firstly, a preliminary problem is the definition, measure and role of *normal* output and *normal* degree of capacity utilization, represented in our simple model by the value of the variable  $u$ , and extensively discussed in the recent debate about different Post-Keynesian theories of growth and income distribution. The appraisal of this problem is postponed to section 12.

Secondly, some relation should be established between the expectations of the entrepreneurs underlying their investment plans and the normal state of the economy that in the model corresponds to the value  $u = 1$ . A critical limit exists to a too strict relation, beyond which the model is brought back to a steady growth model. Consider two simple investment functions of expected variables, that might be combined into one investment function jointly with the rate of capacity utilization in a more general model.

The intuitive idea of the accelerator can be formalized by the assumption that the desired rate of accumulation  $g_k$  is a positive proportion  $\sigma$  of the *expected* rate of growth of national income  $g_Y^e$ :

$$g_k = \sigma g_Y^e. \quad [14]$$

Similarly, the intuitive idea of the *animal spirits* suggests that  $g_k$  is proportional to the expected rate of profit  $r^e$

$$g_k = \eta r^e. \quad [15]^{14}$$

where  $\eta$  is a positive volatile parameter. [14], [15] can be interpreted as distinct components of an investment function. Under the assumption of perfect foresight the expected rates  $g_Y^e, r^e$  are equal to the rates  $g_k, r$  determined in the *normal* state of capacity utilization,  $u = 1$ . Hence:  $r^e = \frac{1-\bar{w}\lambda}{v} = r$ , and  $\sigma = 1$ , and  $\eta = s_\pi$ . It follows the interpretation of the Cambridge equation in which the rate of growth, instead of being affected by the independent decisions of the entrepreneurs, is determined by the proportion of saving out of profits and the normal rate of profit: this means a return to a steady growth model where the role of the effective demand fades out.

Two complementary routes have been explored to avoid the assumption of steady growth and at the same time preserve the extension of the principle of effective demand to the long period.

One consists in assuming that some component of aggregate demand has an autonomous trend. In the next section we shall deal with an open economy subject to a balanced-trade constraint and autonomous exports. The second route maintains the relation between the normal and expected rate of profits,  $r^e = \frac{1-\bar{w}\lambda}{v}$ , but denies that normal prices have to be associated with normal quantities conceived as steadily growing quantities. This issue will be discussed in section 12.

## 9. Growth with balanced foreign trade

A preliminary notion of the foreign trade multiplier with balanced trade  $X = mY$  can be traced back to Harrod<sup>15</sup> and has the form

$$\Delta Y / \Delta X = \frac{1}{m} \quad [16]$$

Harrod was aware that equation [16] is a special case of a more general short period multiplier associated with other autonomous components of demand (consumption, investment and public expenditure), Palumbo (2009, 2011) observes that the same form of the multiplier  $\frac{1}{m}$  that is found in Kaldor (1970) does not neglect such additional items

<sup>14</sup> The same *form* of the proportional investment function [22] has been adopted in Foley-Michl (1990) to formalize in a simple way the role of the animal spirits of the entrepreneurs, subject to the assumption  $g_k = \eta r$ , where the current rate  $r$  may be different from the expected rate  $r^e$ . Marglin-Bhaduri (1990) adopt a more complex investment function where  $\eta r$  is combined with other explanatory variables.

<sup>15</sup> Cfr. Harrod 1957, p. 120.

in demand, but would be derived from Hicks's (1960) supermultiplier  $\frac{1}{s-i+m}$ , in which  $i \equiv I/Y = vg$  is the propensity to invest induced via the accelerator mechanism. Equation [16] obtains in the special case  $s = i$  independently from the level of income and this is a strong departure from Keynes's principle. Furthermore the so-called Thirlwall law (Thirlwall 1979)<sup>16</sup> derives from the equation of balanced trade  $X = mY$  and states that a proportional change of exports *drives* a proportional change in income. Since the income elasticity of imports is equal to one and in the absence of international capital movements, the law in continuous time can be simply written:

$$\dot{Y} / Y = \dot{X} / X \quad [17]$$

with  $\dot{Y} = dY/dt$

Different interpretations and derivations of [16], [17] have been suggested following the evolution of the ideas of Harrod and Kaldor and the development of balance-of-payments-constrained growth models. We shall short-circuit such useful exegetical work and confine to a few analytical considerations.

In principle the condition of macroeconomic equilibrium of the product market can be written as a sum of  $n$  excess demands or "balance of payments" functions  $(D_t(Y) - S_t(Y))_j$  where the aggregate income,  $Y$ , is the only independent variable :

$$(D_{1t}(Y) - S_{1t}(Y)) + \dots + (D_{nt}(Y) - S_{nt}(Y)) = 0 \quad [18]$$

We may assume that the sum of all  $n$  excess demands except the first one is always null independently of the level of  $Y$  and fix the first demand as exogenous variable,  $D_{1t}(Y) = \overline{D}_{1t}$ ; then derive a multiplier for the given  $\overline{D}_{1t}$  and call a growth model which adopts such assumptions a  $D_1$ -led growth model or  $(D_1(Y) - S_1(Y))$ -constrained-growth model.<sup>17</sup> In particular, we may pick the foreign balance of payments  $X_t = mY_t$  as such excess demand and derive an export-led growth model. The critical issue, that cannot be settled without an empirical evidence, arises if we pretend that such a type of demand-led or constrained growth modelling represents a general theory instead of a special case.

<sup>16</sup> Cfr. Thirlwall (2011), McCombie (2011), Setterfield (2011).

<sup>17</sup> Unfortunately the twin connotation of demand-led and demand constrained growth reflects the difficulties of the meaning of causation in philosophy: the counterfactual dependence of the effect from the cause as distinct from the primitive notion of production of the effect by the cause; and the general problem of the selection of a cause from the background conditions within the multiplicity of causal conditions. Cfr. Parrinello (2013a, 2013b).

## 10. Demand-led growth and closed models

Relatively to the equation of macroeconomic equilibrium, [18], we may distinguish two general types of models: 1) models in which some or all addends ( $D_i(Y) - S_i(Y)$ ) are not null in equilibrium (for example the government budget or the trade balance) and have their own autonomous demand; 2) models in which all components  $D_i(Y)$ ,  $S_i(Y)$  are endogenous variables. The closed models of accumulation, in which *no* demand or supply is autonomous, belong to type 2. Marx's scheme of accumulation, Harrod, Von Neumann, Leontief (his closed input-output model), Duesenberry (1958), Smithies (1957) models are examples of closed models of endogenous growth with or without cycles. It might be tempting to call them *supply-led or supply-constrained growth models* by exclusion: they do not possess the demand-led feature due to the autonomous demand. However this does not seem the correct interpretation of closed *growth models*. In particular it is true that growth at the warranted rate in Harrod's model is not *driven* by a limited supply of labour or constrained by natural resources and it might seem to be driven or constrained by the "supply of savings". Yet, suppose to follow Kaldor and assume that the propensity to save out of profits,  $s_\pi$ , is mainly governed by large corporations who decide the amount of non distributed profits and how to allocate them for investment. In this case the saving coefficient  $s_\pi$  simultaneously represents the propensity to save *and* to invest and only our parsimony of language induces to choose one (the former) of the two terms instead of a third one. The only *safe* intuitive connotation of those closed models is that they are models of growth *led* by the search of profits and *constrained* by the technical conditions of the economy. However, "*led*" and "*constrained*" do not mean "caused" within the context of steady growth, because such as a growth path is like an inertial state that is *uncaused* in its continuation, although its initiation may have been caused. The same *safe* feature would apply to the demand-led or demand-constrained growth models: they are led by the search of profits and constrained in a similar way, but we are looking for a distinctive leading factor.

## 11. Different exits to avoid steady growth: normal prices without normal quantities?

According to certain interpretations (Roncaglia 1978, Palumbo and Trezzini 2003, Ciccone 2011) of the long period states of the economy in the Classics and Sraffa, there is an asymmetry between theoretical prices and quantities. It is maintained that the relative speed of adjustment of market prices towards their normal values is *high*, whereas the quantities of the capital goods and the capacity utilization move *slowly* towards the corresponding normal levels or they do not tend at all in the presence of a multitude of interfering forces.

There is a sense in which such a distinction between price and quantity adjustment is obvious on the basis of the feature of the compound growth (cfr. section 2). In fact, any



impulse that changes the productive capacity of the economy at a point of time has a cumulative effect in the next periods on the absolute quantities and therefore a steady growth model is inherently unstable relatively to the quantity *levels*: it is indeed an elementary case of path dependency. The quantity *levels* cannot play the role of “normal” quantities<sup>18</sup>. If some gravitation exists on the side of quantities, it has to be confined to the *proportions* among quantities and to their *rate* of growth. There is another sense in which the distinction between the different speed of adjustment of relative prices and quantity proportions is not obvious. It is not always clear whether the arguments advanced by the participants to the debate<sup>19</sup> are confined to the problem of the gravitation toward normal values and to the sequence of short periods of the convergence process or instead they imply an “intermediate period” analysis between the short period and a long period conceived as a fully adjusted long period equilibrium. The following argument tries to clarify the notion of a *medium period*, that seems suggested by some contributions<sup>20</sup>, still without the help of a formalized disaggregated model.

*A basic discrimination concerns the assumptions about the pace of technical change and the proportion between gross investment and the total stock of fixed capital.* On the basis of the idea of embodied technical progress adopted in the vintage growth models, the adjustment of prices and outputs equally depends on the amount and allocation of *gross* investment, i.e. on the new capacity installed in the different sectors, compared with the size of the existing stocks of fixed capital inherited from the past. *If the annual flow of gross investment is small, compared to the total stock of fixed capital, the diffusion of the best technique embodied in the new fixed capital will be relatively slow.* However, the annual flows of outputs can adjust relatively quickly towards the proportions consistent with a uniform rate of profit on the new capital goods by means of variable degrees of utilization of the total stocks of fixed capital and a corresponding change in the inputs of circulating capital. Instead the proportions among such stocks would adjust slowly towards normal proportions (including a normal degree of capacity utilization) and may even not gravitate at all in the presence of a non-stop flow of supervening new techniques. This picture describes a “medium period” position consistent with the emergence of quasi-rents on the old machines.<sup>21</sup> The normal prices would be determined by the techniques embodied in the gross investment of the latest vintage and they can be assumed as guidelines for the investment decisions. In the type of aggregated model presented in section 7, this idea of medium period can be

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<sup>18</sup> On the basis of this property the idea of a dual center of gravity (natural prices and corresponding *absolute levels* of effectual demand) suggested by Adam Smith, can be applied to a stationary state but cannot be extended to an economy that undergoes a geometric growth.

<sup>19</sup> A useful critical overview of the debate on the role of normal quantities and normal capacity utilization in demand-led growth models can be found in (Palumbo and Trezzini, 2003). The debate includes, beside the early contribution of Steindl (1952), Ciccone (1986, 2011), Duménil, G. and Lévy, D. (1999), Garegnani (1992), Kurz, (1986, 1990), Lavoie (1992), Man Seop Park (2000), Palumbo (2009, 2011), Palumbo and Trezzini (2003), Serrano (1995), Semmler (1990), Trezzini (1998), Vianello (1985).

<sup>20</sup> Cfr. Ciccone (2011).

<sup>21</sup> Cfr. the treatment of obsolete machinery in (Sraffa 1960, ch. 11, section 91).

illustrated assuming for simplicity that the given capital stock inherited from the past is a homogenous good instead of vector of layers of different machines, whereas the new investment is made of a new capital good and is associated with a more profitable technique. The price equation [7] will be replaced by two price equations with two endogenous prices: the normal rate of profit  $r$  on the new capital good and the quasi-rent  $z$  on the old capital that will not be produced anymore.

The Cambridge equation would still hold under the classical saving function, but the rate of profit in this equation should now be interpreted as an *actual* average rate of profit (including the quasi-rents) on the total new and old capital instead of a *uniform* normal rate of profit  $r$  on the supply price of *all* capital goods.<sup>22</sup> Actual income distribution in such a medium period does not coincide with the distribution determined with normal prices by the subsystem of production with best techniques.

*By contrast, if the annual flow of gross investment is not small, compared to the total stock of fixed capital, and granted a correspondence between the normal prices and the choice of best/ cost-minimizing techniques, there seems to be no reason why the composition of the total stocks of fixed capital and the market prices should adjust at different speeds. In particular, the medium period analysis suggested above would become useless if the economy should employ only circulating capital without fixed capital reduced to joint-products.*

In general, the normal prices cannot be completely decoupled from a normal degree of capacity utilization as a result of cost-minimizing choices. Our argument on this issue is a development of some previous reflections of the author ((Parrinello 1990, 1992), and is related to Kurz (1986, 1990) and Lavoie (1992). In the recent literature it is usual the following distinctions of the capital/output ratios,  $K_t/Y_t$  : the *actual* ratio, the *normal* ratio, the *desired* ratio and the *technically minimum/maximum* ratio<sup>23</sup>. The debate is especially focused on the problematic notions of normal quantities and, in particular, on the role of normal capacity utilization. Both the *normal* ratio and the *desired* ratio are problematic. Only a small information of normality is conveyed by a single capital/output ratio. The fraction  $K_t/Y_t$  can be *desired* in period  $t$  but can be hardly said *normal*: only a (either deterministic or probabilistic) *distribution* of quantities over a certain time interval can possess such attribute.<sup>24</sup> This shortcoming is not overcome if the ratio  $K/Y$  applies to a time average of *ex-post* capital/output ratios. The cases of a power plant or a blast furnace can be useful examples of the problem of definition and measure of capacity. What does it mean to measure the degree of utilization of a power plant by a single percentage, say 50%, in terms of output of a certain period of time? The *actual* capital/output ratio in a certain period, except for some abnormal peak or

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<sup>22</sup> Cfr. footnote 11 and Garegnani (1992). The statement in the text is consistent with the existence of a uniform rate of profit in *all activities* only if we *assume* an equilibrium state in which the price of the old capital good,  $p$ , that is not a price of production, is equal to the quasi-rent capitalized at the normal rate of profit:  $p = z/r$ .

<sup>23</sup> In the case of a blast furnace, mentioned below, it is relevant also a minimum degree of capacity, below which the plant irreversibly goes out of order.

<sup>24</sup> Cfr. Parrinello (1990, 1992).

fall, cannot be judged abnormal to the extent that the investment plans become affected. The choice of the type of power plant depends on the expected *distribution* of the demand for kilowatts subject to peaks and floors over some time horizon.

Furthermore the correspondence between “normal” and “desired” is problematic.

The normal degree of capacity utilization is the one that firms have in mind in taking their investment decisions; i.e. the degree that, if attained, would make them – to use Harrod’s famous expression – ‘content with what they are doing’. (Palumbo and Trezzini, 2003, p. 111)

We wonder what is the meaning of the assertion that the investors are “content with” a certain actual  $K_t/Y_t$  instead of others ratios. If we want a non circular answer, we can hardly avoid the conclusion that this preferred ratio, expressed by the word “contentment”, is the result of a search for higher profits. In the previous example, the choice of the initial plant and of the capacity utilization, associated with deterministic and random fluctuations in energy consumption, involves indivisibilities and non linearities and therefore cannot be formulated like the problem of the choice of *linear* techniques used for the criticism of the neoclassical theory of value and distribution<sup>25</sup>. Despite such complexities, the choice of the degree of capacity utilization in the long run, as already stressed by Kurz (1986, 1990), cannot be separated from the cost-minimizing choice of techniques behind the investment plans.

## 12. A corridor (range) of normal degrees of capacity utilization

In the Marshallian theory of the firm, the plant is given in the short period and variable degrees of capacity utilization can be chosen at given prices, one of which is assumed to be profit-maximizing. In a demand-led macroeconomic growth model the “short” period means not only a given maximum capacity of the economy but also given long term expectations underlying the investment plans. We may assume<sup>26</sup> a *corridor* of *normal* degrees of capacity utilization instead of a single value. The degree of capacity utilization can change within the corridor and actual income distribution and market prices may change accordingly, because the demand for some inputs (e.g. circulating capital) will be affected through the cost-minimizing choices of the producers. However, the *normality* of such occurrence means that the long term expectations of the normal prices and the normal rate of profit will not change and therefore investment plans will persist. This seems to concede the flexibility required for the consistency between a driving autonomous demand and normal capacity. Notice that a constant corridor of rates of utilization is consistent with an ever expanding corridor of *absolute* quantity changes in the presence of compound growth. More importantly, in general the norm cannot be defined by a range of quantity *levels*, but by

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<sup>25</sup> Cfr. Kurz and Salvadori (1995).

<sup>26</sup> Cfr. Lavoie (1992: 327-32; 417-22). See also Dutt (1990: 58-60).

a *range of normal* (not necessarily in the statistical sense of a bell shaped distribution) *distributions of capacity utilizations*. Normal accidents should be kept distinct from abnormal accidents<sup>27</sup> and embedded in a *loose notion of normal output proportions*<sup>28</sup>. Despite such a generalization of the notion of steady growth, the normal capacity utilization, conceived as a *range* of distributions of utilization rates, cannot be separated from the choice of techniques that is governed by cost-minimizing and profit-maximizing behavior. A change of capacity utilization within the normal range does not leave the costs unaffected, still it preserves the long term expectations of investment profitability.

The position of the corridor depends on the past and the expected utilization. If, for example, the actual distribution of utilization rates falls outside the corridor, it is plausible that investment plans and normal prices can be affected and the position of the corridor may shift. As already stressed by Nell (1998, pp. 492-493), a *once-for-all* change in the *level* of the autonomous demand should be kept distinct from a persistent change in the *rate of growth* of demand. More precisely, we should keep distinct i) a one-for-all impulse like the subtraction of a mother from a population in the example of Samuelson-Solow, ii) a persistence change in the absolute level of the autonomous demand; iii) a persistent change in the *rate of growth* of the autonomous demand. The changes i), ii) can have a cumulative effect on the growing scale of the economy but they may be consistent with a persistent corridor; instead the latter can also affect such a normal range of values, the investment plans and income distribution.

As a consequence, the problem of consistency is shifted to a higher level but not eliminated. In the presence of a persistent change in the *rate of growth* of the autonomous demand, the normal range of capacity utilization cannot be on the one hand an adjustment variable between the rates of growth of savings and investments without an impact on prices and distribution, *and* at the same time consistent with a cost minimizing choice of techniques.<sup>29</sup>

### 13. A residual characterization of demand-led growth and conclusion

A residual distinctive characterization of growth models is to be found in the conditions that make *effective* (feasible, actual) the autonomous changes in demand.

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<sup>27</sup> See Hannsgen (2013) for a review of the types of heterodox shocks.

<sup>28</sup> Cfr. Parrinello (1990, 1992).

<sup>29</sup> The following conclusion in Garegnani's essay intends to limit the reach of his own argument and seems to become close to that presented in the text.

*"It should finally be stressed that the present argument for which a long-period rise in investment needs not alter distribution in order to generate the corresponding savings, should not be taken to deny the possibility – or indeed the likelihood – of interactions between the real wage and the normal rate of profit on the one hand, and the speed of capital accumulation on the other. We have just mentioned, e.g., the possibility that a rise in the incentive to invest and the consequent improvement in the situation of labor employment might result in a rise of the real wage (rather than a fall which we would expect from the First Keynesian Position [Kaldorian]. What is disputed is only the necessity of the particular effect postulated in the First Keynesian Position"* (Garegnani, 1992, p. 64, [Kaldorian] added).

Such conditions are mainly related to the workings of financial institutions and are implicit or neglected in the mathematical growth models confined to the real economy.<sup>30</sup> Demand-led growth models formulated only in terms of real variables are not well equipped for this task. The source of the asymmetric effectiveness of demand versus supply cannot be reduced only to the existence of margins of excess productive capacity and to the assumption of endogenous money supply. The financial sector should be explicitly specified, alongside the investment function and the saving function, in order to describe the financial (institutional), beside the real (technical), conditions that allow the effective demand to play its driving role in the growth process, leaving the supply to adapt to the former.<sup>31</sup> In the following concluding remarks we surmise three general warnings and a tentative research agenda.

Firstly, it would not be of much help to claim that demand – in particular the demand for investment – plays a *causal* role, whereas the supply of finance and financial investment is a *background condition*.<sup>32</sup> For a growth theory it would be more useful to find, on the basis of the empirical evidence, the locus of persistency/normality/long-term and that of volatility/short term of effective demand. A distinction in this direction is found in Nell (1998), where the plans of investments are supposed to be grounded on persistent expectations and beliefs, whereas the locus of the volatile animal spirits is the timing of investment expenditures, because the latter depend on the vagaries of short-term decisions and on the mutable conditions of the credit market. This suggestive claim should be further developed and supported by empirical evidence. In the text-book *vulgata* of Keynes's theory, the *productive* investment is said to govern savings instead of the other way round. Yet, we should refrain from invoking a one-cause growth model, like the demand-led growth models, or, by reaction, a *finance-led growth* models, although it is tempting to choose such a short way.

Secondly, it should be admitted that a certain representation of the leading role of investment, associated with the autonomous decisions of the entrepreneurs, looks rather exaggerated in the present stage of capitalism. It seems to attribute a prevailing role to the desired plans of the heroic figure of a Schumpeterian entrepreneur who would be capable of transforming inventions into innovations; whilst leaving only a passive role to the credit institutions and the financiers with their short term horizon. This vision should be reconsidered in the light of the development of the modern financial institutions that are not simply passive lenders. By extending the metaphor of Joan Robinson, the *spirits* of the corporate financiers should be superimposed to the *animal spirits* of the entrepreneurs in order to explain the growth performances of the modern capitalist economies.

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<sup>30</sup> A similar focus seems to be suggested by the distinction between investment *decisions* and investment *spending* in Nell (1998, ch. 10-11), and in the assumption of a propensity of saving out of total *wealth* (including the financial wealth), instead of income (cfr. footnote 12).

<sup>31</sup> A recent contribution in the direction suggested in the text is offered by Duménil and Lévy (2013).

<sup>32</sup> Cfr (Parrinello 2013<sub>a</sub>, 2013<sub>b</sub>) about causes, background conditions and normal states in economics and, in particular, in macroeconomic models ,

Thirdly, a long term horizon and volatile factors are not a distinctive feature of real investments. Also the long term expectations of the consumers for durables can be volatile, and even the owners of a stock of an exhaustible natural resource decide the flow of its supply in a certain period on the basis of mutable long term expectations. However, important distinctions remain in the conditions that make their plans effective. The consumers who lack financial assets are highly limited by the request of real collaterals if they want to have access to the bank credit. The flow-supply of oil is limited by its stock in the ground but is unconstrained by financial conditions. Instead the investments plans of the entrepreneurs are subjected to a budget constraint that can be soft to different extent according to the institutional setting.<sup>33</sup>

It should be agreed on the premise that to *become effective*, without crowding out effects, a plan of *productive* investment requires not only the existence of spare productive capacity, but also, so to speak, a capacity of spending that derives from the access to lines of credit, in addition to money inventories and self-financing out of retained profits. In the textbook Keynesian theory of the multiplier both (productive and financial) excess capacities are implicit and are supposed to make the multiplier *effective*. In the invoked extension of that theory to the long run and to the theory of growth, the excess expenditure capacity should be made explicit like the excess productive capacity. In particular, it should be explored whether, by analogy to the productive capacity, a corridor of credit distribution exists such that any change in the realization of such distributions does not affect the normal prices and the income distribution including the long term interest rate; whereas outside that range this influence does materialize and affects the investment plans and the path of accumulation. The analogy is not misplaced because both ranges are supposed to depend on cost evaluations, despite the normal range of productive capacity utilization must also satisfy technical constraints.

## APPENDIX

### The simple model of compound accumulation

The equation (section 3)

$$Y_{t+1} - mY_t = v(Y_t - Y_{t-1}) + X_t \quad [2]$$

is a first order, linear non homogenous difference equation, and can be written in the simple form:

$$Y_{t+1} = aY_t - B_t \quad [a]$$

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<sup>33</sup> It might be objected that the softness of the budget constraint of the investors vis-à-vis the consumers is a question of degree, not a sharp distinction, but since a long time we have learnt that differences in quantity beyond a certain point become differences in quality.

$$\text{with } a \equiv \frac{v}{v-(s+m)}; \quad B_t \equiv \frac{X_t}{v-(s+m)}.$$

Given the initial conditions:  $Y_{t=0} = Y_0$ ,  $K_{t=0} = vY_0$ , the level of income in period  $t$  is determined as the sum of two parts which are the general solution to the homogenous equation  $Y_{t+1} = aY_t$  and a particular solution to the non homogenous equation [a]. The national income in period  $t$  can be determined also recursively from the equation [a] with the initial conditions :

$$Y(t) = Y_0 a^t - \sum_{i=0}^{t-1} a^i B_{t-1-i}$$

and the stock of capital in period  $t$  is determined by  $v Y_t = K_t$ .

If exports have a geometric trend,  $X_t = X_0 b^t$ , the general solution to [a] becomes:

$$Y(t) = Y_0 a^t - B_0 b^{t-1} \left( \frac{(a/b)^t - 1}{a/b - 1} \right)$$

where  $B_0 \equiv \frac{X_0}{v-(s+m)}$ , and  $\left( \frac{(a/b)^t - 1}{a/b - 1} \right) = \sum_{i=0}^{t-1} (a/b)^i$ , the sum of a geometric series.

In the simple case of constant exports,  $X_t = X_0$  the general solution:

$$Y(t) = \left( Y_0 - \frac{X_0}{s+m} \right) a^t + \frac{X_0}{s+m} \quad [b]$$

and

$$Y^* = \frac{1}{s+m} X_0$$

is a stationary solution to [b], a state where *net* investment is zero and  $K^* = vY^*$  is the constant capital stock.

### *Conditions for positive growth*

A necessary condition for a positive rate of growth in the closed Harrod's model is:

$$\frac{s}{v} > 0$$

In the open model this the condition for each  $t$  becomes:

$$\frac{s+m-h_t}{v} > 0; \quad h_t \equiv \frac{X_t}{Y_t};$$

and is satisfied by the assumption:

$$Y_t > \frac{1}{s+m} X_t \quad [c]$$

The autonomous exports in real terms must be less than the amount of product not consumed, so that the capital stock is not eroded during the period. In particular this inequality applies in the initial period:

$$Y_0 > \frac{1}{s+m} X_0.$$

It means that growth in the open model requires that the initial level of income is higher than its stationary level determined by the autonomous demand. Furthermore in each period, the conditions for the survival of the economy must be satisfied:

$$K_t > X_t, Y_t > X_t/v \quad [d]$$

In particular in the initial period  $K_0 > X_0, Y_0 > X_0/v$ .

[d] means that the unproductive autonomous demand, even though it is a source of finance of imports, must not “eat the whole cake”, otherwise the economy will cease to exist. It is plausible to assume  $v > s + m$  and therefore condition [c] implies condition [d].

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