Centro di Ricerche e Documentazione "Piero Sraffa"

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Centro Sraffa Working Papers
n. 20

May 2016

ISSN: 2284 -2845
Centro Sraffa working papers
[online]

Nonsubstitution Theorem, Leontief Model, Netputs: Some Clarifications

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Abstract

The nonsubstitution theorem concerns long-period technical choice and relative prices, and was so understood in its first (1951) formulations, but the modern advanced micro textbooks that present it do not make this clear, rendering the theorem impossible to understand for students. These modern presentations derive from a reinterpretation of the Leontief model as a 'timeless' economy in Walrasian equilibrium, capable of positive production in spite of zero initial endowments of all inputs except labour: an unacceptable interpretation, made possible by a use of netputs, to describe the economy's production possibilities, that is illegitimate in this case even from a strictly neoclassical perspective. The notion of a 'timeless' economy disappears from the textbook presentations of the Leontief model and of the nonsubstitution theorem, but the result is that the nature of the model and of the prices to which the theorem refers is not clarified, inevitably leaving students utterly confused. This note remembers the true nonsubstitution theorem, points out that it had been correctly enunciated by Samuelson (1961), and suggests that the current inability to present it in a correct way is due to the absence of the notion of long-period prices from the theoretical horizon of contemporary neoclassical value theory. The paper opens with clarifications on the meaning of the Leontief model which prepare the ground for the discussion of the problem with netputs.

Keywords: Nonsubstitution theorem, Leontief model, long period, netputs

JEL Codes: B21, D24, D57

1. Introduction

The nonsubstitution theorem is frequently mentioned, but students cannot understand it from the way it is presented in the mainstream advanced micro textbooks that discuss it, or in the older treatises such as Arrow and Hahn (1971) or von Weizsäcker (1971). This note[1] tries to correct this unhappy situation. The

[1] That clarifies and corrects the argument in Petri (2004, Appendix 6A3, pp. 246-251).

nonsubstitution theorem concerns *long-period* relative prices, that is, the relative prices a competitive economy tends towards if time is allowed for competition and entry to bring about prices equal to minimum average cost (inclusive of the normal rate of return on capital). In the recent mainstream presentations of the theorem this characteristic is not made clear. The ultimate reason is the disappearance from neoclassical value theory of the notions of long-period equilibrium and long-period prices[2]. This absence has caused misrepresentations of what the nonsubstitution theorem is about. Without aiming at tracing a history of the interpretations of the theorem, the present note clarifies the roots of these misrepresentations by examining a few episodes in this history. It re-examines the first formulations of the theorem (1951), and finds that the theorem *was* initially intended to refer to long-period technical choices, specifically, the ones associated with a zero interest rate; but subsequent presentations lost clarity on the issue, and the theorem was interpreted as referring to the Walrasian equilibrium of a 'timeless' Leontief economy (where the inputs other than labour earn no interest because they are 'flows' and not capital goods), capable of positive production in spite of *zero* initial endowments of all inputs except labour: this description is made possible, not by an assumed 'Austrian' structure of production (that would deprive the theorem of any generality), but by the representation of production processes via *netputs*[3], a representation that will be argued to be unacceptable in this case even from a strictly neoclassical perspective, and generally dangerous for general equilibrium theory itself.

In recent mainstream textbooks the theorem is presented either as referring to the 'Walrasian' general equilibrium of a Leontief economy, or (in a single case: Mas-Colell *et al.*, 1995, pp. 159-160) as referring simply to choice of 'efficient' netputs. In all of them the reference to a 'timeless' economy disappears, probably because of some consciousness of the nebulous nature of such a notion; but the result is that, in the first type of presentation, the implicitly zero rate of interest remains without justification, and what is assumed about the initial factor endowments of this 'Walrasian' equilibrium is left totally unclear, which means a yawning gap in the presentation of the theorem, that students must find it impossible to fill. The second type of presentation too is misleading and incomprehensible for students, because it uses netputs but does not clarify the framework of the notion of 'efficient' netputs. The present note remembers the true nonsubstitution theorem, points out that it had been correctly enunciated by Samuelson in 1961, and suggests that the current inability to present it in a correct way is due to its being alien to the theoretical horizon of contemporary neoclassical value and distribution theory, which has expunged the notion of long-period prices as part of the refusal to admit the origin of the neoclassical approach in versions attempting to determine *long-period* general equilibria. The initial considerations on the meaning of the Leontief model, besides pointing out the independence of the model from any full-employment assumption, prepare the ground for the discussion of the problem with netputs.

[2] For an introduction to this issue see chapter 1 of Petri (2004).
[3] The term, that stands for net outputs, appears in Varian (1978, p. 3).

2. Clarifications on the Leontief model

The nonsubstitution theorem was born in 1951 with reference to Leontief's open model, notoriously describable through the matrix equations

$\mathbf{q} = \mathbf{x} - \mathbf{A}\mathbf{x}$,

$L = \mathbf{a}_L\mathbf{x}$,

where **q** is the given vector of desired yearly net outputs (if the year is the accounting period); **x** is the vector of yearly outputs that must be produced in order that net outputs be **q**; **A** is the given square matrix of commodity technical coefficients a_{ij}, that indicate the amount of commodity i to be used as input (as circulating capital[4]) per unit of output of commodity j, and $\mathbf{a}_L$ is the given (row) vector of labour technical coefficients; L is the quantity of (homogeneous) labour services (e.g. labour hours) that must be employed to produce **x**, in general different from labour supply. Some observations on the nature of this model will dispel possible mistaken interpretations of what it indicates and what is implied by using it to determine how **x** and L change if **q** or **A** or $\mathbf{a}_L$ change.

First of all, the reader accustomed to assume, as frequently done in the literature on value and distribution, a yearly (or more generally a one-period) production cycle, with production processes started at the beginning of the year and all products coming out at the end of the year, must make an effort to abandon this assumption. The Leontief model is perfectly compatible with this case, but it is more general. Nothing in the definitions of the vector of total yearly outputs **x** or of the vector of yearly net products **q** obliges one to assume that the length of the *production process* that produces one unit of good j with inputs $a_{1j},...,a_{nj}$ is the same as the length of the *accounting period* (e.g. the year) used to calculate **x** or **q**. Outputs can be coming out all along the accounting period; for some commodities even hundreds of production cycles may be performed during the accounting period. This is no impediment to determining **x** as the vector of the total quantities produced during the year, nor to determining **q** as **x** minus the produced inputs used up in order to produce **x**. For example if for all commodities the production cycle takes one month, and in a year twelve production cycles of identical dimension are carried through, **x** will be twelve times the vector of the amounts produced each month, and **q** will be twelve times the net output of each month. The main difference relative to an assumption of yearly production cycles is the following: with a yearly production cycle and all output coming out at the end of the year, the production of one cycle/year is only physically available at the end of the year, so it is not available for use as input, or for consumption, during the year: the goods used as inputs or for consumption during the year must necessarily be goods already available at the beginning of the year; if we admit that output is coming out *during* the entire year, then production and consumption during the year can and will often utilize commodities produced *during* that same year. Thus **Ax** may well for the most part consist of goods produced during the year, i.e. goods that also appear in **x**. The same is true for

[4] A capital good is circulating capital if it disappears in a single utilization. Examples are petrol used for travel services, raw materials transformed into a final product, corn seed used to produce corn.

consumption: when we assume a yearly production cycle with all output coming out at the end of the year, the goods consumed during the year have to be already available at the beginning of the year; now that we admit production during the year, consumption can utilize that production. (All this may seem trivial, but there is a reason why I insist on it, that will soon become clear.)

This difference can also be visualized in terms of what goods one finds in inventories at the beginning and at the end of the year. At the end of the year, the economy with yearly production cycles (all started at the beginning of the year) finds in its inventories the entire production of that year, that is $\mathbf{x}$, not $\mathbf{x}$-$\mathbf{Ax}$; and in order to have produced $\mathbf{x}$, it had to have at least $\mathbf{Ax}$ in its inventories at the beginning of the year[5]. If we admit many production cycles during the year, the economy may need much smaller inventories than $\mathbf{Ax}$ at the beginning of the year, and may find itself with much smaller inventories of produced goods than $\mathbf{x}$ at the end of the year, because much of $\mathbf{x}$ disappears during the year, being used not only as input for further production but also for consumption. For example, let us assume that all goods are produced in short production cycles, a year encompassing 100 of them, and that all production cycles during the given year produce identical quantities. Then each cycle produces the hundredth part of $\mathbf{x}$, and uses the hundredth part of $\mathbf{Ax}$ as inputs. In order to produce $\mathbf{q}=\mathbf{x}-\mathbf{Ax}$ as net product during the year, the economy need have at its disposal as means of production at the beginning of the year only the hundredth part of $\mathbf{Ax}$; the production cycles after the first one can use the products of the preceding cycle.

Thus the shorter the production cycles of the several products, the smaller the amount of goods necessary at the beginning of the year to realize a given $\mathbf{q}$ during the year; this amount may even be so small, relative to $\mathbf{x}$ and $\mathbf{q}$, as to allow considering it negligible *for purposes of statistical computations*. However (here we get to the important point), since inputs must be available some time before the output comes out, some positive inventories of inputs must always be available at the beginning of the year in order for $\mathbf{q}$ to be produced, and this will be argued later to be *theoretically* extremely important. The need for some initial inventories would disappear only if $\mathbf{q}$ were produced directly or indirectly by labour alone according to an ‘Austrian’, or ‘Smithian’, structure of production, and, each period, production started with labour alone, the initial intermediate products being then used, together with additional labour, to produce further intermediate products up to the coming out of the final, net products within the period. But apart from this case—and real economies are not like this—the inventories of circulating capital goods at the beginning of the year can never be all zero. There *must* be not only labour but also some goods available as inputs at the beginning of the year in order for production to start, and their amounts imply

[5] I say ‘at least $\mathbf{Ax}$’, in order to allow for the (totally unrealistic but logically admissible) possibility that there is no consumption during the year except the last day, all consumption being effected out of the harvest the day itself of the harvest (which is the last day of the year), so that no inventories of consumption goods are needed at the beginning of the year. More realistically, at least $\mathbf{z}\mathbf{a}_L\mathbf{x}$ must also be available at the beginning of the year, where $\mathbf{z}$ is the vector of average minimal (‘subsistence’) consumption per labour unit.

constraints on producible quantities additional to the constraint possibly due to labour availability.

Therefore, if one changes net outputs ('final demands', in the terminology of Leontief input-output tables) and uses the model to determine the new activity levels implied by them and argues that these activity levels are what the economy will converge to, then one is assuming not only that labour employment will adapt[6], but also that some adjustment process will ensure that there will be the initial availabilities of goods, needed to start production at those new levels. Unless one assumes excess initial inventories, there must be an adaptation of initial endowments. The 'appropriate' initial endowments of goods are not made clear by the model, and depend on how many 'rounds' of production are contained in one year (as shown by the above example of 100 rounds), but nonetheless they exist; the model implicitly assumes that there is some process that adjusts them, so *they are endogenously determined*. This means that the model, when used for comparative statics, compares 'adjusted' situations in which initially available produced inputs, and industry dimensions, have had time to adapt to demands, similarly to what is assumed with comparative statics of long-period positions[7].

Note finally that the Leontief model implies nothing as to the degree of resource utilization. That the production of **x** requires the utilization of amounts **Ax** of circulating capital goods does not pre-suppose the full employment of labour, nor the full utilization of inventories: it only tells us that in order to produce **x**, quantities **Ax** must be used up, therefore *if* in the subsequent period the economy wants to produce again **x** without running down inventories, then it must reconstitute the used-up goods by dedicating a portion **Ax** of outputs **x** to this task, and therefore it can only consume **x-Ax** during the period. The tendency in most mainstream presentations to consider the Leontief model as describing a full-employment economy is totally unjustified; the model will describe a full-employment situation only if this is specifically additionally assumed, nothing in the model implies it.

3. The nonsubstitution theorem

Leontief's *empirical* input-output tables are in value, each sector produces many different goods, and there is no assumption that all firms producing a commodity use the same production method. But Leontief's *model* specifies production methods in technical terms, assumes that each sector produces only one good, and does assume that in each sector only one method is utilized; so for an economy described by this model

[6] Unless the change in net outputs is chosen such as to leave labour employment unaltered.

[7] A long-period position, or 'normal position' as P. Garegnani prefers to call it (see Garegnani, 2007, especially pp. 226-231), is additionally characterized by long-period prices, the ones associated with the dominant technique corresponding to the income distribution variable (the rate of return on capital, or the real wage) taken as given. Leontief's model need not assume that prices are long-period prices; however, as we will see, this is the assumption made in the discussions of the nonsubstitution theorem.

one can ask, why the adopted production methods are ($\mathbf{A},\mathbf{a}_L$) when other methods could be utilized.

We have seen that the Leontief model assumes that the quantities of inputs adjust to the levels required to produce the given net outputs. The admission of a possibility of technical choice implies that the adjustment must concern not only the level of activity but also the kind of inputs required in each industry. Then the prevalence of a single method in each industry must result from a complete adaptation of all inputs in all firms in the industry to cost minimization, hence to what in Marshallian terminology is the tendency to adopt the optimal long-period production method; this tendency will be operating simultaneously in the several industries, and it may entail multiple changes of method in an industry as the relative input costs of that industry change because of changes in the methods and hence in the output prices of other industries. The final result of cost minimization is indicated by the theory of long-period choice of technique (e.g. Kurz and Salvadori, 1995, ch. 5), that allows us to conclude that methods ($\mathbf{A},\mathbf{a}_L$) belong to the technique which, at the given rate of profits or of interest r, maximizes the real wage (or at the given real wage maximizes r), that is, the technique which for that rate of profits is on the outer envelope of the $w(r)$ curves corresponding to the alternative techniques available to the economy[8]. Since—owing to the assumptions of constant returns to scale (CRS), no joint production (only circulating capital), and only one paid 'primary' factor (labour)—the $w(r)$ curves only depend on technical coefficients and not on the quantities produced, we conclude that if the quantities demanded change and sufficient time is allowed for the quantities of 'intermediate' goods to adapt to the changed demands, then as long as income distribution does not change the technical coefficients will, after the transition, return to being ($\mathbf{A},\mathbf{a}_L$).

The conclusion just reached is the nonsubstitution theorem. More precisely:

Nonsubstitution Theorem[9]. *Assume an economy where*

(i) there exists only one primary factor, labour; all other inputs are produced goods i.e. capital goods, and their amounts adapt to the demand for them;

(ii) all processes of production are perfectly divisible with CRS, and have the same production period (which is taken as the time unit);

(iii) each process produces one perfectly divisible commodity (no joint production), with fixed coefficients of capital goods and of labour, at least some of which are positive;

[8] A technique is a set of production methods, one per industry. The notion of outer envelope of the w(r) curves, or wage-profit frontier, is well known, see e.g. Sraffa (1960, p. 85), or Kurz and Salvadori (1995, p. 148); it indicates the maximum long-period real wage associated with each level of the rate of profit (or of interest, if by neglecting risk etc. one identifies the two rates). Assuming production processes all of equal length (one period), and having chosen a numéraire, for each ($\mathbf{A},\mathbf{a}_L$) technique one derives the corresponding $w(r)$ curve from the system of equations $(1+r)\mathbf{pA}+w\mathbf{a}_L=\mathbf{p}$; the theory of long-period choice of techniques proves that cost minimization will finally bring firms to adopt the technique whose $w(r)$ curve is the outermost for the given r (or for the given w, if it is the real wage that is taken as given).

[9] See Salvadori (1987), integrated by Kurz and Salvadori (1994) for condition (viii), which concerns an extreme case of little relevance: see Kurz and Salvadori (1995, p. 155, Exercise 8.7) for a numerical example that illustrates this curious case.

(iv) for each commodity there exists at least one process producing it;

(v) each commodity requires labour for its production, either directly or indirectly;

(vi) the price of capital goods is the same at the beginning and at the end of each production cycle;

(vii) the price of each produced commodity equals the costs of the inputs plus a uniform and given *rate of interest (rate of profit) on that part of that cost which is paid in advance (i.e. at the beginning of the production cycle);*

(viii) the rate of interest (rate of profit) is less than the maximum one corresponding to a zero real wage, or, if there exists a maximum rate of interest and the technique yielding this maximum rate of interest is not unique, there exists a commodity that is basic in all the alternative techniques that are equally profitable at the maximum rate of interest.

Then for each admissible value of the rate of interest, cost minimization implies a unique vector of long-period relative prices of products and a unique wage rate (once a numéraire is chosen), independent of the composition of final demand. At those prices and wage rate, either the process chosen in each industry is unique, or the industry is indifferent among alternative processes which yield the same (interest-inclusive) unit cost.

This theorem concerns the nature of *long-period* choice of technique when there is no joint production and no scarce natural resources[10]; it assumes nothing as to labour employment, which may well be far from the full employment level. But nowadays the result is presented in advanced microeconomics textbooks in a way that totally obscures the situations to which it refers. Some aspects of the history of the theorem help to understand the roots of this situation.

4. The 1951 nonsubstitution theorem

The first version of the theorem is generally considered to be the one advanced in the 1951 papers by Paul Samuelson and Nicholas Georgescu-Roegen in a volume (Koopmans 1951a) concerned with a *normative* problem of 'efficiency' in choice of

[10] Long-period competitive analysis assumes that in all industries the Marshallian long-period adjustment of supply to demand has been completed, so that for all produced goods price equals minimum average cost (inclusive of the normal rate of return, or of interest, or of profits, on the capital advanced). This adjustment includes variation of number and type of plants in each industry, a process taking considerable time, therefore entailing an endogenous determination of the quantities of the several capital goods present in the economy. Once this adjustment is completed, the changes that relative prices may be undergoing over time can be assumed to be either so slow as to be negligible (hence (vi) in the statement of the theorem), or to be once-for-all changes (e.g. due to technological innovations) to be analysed through the method of comparative statics (Petri, 2004, pp. 21, 25, 35-38). The non-substitution theorem can be extended to include nontransferable durable capital goods, and even land as long as the changes in the composition or level of production do not alter the no-rent land; but these extensions are unnecessary for the argument of this paper.

technique, within the framework of the open Leontief model with choice among alternative production methods. The problem was, what choice of production methods will maximize a hypothetical planner's monotonic utility function *defined on the net outputs per period of the economy,* if the given supply of labour is considered the sole constraint on producible quantities (and therefore – but this was not made clear in the volume – one accepts an endogenous determination of the quantities of produced inputs available for production, including their quantities available at the beginning of the period). Samuelson's brief paper – commented upon in the same volume by Tjalling Koopmans (1951c) and Kenneth Arrow (1951) – answers the question indirectly, by arguing that the Leontief model need not be interpreted as assuming that for each good only one fixed-coefficients method of production is known; the observed methods can be interpreted as the optimal ones, because resulting from competition-induced choice of technique, and are independent of demand: "With labor the only primary factor, *all desirable substitutions have already been made by the competitive market*, and no variation in the composition of final output or in the total quantity of labor will give rise to price change or substitution" (Samuelson, 1951, p. 143, italics in the text).

To understand how Samuelson's argument answered Koopmans's problem, it is important to note that the normative problem studied in the 1951 volume is characterized by Koopmans (1951b, p. 42) as including the assumption of "a state of saturation with regard to reproducible capital": this implies an endogenous determination of the vector of capital goods per unit of labour[11], hence a long-period framework; and, given the dominance at the time of neoclassical theory and of the conception of the several capital goods as embodying quantities of the single factor 'capital', Koopmans clearly means that the amount of 'capital' (as well as of each capital good) is assumed to be so abundant that its (net) marginal product is zero, which implies a zero equilibrium rate of interest. Georgescu-Roegen (1951, pp. 166, 171) makes the long-period framework explicit by using the term "competitive long-run equilibrium" for the situation assumed by Samuelson and himself; neither he nor Samuelson explicitly say that the rate of interest is zero, but the thing is implicit in Georgescu-Roegen's observation (*ibid.*, pp. 172-3) that prices are equal to direct and indirect wages (we might say, 'wages embodied'); as to Samuelson, he will explicitly admit in 1961 (see below) that in 1951 he was assuming a zero rate of interest.

Then the connection between the Samuelson–Georgescu-Roegen's theorem and the above normative problem is easily grasped, on the basis of what is known about *w(r)* curves. It is known that, when the rate of interest (rate of profit) is zero, long-period competitive choice of technique selects the technique whose *w(r)* curve has the

[11] Koopmans implicitly admits that production needs initial inventories of the circulating capital goods and that these are endogenously determined as the ones required to maximize production per unit of labour, by continuing: "Among the limitations ... on primary resources we have not imposed any limitations on the amount of accumulated products of past flows of primary resources used to increase the productivity of present flows" (1951b, p. 42). However, neither Koopmans nor the other contributions in the 1951 volume explain why one should be interested in efficiency of production under such an assumption.

highest vertical intercept[12]: for each chosen numéraire, the vertical intercept of the *w(r)* curve generated by a technique indicates that technique's net product of that numéraire per unit of labour employment[13]; since which *w(r)* curve yields the highest *w* for a given *r* is independent of the numéraire, the same technique has the highest vertical intercept whichever the numéraire, hence this technique is the one that maximizes *all* net products per unit of labour. (The result can also be expressed by saying that the same technique minimizes all labours embodied, because the labour embodied in a commodity is the total labour employment when that commodity is produced as net product, and the technique that produces the greatest net output of a commodity per unit of labour is also the technique that minimizes labour employment per unit of net output of that commodity.) Samuelson and Georgescu-Roegen discover this result (with different tools), and in this way they also find the solution to the planner's 'efficiency' (or utility maximization) problem, because whatever the desired composition of net output, the optimal technique will be the same, the one associated with the maximization of the net output per unit of labour of whatever basket of goods.

In order to understand the way the nonsubstitution theorem is presented in Mas-Colell *et al.* (1995), to be discussed in Section 13, it is useful to add that, in their generalizations of Samuelson's proof, Koopmans (1951c) and Arrow (1951) concentrate on the planner's 'efficient' choice of technique as purely a maximization problem, *without* arguing that the 'efficient' choice is brought about by market competition, and therefore without mentioning prices or the interest rate. In this way they abstain from evaluating Samuelson's debatable implicit claim that Leontief's analysis referred to an economy in long-period full-employment equilibrium with a zero rate of interest[14].

5. The DOSSO presentation

So the 1951 nonsubstitution theorem, when referred to the results of competitive markets, concerns *long-period* choice of technique when the rate of interest is zero because there is capital saturation, and even when formulated as a pure maximization problem it still implicitly assumes a long-period framework, with the endowments of circulating capital goods endogenously determined so as to maximize net output per unit of labour whatever the composition of net output. But these characteristics are thoroughly obscured in the 1958 book by DOSSO (the usual shorthand for the authors Robert Dorfman, Paul Samuelson and Robert Solow), *Linear Programming and*

[12] I leave aside the possible fluke of two or more techniques having *w(r)* curves with the same vertical intercept.

[13] The vertical intercept of a *w(r)* curve indicates the real wage in terms of the chosen numéraire when r=0 and therefore all the net product goes to labour, hence it indicates the net product per unit of labour if the net product consists of the numéraire good (or basket of goods).

[14] The full employment of labour is implied by the assumption of a 'saturation' of capital that justifies the zero interest rate. Leontief (1951) clearly states that he is assuming long-period prices (e.g. p. 36, where the economy is assumed stationary), but he is also clear that the costs determining these prices include "capital and entrepreneurial services", that is, interest, and entrepreneurial profits (e.g. p. 24).

Economic Analysis, that paves the way to subsequent misinterpretations. In this book, there is no reference to capital saturation as the explanation why there is no interest rate in the competitive equilibrium of the Leontief economy; the suggestion is rather that it must be so in a Walrasian equilibrium if labour is the sole primary input (p. 204: "the theory of input-output ... provides the simplest form of Walrasian general equilibrium"; p. 225: "there is only one thing to be economized, labor"). But this immediately creates a problem, because a Walrasian (or neo-Walrasian[15]) general equilibrium assumes *given* initial endowments of all physically specified factors of production including all capital goods, even circulating capital goods; for example if labour and corn seed produce corn, the equilibrium's data must include given endowments of labour and of corn seed; how can this apply to Leontief's open model, where there are no given initial endowments of the several circulating capital goods? No explicit answer is supplied in the book; the implicit answer seems to lie in the characterization of Leontief's model as a black box in which only labour services enter, and only net outputs come out:

> "The interindustrial sales have no "welfare" significance at all. Social benefits come from final consumption, and social costs come from the use of labor. The economy can be viewed as a machine that uses up labor (and has 50 units of labor per year at its disposal) and produces final consumption. ... Part of our problem will be to calculate what *other* menus of final consumption society could produce with its 50 units of labor and its present technology." (DOSSO 1958, p. 207).

No admission appears of the need, for the "machine" to work, of endowments (continually renewed, but still endowments, that must be present at the beginning of each production cycle) of the several 'intermediate' goods (circulating capital goods) appropriate to the activity levels of the various sectors, nor of the need for a change of these endowments if the "menu of final consumption" changes. The justification appears to lie in a peculiar interpretation of Leontief's static model of 'flows'. In a striking footnote, Leontief's "statical" model is contrasted with "a dynamic model in which production takes time": in the latter model "the stocks of coal to be used in coal mining must be available before any new coal can be produced" (p. 205, fn. 2). This implies that in Leontief's static model production takes no time and there is no need for the coal to be used in coal mining to be available *before* coal production comes out – nor, then, for corn seed to be available months before the corn harvest! This is difficult to make sense of, and it is not what Leontief assumes. When he comes to the introduction of dynamic elements, Leontief admits (1951, p. 211) that his analysis has been "static" up to then, with changes analyzed only through comparison of static positions, that is, with no attention to the actual transition dynamics; the analysis of the

[15] On why the intertemporal and the temporary equilibria of Lindahl, Hicks, Debreu are very different from Walras's own model and therefore should be called neo-Walrasian rather than Walrasian, see Garegnani (1990), or Petri (2004, ch. 5).

latter dynamics, he goes on to note, requires the explicit consideration of inventories and of stocks of durable capital goods, because a change in the rate of output of an industry will generally require a change in the dimension of the inventories of raw materials etc. held by the industry, and therefore a process of accumulation or decumulation of inventories, plus a process of variation of productive capacity (durable capital goods, buildings etc.), both neglected in the comparative statics of flows to which his analysis has limited itself up to that point. Thus Leontief's distinction between his analysis up to then, based on flows only, and the analysis of dynamic transitions that has to include consideration of inventories and of stocks of durable capital goods, in no way implies that the analysis based only on flows was 'timeless', it was simply static, that is, neglecting changes in the data, because assuming a constancy of already adjusted flows. DOSSO on the contrary write on p. 249 of "a simple Leontief system, which abstracts from time", and on p. 266 write that "the introduction of a time dimension and stocks of capital" is what distinguishes a dynamic model from a "statical" one". Thus, one must infer, as long as the analysis is "statical" and the only inputs (apart from labour services) are "flows of raw materials or current inputs" (p. 284), one can assume that there is no "time dimension" and this apparently justifies—but no explanation is supplied as to why—the treatment of these flows as unproblematically adjusting to the needs of production, posing no constraint to producible quantities additional to the constraint due to the limited availability of labour.

The same idea appears later in the book (p. 355) in the description of how intermediate goods can be introduced into the Walras-Cassel general equilibrium model, which is classified as "statical" (p. 204). The thing can be concisely explained by using vectors and matrices. Assume m "resources or factors of production" (p. 351) in fixed supply $r_1,...,r_i,...,r_m$, and n produced commodities with total outputs $\mathbf{x}=(x_1,...,x_j,...,x_n)^T$ and final demands $\mathbf{y}=(y_1,...,y_n)^T$. Production of the commodities requires *both* the m resources according to fixed technical coefficients a_{ij} *and* the commodities themselves as intermediate inputs according to fixed coefficients that we can indicate as b_{ij}, that form a Leontief matrix $\mathbf{B}$, with Leontief inverse $(\mathbf{I}–\mathbf{B})^{-1}$ that DOSSO indicate as $\mathbf{A}$, with elements A_{jk}. The total demand for outputs implied by final demands $\mathbf{y}$ is determined as $\mathbf{x}=\mathbf{A}\mathbf{y}$; the conditions of equality between supply and demand for resources are accordingly written (p. 355) as $r_i=\Sigma_j a_{ij}x_j= \Sigma_j a_{ij}(\Sigma_k A_{jk}y_k)$, i=1,...,m. The production of coal requires not only labour and land but also coal, and the production of corn requires corn seed, but this poses no constraint, and the data of equilibrium do *not* include given initial endowments of these circulating capital goods. (See sections 9 and 10 below for criticism of these ideas.)

Thus, there is definitely in DOSSO a blindness to the fact that circulating capital goods, 'intermediate' goods, are capital goods too[16] and that a Walrasian equilibrium

[16] The tendency to exclude circulating capital (intermediate goods) from capital, and to consider capital as including only durable capital goods, is frequent in neoclassical analyses, and is confirmed for example by a recent statement by Duncan Foley who writes of "the view, shared by Ricardo, that the advance of

would have to include, among its data, some given initial endowments of them. But the notion of a 'timeless' economy is anyway needed to circumvent the problem of how the flows of intermediate goods adjust to production needs. The same notion is also helpful to explain why corn seed has the same price as the corn it produces. In the DOSSO book the notion of "long-run competitive equilibrium" (p. 352; see also p. 207) is identified with price equal to average cost, but not with the result of time-consuming adjustments, that would entail an endogenous composition of capital. The introduction of time is reconciled with the idea of equilibrium only through the notion of intertemporal equilibrium (see especially pp. 317-321), where input price generally differs from output price for the same good, and where there is no room for time-consuming adjustments, an idea totally absent from the book, which is, for this aspect, fully neo-Walrasian. The conception of the economy as "timeless", producing *without* "a time dimension", is then the *deus ex machina* that justifies the absence of dated quantities and discounted prices and the absence of an interest rate, and somehow reconciles the need to admit an adaptation of the amounts of intermediate goods to the needs of production, with the absence of a notion of equilibrium as a centre of gravitation of time-consuming trial-and-error adjustments.

6. Samuelson's 1961 nonsubstitution theorem

The notion of a 'timeless' economy reappears even more explicitly in a subsequent article by Samuelson, which starts precisely with a "model in which all effects of time are nonexistent or ignorable" (Samuelson 1961, p. 407). The article makes no attempt further to explain this notion or to argue its logical conceivability[17]; only toward the end of the article it becomes clear that the role of this notion is essentially that of justifying the absence of a positive interest rate; then, if labour is the sole "primary" factor, prices are proportional to labours embodied (that is, equal to wages embodied, if the wage is the numéraire).

But after a couple more pages of considerations premissed by "Still ignoring all time relations", Samuelson admits that "we must come to grips with the problem of time" (1961, p. 410) and then, implicitly recognizing the correctness of Sraffa's price equations (he has now read Sraffa's *Production of Commodities by means of Commodities* and he writes, p. 412, that "some of us have been Sraffian without realizing it"), he admits a positive rate of interest, and on p. 415 he formulates the

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wages are a part of capital, in contrast with neoclassical production functions, which include only the value of fixed capital in measuring capital input" (Foley, 2004, p. 4 fn. 2), forgetting that corn seed, for example, is not fixed capital and yet no doubt is part of capital advances and would have to be considered as one of the inputs by any neoclassical production function describing corn production.

[17] Thus, for example, there is no attempt to explain how one can conceive of production as using labour services if all effects of time are "nonexistent or ignorable": doesn't the measurement of labour services inevitably require considering for how long labour has been exercised? So, it is perhaps not by chance that later in the 1961 article Samuelson admits another (and less fairy-tale) possible justification of a zero interest rate (see here the last lines of Section 6, and Section 7).

nonsubstitution theorem in a form essentially equivalent to the one given in Section 3 above, that is, as referring to “long-run conditions” and as stating that (in the absence of joint production and of scarce natural resources), if the rate of interest is given, then choice of technique, relative prices and real wage are univocally determined independently of the composition of the demand for net outputs. Samuelson then characterizes the 1951 theorem as applying to “that special case where the stipulated interest rate is zero, as in a timeless system or a ‘capital-saturated’ system” (1961, p. 418).

7. Endogenously determined capital endowments

Thus, if one leaves aside the nebulous notion of a “timeless system”[18], there is little to criticize in the presentation of the nonsubstitution theorem in Samuelson’s 1961 article; in particular, the long-period nature of the nonsubstitution result is explicitly admitted, as well as its applicability to economies where the rate of interest is positive. Furthermore, the expression “or a ‘capital-saturated’ system”—a clear allusion to Koopmans’ (1951b, p. 42) “state of saturation with regard to reproducible capital”—, although jumping out of the blue and not further explained, admits that there is no need to assume the mysterious “timeless system” in order to justify a zero interest rate: one may well refer to an economy in time, it suffices (if one is a neoclassical economist) to admit an endogenous determination of capital goods that renders the net marginal product of capital zero; and the allusion to Koopmans amounts to admitting that this, and not the ‘timeless economy’, was the framework of the 1951 theorem. By implication, Samuelson admits that a given rate of interest, be it zero or positive, necessarily implies *endogenously determined* capital goods per unit of labour, differently from the case with neo-Walrasian equilibria that must take the vector of initial capital endowments (even the circulating ones) as given.

But, strikingly, nothing of all this reappears in subsequent neoclassical discussions of the nonsubstitution theorem. The subsequent mainstream presentations of the nonsubstitution theorem (at least, the ones of which I am aware) present only the 1951 version, and with no recognition of its long-period framework nor of its implicit assumption of a zero rate of interest; rather, fully accepting the suggestion of the DOSSO book (and of the first pages of Samuelson (1961), which however is never mentioned), the presentations of the theorem that characterize it as describing the competitive equilibrium of the Leontief model qualify the latter model as ‘timeless’, and

[18] After the present paper had been accepted as a Centro Sraffa Working Paper, Prof. Christian Gehrke kindly informed me that the 1961 article started a correspondence between Sraffa and Samuelson (to be soon published as part of the forthcoming volumes on Sraffa’s unpublished manuscripts), and he let me have access to it. A discussion of this correspondence here would not only be premature, but it would also go beyond my competences and beyond the purpose of this note; anyway it seems not to affect my arguments, and to confirm my evaluation of the notion of a ‘timeless system’ as nebulous or worse: Sraffa criticizes it, and Samuelson finds it difficult to defend it.

the equilibrium as Walrasian. This is made possible by a treatment of labour as the only factor endowment in the equilibrium of the Leontief economy; this aspect, which in the DOSSO book could only be indirectly inferred, is now explicit, and formally justified in a way that deserves careful discussion and, it will be concluded, must be rejected.

8. Arrow and Hahn

The thing emerges most clearly from the presentation of the 1951 nonsubstitution result in the treatise by Arrow and Hahn (1971, pp. 40-46). The result, presented without naming it 'nonsubstitution theorem', is described as applying to the temporary or short-period general equilibrium (that is, equilibrium only for the 'current period') of a "Leontief economy" defined by constant returns to scale, no joint production, no durable production goods, only one non-produced good (labour services), and, importantly, households who are net suppliers of no producible good, that is, who *supply only labour services to firms* (Assumption 14, p. 44).

Now, the authors have earlier stipulated (p. 18): "If there are any quantities of goods available in the economy before there is any production or market exchange, then we shall take it that these goods are owned by households." Then Assumption 14 means that even if there are some initial endowments of produced goods, these are in the hands of households and are not offered to firms i.e. are directly consumed by households, so firms do not own any initial endowment of produced goods nor do they receive produced goods from households[19]. So the authors are assuming that *in this economy it is possible for firms to produce without holding, or obtaining from households, any initial endowment of produced goods*. How can this be? As noted above (Section 2), nonzero initial endowments of some goods are indispensable for production to start[20] (unless production processes are 'Austrian', but this is not what is assumed).

The trick lies in how the production possibilities of the entire economy are described. It is assumed that "the production process of each firm can be completed in the current period" (p. 36), goods used as inputs and goods produced in the period are simply both goods *of* the current period, so inputs and outputs can be netted out, and the *netputs*[21] that represent the production processes available to each industry, when summed, one per industry, to obtain the production processes available to the entire economy, can yield vectors positive in *all* elements referring to produced goods (that is,

[19] So the authors might as well have directly assumed zero initial endowments of produced goods. And indeed the authors comment on Assumption 14 as follows (1971, p. 47): "This assumption has no immediate appeal unless it is argued that we may take households not to hold any quantities of the producible goods".

[20] Georgescu-Roegen explicitly admits it by stating that "the current production undeniably requires some preexisting stock" (1951, p. 100). For this reason he dislikes netputs and prefers Von Neumann's representation of a production method as a vector of inputs and a vector of outputs.

[21] Net output vectors, with negative elements indicating net demands for inputs and positive elements indicating positive net supplies of outputs.

in all elements except the one corresponding to the input of labour services). The thing requires detailed comment.

9. Netputs

In modern formalized general equilibrium theory, the universally adopted formalization of a production process is precisely as a vector of ***netputs***, a vector with N elements if the economy has N goods and services, where negative numbers indicate *net* inputs and positive numbers indicate *net* outputs of goods. The production possibility set of the economy is the set of all netputs available to each firm, or to several firms combined (the sums of one netput from each firm), or even to *all* firms combined (that is, to the entire economy). Note that the notion of netput implies that each element of a netput indicates either a (net) input or a (net) output. This creates no problem if production does not use capital goods, because then inputs are services of 'primary' factors (labour and lands) while outputs are consumption goods (then the qualification 'net' is superfluous). But netputs are also used to describe the available production methods in economies that use and produce capital goods. Then cases like corn used as seed to produce corn can be accommodated in the netput representation by distinguishing goods according to their date (i.e. instant) of availability, so that corn seed at date/instant t is a different good from corn harvested at date/instant t+1. In this case to talk of *net* outputs is useful for situations like the following: consider a production plan including an output of 100 units of a circulating capital good at date t, and also the re-utilization of 80 of those units at date t to obtain other products at date t+1; one says then that the planned netput of that capital good at time t is 20 (the other 80 units, 'proper' intermediate products, being re-utilized within the same firm, need not be rendered explicit); it's what the firm can sell of that date-t capital good to others, given its intended outputs for date t+1. Netputs are convenient here because if **y** is a netput and **p** the vector of discounted prices of the goods in **y**, the inner product **p·y** yields the discounted (neoclassical) profit from adopting that production plan, with negative netput entries (amounts of inputs) contributing to (discounted) cost and positive netput entries (amounts of outputs) contributing to (discounted) revenue.

But troubles arise when one nets out the total use of each good as input *during an entire period* from the total production of the same good during the same period, and one treats the resulting positive or negative net outputs as what the ensemble of firms supplies to, or demands from, the rest of the economic system in that period. Let us consider the example from p. 64 of Arrow and Hahn (1971): the economy has 2 products (e.g. corn and iron) that are consumption goods and also circulating capital goods; in one period, industry A produces 2 units of commodity 2 (iron) using 1 unit of commodity 1 (corn), and industry B produces 2 units of corn using 1 unit of iron. This means that the entire economy's netput is (1,1); that is, since the production possibilities set of the economy is *defined* to consist of all netputs obtainable by addition of those available to the different firms, one concludes that in order to be capable of producing (a net output of) one unit of each good, this economy ***needs no input at all***.[22] But this would mean the possibility of infinite production. So let us introduce labour as good 3, and assume that both the above production processes also need one unit of labour. Then the netputs of the two industries are (-1, 2, -1) and (2, -1, -1), and the economy-wide netput is (1,1,-2): in order to produce one unit of each good the firms' sector needs only to obtain 2 units of labour services from the household sector. Labour availability is then the constraint, and the sole constraint, on production possibilities.

Now, there is nothing wrong in this representation of production possibilities if one is interested in possible net outputs per unit of labour per period in stationary situations in each one of which the quantities of intermediate goods (circulating capital) are assumed adapted to requirements. But this representation is of no help for the determination of a supply-and-demand equilibrium. The aggregate production process resulting from the sum of the processes of the several firms is needed to determine the demands for inputs and the supplies of products of the aggregate firm sector, to be then confronted with the supplies of inputs to this sector, and the demands for its products, so as to determine whether the economy is in equilibrium; the netput vector (1,1,–2) does *not* represent these demands and supplies of the firm sector, it only tells us the *net* productions of the period (according to the usual definition of net products adopted in national accounting); but these can result from operations at different moments; equilibrium requires equality of supplies and demands at each one of these different moments, and netputs do not supply the data necessary for establishing it.

Suppose for example that both production processes take the entire current period, are started at the beginning of the period, and are completed at the end; this means that the firm sector demands, besides labour, 1 unit of each good as inputs at the beginning of the period, and supplies 2 units of each good at the end of the period; there will be equilibrium if there is the corresponding supply of inputs at the beginning of the period, and the corresponding demand for the products at the end of the period. On the contrary, the netput representation makes it look as if equilibrium required only a household supply of 2 units of labour, and a household demand for 1 unit of each product: which would be the case only if the aggregate production process were 'Austrian', started by labour alone, internally producing and re-utilizing one unit of each good, and ending up with final outputs of 1 unit of each good. And yet, this netput representation is taken by Arrow and Hahn correctly to represent input demands and output supplies of the firm sector: they define the economy-wide excess demand vector as the vector of households' demands minus the vector of households' endowments plus the firm sector's aggregate *netput* vector (see the definition of **z** on p. 37). So, for the economy of the above example, there is no doubt that Arrow and Hahn would consider equilibrium to require only an aggregate household supply of 2 units of labour, and an

[22] Arrow and Hahn avoid mentioning the possibility that production of a good may use only that same good as input (corn produced by corn, a famous assumption), in which case already at the level of a single industry one would obtain an entirely positive netput – production out of nothing!

aggregate household demand for 1 unit of each product.

It may be useful to stress that the determination of the beginning-of-period demand for corn and iron as inputs need not be one unit of each, because the assumption need not be made – although it makes life so much simpler – that within the unit time period only one production cycle takes place. Suppose that the two productions of our example result from the repetition 100 times, within the period, of a two-stage production process consisting of a first stage in which industry B produces 0.02 units of corn with 0.01 units of iron and of labour, immediately followed by a second stage in which industry A produces 0.02 units of iron with 0.01 units of corn and of labour. Then the first production process needs only a positive initial endowment of 0.01 units of iron: small, *but positive*. An initial endowment of some produced input besides labour is ineliminable. On the contrary, according to Arrow and Hahn, this economy *does not need initial endowments of corn or iron* to carry out those productions; it only needs the two units of labour.

10. Arrow and Hahn's defence

Arrow and Hahn try to justify this thesis by assuming "that production and all other economic activity is timeless; inputs and outputs are contemporaneous" (p. 53)[23]. The meaning of the nebulous adjective 'timeless' is not made any clearer by being accompanied by the notion of 'contemporaneous' inputs and outputs, a notion that implies the flow of time, but is itself unclear. But on p. 64 the authors write: "Alternatively, if production takes time and differently dated commodities are distinguished...", so apparently they were assuming that production was, not timeless, but rather instantaneous: in zero time corn produces iron which can be instantaneously transferred to the corn industry to instantaneously produce corn which in zero time can be transferred to the iron industry to instantaneously produce iron, and the thing can be repeated indefinitely; then, were it not for the constraint due to the limited supply of labour, even an extremely small initial endowment of corn or iron would allow, by infinitely fast indefinitely repeated instantaneous production processes, the production of any amount of output. But even this assumption, absurd as it is, does not save the thesis that only a labour endowment is needed, because if the initial endowments of corn and iron are truly zero, neither industry can start producing, production is zero.

[23] The Leontief model is analogously described as timeless by Weizsäcker (1971), in a chapter that starts with the assertion that it will present "certain results of price theory which are independent of the introduction of the concept of capital" (p. 4); among these results there is the 1951 nonsubstitution theorem, in the competitive market equilibrium version. So Weizsäcker implies that there is no capital, not even circulating capital, in the Leontief model, because in it there is no time (the absence of an interest rate is identified with the absence of time). The analogy is clear with the considerations in the DOSSO volume. But Weizsäcker does not write down the supply-and-demand equilibrium conditions of the Leontief model, therefore he is less clear than Arrow-Hahn on what is being assumed as to factor endowments, and on the role of netputs.

So labour *cannot* be the only initial endowment; in a Leontief model the aggregate economy's netputs do *not* correctly indicate the firm sector's demands for inputs, nor its supplies of goods to the rest of the economy[24].

11. The basic misunderstanding

So *if* one tries to determine the neo-Walrasian temporary equilibrium of a Leontief economy, *then* labour supply cannot be the sole constraint on the production possibilities of the economy; some given initial endowments of coal, corn etc. *must* appear among the data of equilibrium, and the nonsubstitution theorem *does not apply*.

Clearly, the basic misunderstanding consists of not grasping that the theorem presumes, not the *absence* of any endowment of produced inputs, but their *endogenous determination* because referring to *long-period* choice of production activities. We find here another effect of the disappearance of the notion of long-period or 'normal' position from the theoretical horizon of modern neoclassical theorists: unable to understand that the absence of a dating of commodities in traditional analyses is due to the fact that, in those analyses, equilibrium is intended to represent a persistent situation in which prices can be treated as sufficiently close to constant (which implies an endogenous determination of the quantities of produced means of production), the modern neoclassical economist opts for the idea that if commodities are not dated the reason must be that the economy is assumed to be 'timeless', whatever that may mean.

12. Three advanced micro textbooks

Matters do not improve if one looks at the presentations of the theorem in recent advanced microeconomics textbooks. I will comment on the four ones that, as far as I know, do present the theorem: three where the nonsubstitution theorem is presented as referring to the general equilibrium of a Leontief economy[25] (Luenberger (1995), Blad and Keiding (1990), Varian (1992)); and one, Mas-Colell, Whinston and Green (1995), already mentioned in Section 1. More recent textbooks do not mention the theorem.

[24] The considerations advanced in this Section appear to have a relevance that goes beyond the discussion of the nonsubstitution theorem. For neoclassical theory itself, netputs are a dangerous way to describe production processes, especially production processes of the entire economy, because as shown in the text they easily entail a mistaken representation of input demands and output supplies of the aggregate firm sector. Therefore much more attention should have been given by neoclassical authors using netputs to answering questions such as, when is it legitimate to represent production processes via netput vectors, and when is this representation compatible with the determination of equilibria between supply and demand. An intertemporal formulation in which goods are dated is not sufficient to avoid the misrepresentation of firms' demands and supplies illustrated in the text, unless a date corresponds to a period so short that no productive process can use its own output as an input within the same period.

[25] In all three presentations the theorem is the 1951 Samuelson-Georgescu Roegen theorem. As in Arrow-Hahn, Samuelson (1961) is never mentioned.

A common characteristic of the first three presentations is that the prices that the theorem shows to be uniquely determined independently of the composition of demand are characterized, not as long-period prices (the notion of long-period price is absent from these textbooks), but rather as general equilibrium prices, and the sole notion of general equilibrium with production presented in these textbooks is the neo-Walrasian one, which determines equilibrium prices on the basis of *given* initial factor endowments. But no explanation is given as to how the initial endowments are to be treated in the equilibrium of the Leontief economy: it is not explained whether only the labour endowment is given; so the thoughtful student cannot but be left wondering how the non-given initial endowments of produced means of production are to be reconciled with the notion of Walrasian equilibrium, if only the labour endowment is given; and, if that is *not* the case, how wheat employed as seed at the beginning of the year (and in given endowment) can have the same price as wheat coming out at the end of the year, something which is generally not the case in intertemporal equilibria (no assumption of stationary state or steady growth is made in these presentations). The difficulty of the reconciliation of the notion of (neo-)Walrasian equilibrium with the Leontief model is what had induced Arrow and Hahn[26] to speak of a 'timeless' economy; in these three textbooks the notion of a 'timeless' economy is not introduced, probably because of some perception of its difficulties, but the solution consists simply of not mentioning the problem, which must leave the attentive student terribly confused.

The thing is particularly striking for the Luenberger textbook, because the author (Luenberger 1995, pp. 241-3) illustrates the Leontief model with an example where wheat, iron and labour produce wheat and iron[27], an example where the presence of wheat implies that production processes take considerable time. Luenberger proceeds to argue that, because of CRS, profits (in the neoclassical sense) must be zero under perfect competition, hence, taking the wage as numéraire (w=1), prices must satisfy (in our symbols) $\mathbf{p}=\mathbf{pA}+\mathbf{a}_L$.[28] No justification is given for the assumption that value added consists only of wages, that implies (but the thing is not made clear) that the rate of interest is zero; nor for the assumption that wheat seed has the same price as wheat output, with an implication of unchanging relative prices that, again, is not made clear and therefore is given no justification. In 1951 there had been an explicit assumption of 'saturation' of capital (with an implicit suggestion of stationary state), but here no such assumption appears, indeed the presence of capital is never mentioned. One is given to suspect that Luenberger, like DOSSO, is unable to view the inputs other than labour of the Leontief industries as indeed capital goods albeit circulating: had he recognized that wheat and iron when used as inputs are capital goods and require an anticipation of

[26] Or Weizsäcker.

[27] It is perhaps not a coincidence that this is the economy studied in the first chapter of Sraffa (1960). However, Luenberger does not assume that the economy "produces just enough to maintain itself".

[28] The nonsubstitution theorem (1951 Samuelson version) is then presented (p. 254) as an exercise, the student is asked to find the proof that, if in the Leontief economy there is choice of technique, under perfect competition relative prices are uniquely determined (which of course would not be the case if the rate of interest were not implicitly assumed to be zero).

capital funds in order to be utilized for production, then neoclassical theory would have suggested that there ought to be a reward—a positive rate of interest—for the savings that allow these capital goods to come into being, and the zero rate of interest would have needed discussion. And yet, at least for wheat production (that takes several months to yield its output), the need for capital anticipations (and for a pre-existing endowment of wheat seed) should have been evident. Analogously evident should have been the need to explain whether $\mathbf{p}=\mathbf{pA}+\mathbf{a}_L$ implied stationary relative prices, and if so, what kind of equilibrium one was then describing (with what assumptions as to savings, etc.).

Blad and Keiding (1990, pp. 177-183) are similar to Luenberger in that no clarification is offered as to why value added consists only of wages, or as to whether the fact that input prices are the same as output prices implies a stationary economy, or as to what is assumed about the endowments of the inputs other than labour. The main difference is that the authors explicitly note that the labour theory of value holds for prices obeying $\mathbf{p}=\mathbf{pA}+\mathbf{a}_L$, concluding that therefore the theorizing of classical economists "was after all not so unreasonable as later generations would have it" (p. 180). This reference to classical economics makes it all the more surprising that nothing should be said on the absence of a rate of profits (or rate of interest) in the price equations, since Ricardo assumed a positive rate of profits, and it is precisely the positive rate of profits that creates difficulties to the labour theory of value. Also, it is not pointed out that the labour theory of value was supposed to explain *natural* (normal, long-period) prices, differently from neo-Walrasian theory; in this way students are taught that the classical natural prices and modern general equilibrium prices are the same kind of prices, and remain ignorant of the fact that the former are long-period prices while the latter are very-short-period prices.

Varian's *Microeconomic Analysis*, probably the most widely used advanced microeconomics textbook in the 1980s and 1990s and still now much in use, is even worse on this topic, because the framework of the theorem is left much vaguer than in the other two textbooks (see Varian 1992, pp. 350, 354-6; the treatment is essentially unchanged from the first, 1978 edition). The clarifications indicated above as missing both in Blad and Keiding and in Luenberger are also absent in Varian; but in addition the Leontief model is not explained; the theorem is introduced after saying only that there are n single-output CRS industries, and a single nonproduced input (labour) necessary to each industry and *thus* with positive equilibrium price (this debatable 'thus' remains unexplained). Output prices enter the cost functions as input prices but it is not explained how that can be: it is left to the reader to understand that the several industries use one another's products as inputs; it is not even explained that these inputs are assumed to disappear in a single usage[29]. The price equations are not written down. Furthermore, the uniqueness, and independence from demand conditions, of equilibrium prices is presented as referring to the *Walrasian* equilibrium of this economy, but up to

[29] The book nowhere explains that the assumption of no joint production implies no durable capital.

that point Varian has discussed only the equilibrium of production economies without capital (only labour and land as factors, and only consumption goods as products), so the student has no idea how to formulate the 'Walrasian' equilibrium of an economy with produced means of production, nor is the thing further discussed by Varian later. So the knowledgeable reader recognizes here the Leontief model and the zero-interest 1951 version of the nonsubstitution theorem, but the students can only find these pages utterly incomprehensible.

13. A different textbook

A different presentation of the nonsubstitution theorem—but again a misleading one, and nearly impossible for students to understand—is supplied in the currently dominant graduate microeconomics textbook: Mas-Colell, Whinston and Green (1995). Here too the presentation is of the 1951 theorem, but the authors choose to present it in the pure optimization form adopted by Koopmans (1951c) and Arrow (1951), that is, *without* connecting it to prices or to competitive equilibrium. The theorem is described (p. 159) as stating that *all efficient production vectors with positive net outputs of all commodities can be generated from the same set of activities, one per commodity*[30]. Since in this textbook 'production vector' or 'activity' means 'netput' (see p. 128), the reason for this result will be evident in view of what was explained in Section 4. An economy-wide netput positive in all elements except labour is possible for the same reason as in Arrow-Hahn[31]; it simply means that the economy is assumed to produce a positive net output of all commodities, $\mathbf{q}=\mathbf{x}-\mathbf{Ax}>0$, where $\mathbf{q}$ is not a netput, the economy-wide netput is of the form $\mathbf{y}=(q_1,...,q_n,-L)^T$ with $L=\mathbf{a_L}\cdot\mathbf{x}$. A production vector or netput $\mathbf{y}$ is defined *efficient* (p. 150) if in the production possibility set of the economy there is no $\mathbf{y}'$ greater than $\mathbf{y}$ in at least one element (where, for an input, greater means smaller in absolute value), and not smaller in all other elements. Since the methods of the zero-interest-rate wage-maximizing technique minimize the labour embodied in each commodity, they permit the maximum net output of a commodity if all other commodity net outputs are given and total labour employment is given; and they permit the minimum labour employment if all commodity net outputs are given; therefore all efficient economy-wide netputs are generated from the methods of that technique, by changing industry dimensions. This is the 1951 theorem of Koopmans and Arrow; but, without a clarification of its framework such as was supplied above in

[30] Thus, Mas-Colell *et al.*, differently from Samuelson or Georgescu-Roegen, do not go on explicitly to add that such a choice of production methods is what competitive equilibrium brings about, although they do have some hints in this direction (1995, p. 160). But in this way they, like Koopmans, leave unexplained why one should be interested in an efficiency criterion for *net output* vectors, that amounts to minimization of labours embodied. For example, if the purpose were to maximize per capita consumption in an economy growing at a growth rate equal to the rate of interest (the 'golden rule'), efficiency would require a different choice of technique.

[31] Fig. 5.AA.3 on p. 158 of Mas-Colell *et al.* admits the possibility of a totally positive economy-wide netput resulting from the sum of the netputs of different industries, as in Arrow-Hahn.

Section 4, the student cannot understand anything about its meaning, because the peculiar meaning that the definition of efficiency acquires when referred to economy-wide netputs, its connection with *long-period* competitive choice of technique at a zero rate of interest, or the fact that the production of these 'efficient' netputs requires an endogenous determination of the endowments of capital goods, remain entirely hidden.

Of course, on top of all these difficulties with grasping what these textbooks are talking about, the student is not presented with the real nonsubstitution theorem.

14. Conclusion

Having, hopefully, made clear how and why the recent and not-so-recent mainstream presentations of the nonsubstitution theorem misrepresent it, there remains to discuss the relevance of the theorem correctly understood. This goes beyond the purpose of this note. Let me only suggest that any such discussion will do well to give attentive consideration to Garegnani (2007, pp. 187-8).

References

Arrow, Kenneth J., 1951. "Alternative Proof of the Substitution Theorem for Leontief Models in the General Case", in Koopmans 1951a, pp. 155-164.

Arrow, Kenneth J., and F. H. Hahn, 1971. *General Competitive Analysis*, Edinburgh: Oliver and Boyd.

Blad, Michael C., and Hans Keiding, 1990. *Microeconomics. Institution, Equilibrium and Optimality*, Amsterdam: North-Holland.

Dorfman, Robert, Paul A. Samuelson, and Robert M. Solow (DOSSO), 1958. *Linear Programming and Economic Analysis*, New York: McGraw-Hill.

Foley, Duncan K., 2004. "Value, Distribution and Capital. A Review Essay", *Review of Political Economy*, 2001, vol. 13, as reprinted in D. K. Foley, P. Garegnani, M. Pivetti, F. Vianello, *Classical Theory and Policy Analysis: A Round Table*, Materiali di Discussione no. 1, Centro Sraffa (Rome: Aracne), pp. 3-21.

Garegnani, Pierangelo, 1990. "Quantity of Capital", in J. L. Eatwell, M. Milgate and P. Newman (eds.), *The New Palgrave: Capital Theory*, London: Macmillan, pp. 1-78.

Garegnani, Pierangelo, 2007. "Professor Samuelson on Sraffa and the Classical Economists". *European Journal of the History of Economic Thought*, 14:2 (June), pp. 181-242.

Georgescu-Roegen, Nicholas, 1951. "Some Properties of a Generalized Leontief Model", in Koopmans, 1951a, pp. 165-173.

Koopmans, Tjalling C. (ed.), 1951a. *Activity Analysis of Production and Allocation*. New York: John Wiley and Sons.

Koopmans, Tjalling C., 1951b. "Analysis of Production as an Efficient Combination of Activities", in Koopmans 1951a, pp. 33-97.

Koopmans, Tjalling C., 1951c. “Alternative Proof of the Substitution Theorem for Leontief Models in the Case of Three Industries”, in Koopmans 1951a, pp. 147-154.

Kurz, Heinz D., and Neri Salvadori, 1995, *Theory of Production*, Cambridge University Press.

Leontief, Wassily W., 1951. *The Structure of the American Economy, 1919-1939*. New York: Oxford University Press.

Luenberger, David G., 1995. *Microeconomic Theory*. New York: McGraw-Hill.

Mas-Colell, Andreu, Michael C. Whinston, and Jerry R. Green, 1995. *Microeconomic Theory*. New York: Oxford University Press.

Petri, Fabio, 2004. *General Equilibrium, Capital and Macroeconomis. A Key to Recent Controversies in Equilibrium Theory*. Aldershot, UK: Edward Elgar.

Salvadori, Neri, 1987. “Non-substitution Theorems”, in J. Eatwell, M. Milgate, P. Newman (eds.), *The New Palgrave. A Dictionary of Economics*, vol. 1. London: Macmillan, pp. 680-2.

Samuelson, Paul A., 1951. “Abstract of a Theorem concerning Substitutability in Open Leontief Models”, in Koopmans 1951a, pp. 142-6.

Samuelson, Paul A., 1961. “A New Theorem on Nonsubstitution”, in Hugo Hegeland (ed.), *Money, Growth, and Methodology. In Honor of Johan Åkerman*. Lund: CWK Gleerup, pp. 407-424.

Sraffa, Piero, 1960. *Production of Commodities by Means of Commodities*. Cambridge: Cambridge University Press.

Varian, Hal R., 1992. *Microeconomic Analysis*, III edition (I edition: 1978). New York: W. W. Norton & Co.

Walras, Léon, 1954. *Elements of Political Economy*, Jaffé transl., Homewood, Ill.: Richard D. Irwin.

Weizsäcker, Carl Christian von, 1971. *Steady State Capital Theory*. Berlin: Springer-Verlag.

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