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Centro Sraffa Working Papers n. 41

May 2020

ISSN: 2284-2845 Centro Sraffa working papers [online]

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NOTES ON THE CLASSICAL THEORY OF NORMAL PRICES: EXHAUSTIBLE NATURAL RESOURCES AND NUMÉRAIRE DEPENDENCE

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Abstract: The present paper contains two notes. The first one resumes and expands the classical approach to cope with the existence of exhaustible natural resources in the context of the theory of normal prices. Alternative closures of the model are envisaged: either a given supply of the resource or a given royalty. The fixed-supply alternative, suggested in (Parrinello 2004), rests on the method used in (Sraffa 1960) to deal with the case of intensive land cultivation. The fixed-royalty assumption reflects the position of Piccioni and Ravagnani (2002) and Ravagnani (2006) about the theory of absolute rent. The second note addresses the problem of numéraire dependence, which has been stressed as a criticism of models of general equilibrium without overall perfect competition, and argues that such a property may concern also the theory of normal prices with exhaustible natural resources. It is suggested that a way out from the impasse of the numéraire dependence should be found in an extension of the theory of normal prices to a monetary economy.

Keywords: classical theory; Sraffian approach; exhaustible natural resources; numéraire dependency; price normalization

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Acknowledgements: the present notes have benefited from discussions between the author and Christian Bidard and comments from Saverio Fratini, under the usual exhemption from responsibility for errors and omissions.

NOTE I. THE GIVENS IN THE THEORY OF NORMAL PRICES WITH EXHAUSTIBLE NATURAL RESOURCES

I.1 A "solution" to the problem of exhaustible resources in the classical theory

Since the early Eighties, different arguments and approaches have been offered in response to the challenge raised to the classical theory of normal prices by the existence of exhaustible natural resources, which receive a positive price, despite the fact that only a fraction of the amount in the ground is used for current production¹. In the available literature on this subject, even within the same author's contributions, we often find a mix of arguments ranging from the history of economic thought (typically, the real meaning is questioned of Ricardo's cursory treatment of mines)², the description of the institutional features of some key-market (in particular the oil market)³ up to mathematical models of intertemporal equilibrium⁴. This state of affairs makes reading difficult at times and certainly not ideal for the layman. We believe that the (or, at least, some) basic point – the subject of this note - can be isolated and presented in a simple way.

Let us call in short "resource" an "exhaustible natural resource". For the sake of argument, we may safely disregard the costs of extraction and assume that each resource in the ground is freely available. The economy can be assumed to be endowed with heterogeneous resources in certain conjectured amounts. The argument will be confined to the case of one resource for simplicity. Free competition rules over all industries and a uniform rate of profit is paid on the capital invested in the production of each commodity. We may envisage two theoretical regimes in order to formulate a determinate system of price equations for such economy. Let us call them the fixed-flow and the fixed-royalty regime, respectively. The first one, which was suggested by the author (Parrinello 2004), assumes that the quantity of the resource available for production is a given flow. The second regime, a hint of which is found in Ravagnani (2006, 2008), assumes instead that the price (royalty) of the resource is

¹ A comprehensive record of the development of the theory at issue, from Parrinello (1982, 1983) to Bidard and Erreygers (2020), can be found in the references of the latter contribution.

² Cf. Kurz and Salvadori (2009).

³ Cf. Roncaglia (1985, 2016), Piccioni and Ravagnani (2002), Ravagnani (2006, 2008).

⁴ Cf. Bidard (2004), Kurz and Salvadori (2011, 2014), Bidard and Erreygers (2020).

independently given. The fixed-flow describes a limit, governed by the owners of the resource, to the effectual demand for produced commodities and, as a consequence, the resource receives a price similar to a rent. In this case, the model can be closed by assuming the co-existence of two methods of production in the same industry; in particular the industry which uses the resource in short supply. This closure follows the approach adopted in Sraffa (1960) to determine the normal prices of produced commodities, jointly with the intensive rent on land of uniform quality. Instead, in the fixed-royalty regime, a single – the less costly – method of production implementing the resource is chosen and the quantity of resource in use becomes a derived variable, determined by the effectual demand for commodities, in the same vein as the amount of employment depends on that demand, given the rate of profit or the real wage rate and assuming a not binding supply of labour. Therefore, the adoption of the fixed-royalty enters a model with three distributive variables (the rate of profit, the real wage rate and the royalty), one of which is definitely assumed as given (the royalty) and the other given can be either the rate of profit or the wage rate. The two regimes represent the basic logical alternatives which seem consistent with the classical theory of normal prices and a model with many resources may adopt the fixed quantity for some resources and the fixed royalty for others.

It should be noticed that whatever regime is chosen, the total and possibly unknown amount of the resource in the ground is not accounted among the determinants of production prices; only the flow of resource is at issue. Furthermore, the approach does not presuppose perfect foresight and no intertemporal link is acknowledged among the prices of different periods, because the prices are determined in a self-contained production period. All this host of assumptions is open to a critical appraisal and requires a plausible explanation, beyond the logical exercise of finding possible closures of the price model. Let us narrow our task by assuming that the pros and cons of the classical method of long period equilibrium, applied to an economy without exhaustible natural resources, are common knowledge. What new difficulties emerge in the presence of the resources dealt with the two hypothetical regimes above? Concerns and objections can be grouped from two different points of view.

From the side of the contributors to the revival of the classical method, one could question: 1) the lack of relative persistence of the given flow or the given royalty; 2) the introduction of a sort of *deus ex machina* in the fixed-flow regime, consisting in the assumption of the co-existence of methods of production borrowed from Sraffa's theory of rent. Instead the advocates of the intertemporal equilibrium models would object that the recommended approach is not consistent with the need of appreciation of the resource, which is a corollary of the condition of a uniform rate of profit. In particular the Hotelling rule prescribes that in a moving equilibrium an exhaustible natural resource must appreciate at a rate equal to the prevailing rate of interest, which in the classical approach is subsumed under the definition of the rate of profit. Let us face such objections in the order presented above.

I.2 The "persistence" objection

As many Post-Keynesian economists have contended, the classical method of long period positions would have a too narrow scope due to the fact that its givens are not sufficiently persistent in the real world to sustain the role of normal prices as attractors of market prices, even in the absence of exhaustible natural resources. Especially the investments are volatile components of the demand for commodities and the ever-increasing pace of technical innovations would prevent that role. It can be responded that the speed of gradual or *una tantum* shocks of technical progress affects not only the change of the determinants of the normal prices, but also the adjustment of market prices toward such prices; in particular the speed of information diffusion at present is much higher due to the advance of informatic technology. The combined effect is not clear-cut to the extent of supporting the argument of a relative obsolescence of the method of long period equilibrium. Taking into account the existence of exhaustible natural resources, by means of one of the two regimes suggested above, does not tip the scale in favour of the criticism of the long period method. In fact, the quantity of the resource or its own royalty in most real cases is regulated by institutional arrangements and the oligopolistic bargaining among national Governments and multinational companies⁵. This state of affairs is characterized by long lasting arrangements, which are only occasionally subverted by geopolitical disruptions and the impact of which on the normal prices is hardly different from that attributed to technical innovations. Therefore, the supply of the resource does not seem more volatile than the level of investment flows; whereas it is debatable whether the empirical correlate of a given royalty is less persistent than that behind a given real wage rate, in the case in which the latter is assumed to be one of the exogenous distributive variables.

⁵ See Roncaglia (1985, 2016), Piccioni and Ravagnani (2002), Ravagnani (2006, 2008).

I.3 The "deus ex machina" objection

The fixed-flow regime encounters the same theoretical limits attributed to the theory of land under intensive cultivation presented in the chapter "land" of Sraffa's book.

We quote:

If land is all of the same quality and is in short supply, this by itself makes it possible for two different processes or methods of cultivation to be used consistently side by side on similar lands determining a uniform rent per acre. (Sraffa 1960, p. 89) Under these circumstances there would be room for two different methods producing the crop

in question on that land (Sraffa 1960, p. 91).

It may be objected that "making it possible" or "being room for" does not guarantee that the "seat is taken". It may happen that no pair of available methods satisfies

the economic condition of not giving rise to a negative rent: which implies that the method that produces more corn per acre should show a higher cost per unit of product, the cost being calculated at the ruling levels of the rate of profits, wages and prices (Sraffa 1960, p. 91).

The assumption of the co-existence of two methods in use appears as a sort of *deus ex machina* which serves to determine the price of a resource of uniform quality in short supply: either a Ricardian land or an exhaustible resource. However, such an assumption is not as peculiar as it looks at a first sight. The observable sign of scarcity, reflected by the coexistence of two processes, is not necessarily located in the industrial or agricultural sector in which the resource enters as an input. It may emerge elsewhere in a system of connected industries. For example, let us assume an economy where gasoline is produced by means of crude oil, transportation services are produced by gasoline; and labour is their common input. Under the assumptions of circulating capital and advanced wages, we may find the following alternative systems of price equations, with L, O, G, T denoting quantities of labour, oil, gasoline and transportation; and where the meaning of the other symbols is self-evident.

$$(1+r)L_{g}^{I}w + O_{g}^{I}\rho = G^{I}p_{g} (1+r)L_{g}^{II}w + O_{g}^{II}\rho = G^{II}p_{g} (1+r)L_{t}w + G_{t}p_{g} = Tp_{t}$$
 [1]

$$(1+r)L_g w + O_g \rho = G p_g$$

$$(1+r)L_t^I w + G_t^I p_g = T^I p_t$$

$$(1+r)L_t^{II} w + G_t^{II} p_g = T^{II} p_t$$

$$(2]$$

Given the numeraire equation and the rate of profit r, both [1] and [2] are determinate systems of equations, subjected to the positivity condition on ρ in [1] and p_g in [2], as reminded in Sraffa. System [1] reveals the scarcity of oil in a direct way, by the co-existence of two methods which implement oil of the same quality. Instead system [2] reveals scarcity through a quantity of gasoline in short supply, which is used by two methods side by side. In both cases it is not a matter of physical scarcity, because the supplies of oil and gasoline are the result of economic decisions related to given long period expectations. Furthermore, in the process of substitution of methods of production, due to the running down of the resource, it cannot be excluded a change in the location of the signal of scarcity, which would be revealed by a switch between systems of type [1] and [2] before the complete interruption of the flow of the resource. In particular, a change of location can be the result of a hypothetical change of a given rate of profit. The meaning and scope of the normal prices do not vanish because of the progressive depletion and substitutions of resources accompanied by a change of techniques. Such a substitution process echoes Sraffa's description of a progressive cultivation of land governed by the dynamics of population and the ensuing "spasmodic" substitution among methods of cultivation on land in short supply.

I.4 The objection of intertemporal inconsistency

It is admittedly difficult to refrain from looking for a more analytical explanation of the quantity flow and price of the resource, instead of confining ourselves to plausible hints, like the suggestion of a catch-all institutional factor. Constructive contributions in this direction can be found in Kurz and Salvadori (1997, 2000), Bidard and Erreygers (2001a, 2001b), Schefold's criticism (2001) and they are scrutinized in the recent critical appraisal by Bidard and Erreygers (2020). Such reformulations aim to explain the path of depletion and the prices of a resource in terms of pure economic reasoning, by adapting the neoclassical intertemporal equilibrium approach which obeys the Hotelling rule; still maintaining a classical flavour due to the assumption of an exogenously given distributive variable (the own rate of interest on the standard of value). We do not want to stress a certain weakness of such models, due to the fact that they mainly rest on the assumption of perfect foresight. This proviso can be relaxed, perhaps at the cost of allowing a host of special cases according to different assumptions about the formation of expectations. Instead we want to reiterate an answer (Parrinello 2004) to the

objection of inconsistency which might be raised by the advocates of the intertemporal equilibrium to the classical approach reformulated in this paper for the case of depletable resources. The objection is quite straightforward: if we take any sequence of price equations (for example, a sequence of the previous systems [1], [2]) of the same economy over distinct long periods, the exogenous succession of the givens and the corresponding endogenous prices do not guarantee that the value of the resource in short supply appreciates at rate equal or related to the prevailing interest rate, contrary to the Hotelling rule. We do not respond to the objection by contending that such a rule is empirically unimportant as many case studies have revealed. The rule can be confirmed or disconfirmed by observing a sequence of states of the economy with their own prices, rents and royalties; but the classical theory of value is not a theory of normal prices of assets like agricultural land, coal mines and oil deposits. It can offer a theory of the extensive or intensive rent like in Sraffa's book, but it would be arbitrary to claim that, if ρ^* is the rent associated to the normal prices of produced commodities, the "normal" price of land is equal to the ratio $\frac{\rho_*}{r}$, interpreted as a perpetual constant annuity ρ^* capitalized at a constant interest rate r. This would lead us to unacceptable results, because any plot of land, which is not yet cultivated and receives a rent equal to zero, would become a free good even in a progressive economy, where its cultivation is expected to accrue in the future. Furthermore, the price of land would not even be defined if the rate of interest is equal to zero and the rent is positive. The ratio $\frac{\rho^*}{r}$ can be interpreted as an equilibrium price of land under additional and restrictive assumptions, but it is not an attractor of the market prices of land, on the same footing as the prices, rents and royalties determined by the normal price equations⁶. Notice that Sraffa in his book deals with rents, but he never mentions the price of land.

A similar argument can be referred to the existence of deposits of exhaustible natural resources, some of which are in short supply and receive a royalty and others are not yet "cultivated". The fixed-flow and fixed-royalty assumptions do not serve a theory which determines a sequence of normal values of the resource in situ. In conclusion, the approach suggested in this note neither denies nor complies with the Hotelling rule and therefore it cannot be charged with inconsistency on this ground. The same argument, focused on the fixed-flow regime, has been advanced in Parrinello (2004 section 3.1. et seq.) and can be extended to the fixed-royalty regime.

⁶ The classical theory of normal prices describes a long period equilibrium of the economy, but not an actual economy in an exact state of steady growth. Note that a "small" percentage deviation between an average of actual rents and the normal rent ρ^* would be associated with a "large" percentage deviation between an average of the actual prices of land and the ratio $\frac{\rho_*}{r}$, due to the fact that *r* is "small".

I.5 A conclusive remark

We have argued that the fixed-flow/fixed-royalty regimes can be adopted in a theory of normal prices with exhaustible natural resources. If we remain at the level of abstraction of Sraffa's theory of normal prices, where the quantities of commodities and certain prices are taken as givens, no additional mathematical effort is needed to deal with the depletion of some natural resources. This conclusion, which may sound rather odd and disappointing to a mathematical economist, does not mean that a lot of valuable analytical work has not been already done and should not be developed even more in order to generalize the theory of normal prices.

For this purpose we should move to a different level of abstraction, where the previously given quantities and given prices are explained; in particular those related to the sector of exhaustible resources. However, a useful extension of the theory in this direction can be hardly confined to economies in which free competition pervades the whole economy, including the sector of exhaustible resources. In the next note it is argued that a problem of "numéraire dependence" may arise as soon as the theory trespasses the safe territory of free competition. It will be suggested that this puzzling property of equilibrium becomes less important if an explicit monetary standard is introduced in the theory of normal prices.

NOTE II. NUMÉRAIRE DEPENDENCE

II.1 An overview of the "numéraire problem"

The meaning of the words "numéraire", "standard of value" and "price normalization" is undisputed in the theories of general equilibrium of Walrasian type. Let us take the equilibrium equations in terms of the absolute (*nominal*) prices $(p_1, p_2, ..., p_n)$ of *n* commodities and add

$$\alpha_1 p_1 + \alpha_2 p_2 + \dots + \alpha_n p_n = 1,$$
^[3]

where $\alpha_1, \alpha_2, ..., \alpha_n$ are given coefficients. Equation [3] is said to *normalize* the prices⁷. In the special case in which $\alpha_1 = 1, \alpha_2 = \cdots = \alpha_n = 0$, equation [3] becomes $p_1 = 1$, and we say that commodity 1 is chosen to be the numéraire. The normalization of prices usually serves to hit two birds with one stone. It provides i) a standard of value of heterogeneous commodities from the perspective of the observer-economist and ii) a standard of value that enters the choices of the agents explained by his/her theory. The general equilibrium theories mentioned above are characterized by *numéraire independence*. This expression means that the equilibrium quantities and relative prices do not depend on the choice of the *numéraire* or the way of price normalization. Formally, such a *real* (opposed to nominal) equilibrium configuration of the economy does not change if some coefficients $\alpha_1, \alpha_2, ..., \alpha_n$ in [3] are changed. This feature characterises not only the Walrasian theories of general equilibrium, but also Sraffa's theory of normal prices. Therefore the economic analyst, within such theories, is free to choose the price normalization which is more convenient for his/her own theoretical interest, even beyond the additive form [3].⁸

Two notions of *numéraire dependence* have emerged in the literature since the early Eighties. The first one means that a change of the numeraire makes a change in the equilibrium real configuration of the economy. The second describes a situation in which the qualitative properties of the *relations* among the real equilibrium variables, obtained by the method of

⁷ The sum over *all* prices has the advantage of setting no price strictly positive *ex ante*.

⁸ Occasionally, mathematicians, instead of equation [3] which defines the *simplex* normalized prices, set the Eucledian norm $[(p_1)^2 + ... + (p_n)^2]^{1/2}$ equal to one.

comparative statics, depend on the choice of the numeraire⁹. We shall focus on the first notion of numeraire dependence, which has been emphasized in the critical appraisal of certain hybrid general equilibrium models, where the original Walrasian assumption of competitive markets is combined with that of some sector of the economy subjected to non-perfect competition.¹⁰ For the sake of argument, let us consider a simple case of numéraire dependence borrowed from the criticism addressed by Srinivasan and Kletzer (1994) to the "Walras- Cournot" models of general equilibrium.

II.2 A basic equation

Let us assume an economy with *m* single product firms and *n* commodities produced and used as means of production. Let us denote, omitting the suffix of reference to the firm:

 $\mathbf{p} = (p_1, p_2, ..., p_n) \text{ the vector of the absolute prices;}$ $\mathbf{x} = (x_1, x_2, ..., x_n) \text{ the vector of the } n \text{ inputs used by the firm producing commodity } j;$ $f_j(\mathbf{x}) \text{ the differentiable production function with output } j;$ $\Pi = p_j f_j(\mathbf{x}) - \mathbf{p}\mathbf{x} \text{ the profit measured by absolute prices}$ $\Pi^{(1)} = \Pi/p_1 \text{ the profit if the numéraire is commodity } 1$ $\Pi^{(n)} = \Pi/p_n \text{ the profit if the numéraire is commodity } n.$

The real profit functions are related by the equation (an identity):

$$\Pi^{(n)}(\mathbf{p}, \mathbf{x}) = \frac{p_1}{p_n} \Pi^{(1)}(\mathbf{p}, \mathbf{x})$$

A basic relation follows between the first-order conditions for profit maximization¹¹, corresponding to the alternative choice of the numéraire:

$$\frac{\partial \Pi^{(n)}}{\partial x_j} = \frac{p_1}{p_n} \frac{\partial \Pi^{(1)}}{\partial x_j} + \Pi^{(1)} \frac{\partial (p_1/p_n)}{\partial x_j} = 0, \quad j = 1, 2, \dots, n$$
[4]

Notice that the partial derivative $\frac{\partial (p_1/p_n)}{\partial x_j}$ means a price change that the firm *perceives*. The equations [4] imply a noteworthy property. *The conditions of profit maximization are numéraire*

⁹ Cf. Opocher and Steedman (2009).

¹⁰ Such models are inspired to Negishi's (1961) seminal contribution.

¹¹ The first order conditions in the text refer to an internal solution for simplicity.

dependent if the firm is a price maker, i.e. $\frac{\partial (p_1/p_n)}{\partial x_j} \neq 0$, and its net profits are positive, $\Pi > 0$. Instead the profit maximizing choice is numeraire independent if the firm is either a price taker or its net profit is zero or if both conditions occur.

II.3 Different reactions to the numéraire dependence

We surmise three alternative reactions and tentative answers to the disturbing case of numéraire dependence. We may conclude that the equilibrium of the firm and the general equilibrium of a production economy based on such equilibrium become inherently undetermined. This result sounds as an irremediable shortcoming of the theory, especially for applications of the theory of general equilibrium to policy issues. Alternatively, we may replace the assumption of profit maximization by a substitution of the maximand. Instead of the profits in terms of an arbitrary numéraire, we may take the utility attributed to the choice of commodities by the owner or the shareholders of the firm. However, it would be implausible to assume that the owner or the shareholders behave as price takers, whereas their "firm" perceives the price making effects of their choices.¹² As a result, the theory of the firm should become a theory of utility maximization on the side of agents which behave as price-makers. This route does not seem to be useful for the sake of applied theory, because it would prescribe many different subjective standards of value across firms and individuals. However, it suggests the need for a distinction to be adopted shortly. The choice of the standard of value, from the perspective of the analyst, should be kept distinct from the standard(s) of value which enters the economic decisions of the agents observed.

A third and more constructive reaction derives from looking at the institutional setting of real capitalistic economies. The triad of money roles – unit of account, medium of exchange and store of value – should be reconsidered. Let us introduce the *augmented* equation of price normalization

$$\alpha_1 p_1 + \alpha_2 p_2 + \dots + \alpha_n p_n + \alpha_M p_M = 1,$$
^[5]

where p_M denotes the absolute price of fiat money. If $\alpha_1 = \alpha_2 = \cdots = \alpha_n = 0$ and $\alpha_M = 1$, nominal money becomes the standard of value, a case of full money illusion. Suppose that the

¹² The same argument has been already noticed in passing by Srinivasan and Kletzer (1994, p.13).

economy is divided in two partitions, A and B, which correspond to the areas of free competition and non-free competition, respectively. Assume that the firms in partition A make choices which are numéraire independent; instead the choices of the firms in partition B are numéraire dependent, according to the criterion established by equations [4]. The selection of the coefficients $\alpha_1, \alpha_2, ..., \alpha_n, \alpha_M$ of [5] can be left to the *wisdom* of the economist relative to his/her analytical needs. By contrast, we may assume that the numéraire adopted by the firms in partition B for their own choices conforms to a standard of value which is institutionally determined in a monetary economy and must be part of the theory, instead of being an arbitrary selection by the external observer. Let us denote such standard of value as a deflator of the absolute prices to obtain the relative prices:

$$p_j(\beta_1 p_1 + \beta_2 p_2 + \dots + \beta_n p_n + \beta_M p_M)^{-1}, \ j = 1, 2, \dots, n, M$$
[6]

where the coefficients $\beta_1, \beta_2, ..., \beta_n, \beta_M$ are not arbitrary, unlike coefficients $\alpha_1, \alpha_2, ..., \alpha_n, \alpha_M$. In particular, the ratio $p_M/(\beta_1 p_1 + \beta_2 p_2 + ... + \beta_n p_n + \beta_M p_M)$ can be interpreted as a real price of money. Both [5] and [6] can be consistently added to the model, under the proviso that the choices of the agents of partition *B* are related to the relative prices [6], whereas those of the agents in partition *A* can be determined by $p_j(\alpha_1 p_1 + \alpha_2 p_2 + ... + \alpha_n p_n + \alpha_M p_M)^{-1}$ derived from the arbitrary choice of $\alpha_1, \alpha_2, ..., \alpha_n, \alpha_M$, which may include the case $\alpha_1 = \beta_1, \alpha_2 = \beta_2, ..., \alpha_n = \beta_n, \alpha_M = \beta_M$. The approach can be generalized by assuming that a bounded set of alternative coefficients $\beta_1, \beta_2, ..., \beta_n, \beta_M$ represents indifferently those institutional features, yet conceding that the ensuing equilibrium configurations become β -dependent.

II.4 The case of intertemporal equilibrium with fixed interest rate¹³

A certain controversial interpretation of some models of intertemporal equilibrium, which assume a fixed rate of interest and deal with exhaustible natural resources¹⁴, can be easily avoided by applying the distinction between a specific standard of value and the numéraire

¹³ This section resumes the arguments advanced by the author (Parrinello 2011, 2014) about the controversial notion of numéraire dependence in the context of intertemporal equilibrium analysis and tâtonnement processes. ¹⁴ See Bidard (2004, 2020), Kurz and Salvadori (2011, 2014).

illustrated in the previous section. An intertemporal equilibrium, outside a stationary state, is characterized by a path of variable relative prices, which include the endogenous and not equal own-rates of interest of the commodities. Leaving aside the deeper notions of the own-rate of interest found in Keynes and Sraffa, let us confine to the simple definition:

The 'own-rate of interest' of a commodity is defined as the ratio of a definite quantity of a commodity, say wheat, available at a future date (t + 1), exchanged against a definite quantity of the same commodity at date (t). (Fisher, 1896, pp.8 ff).

As a matter of fact, such sentence defines the own-factor of interest, from which the own-rate of interest derives by subtracting one. Since the own-factor of interest of a commodity is a relative price, we can cast the controversial argument about the models of intertemporal equilibrium with a fixed own-rate of interest as if one relative price would be fixed at a given value in a static equilibrium model of the economy, subject to the price normalization [5]. For instance, suppose that p_1 is set equal to a given value p and that the equilibrium model remains consistent. This means not only that we have set the absolute price of commodity 1, but also its own relative price $p_1(\alpha_1 p_1 + \alpha_2 p_2 + \dots + \alpha_n p_n + \alpha_M p_M)^{-1}$ equals p. It is true that, if we change the numéraire by changing one of the coefficients α_i , the equilibrium of the economy in general will change. However, this result does not prove that the equilibrium is numéraire dependent. Instead, the equilibrium changes because we substitute one of the (composite) commodities, the *relative* price of which is fixed, with another commodity. A similar result happens if the structure of the model is changed by replacing e.g. the equation $p_1/p_2 = p$ with $p_1/p_3 = p$, still keeping the price normalization [5], i.e. in the absence of a change of the numeraire. The previous argument can be adopted for the interpretation of the models of intertemporal equilibrium with a fixed interest rate. The theory can prescribe the good on which the own-rate of interest is fixed (in particular the own-rate on money), leaving the others among the endogenous variables, and the choice of price normalization or numéraire ad libitum of the theoretician. Therefore, we can say that the intertemporal equilibrium is at the same time dependent on the choice of the specific standard (a dated commodity) implicit in the given interest rate and also not dependent on the choice of the numéraire, because a *pure* change of the numéraire (a change of the coefficients of equation [5]) does not affect the equilibrium path of the economy, unless this change is accompanied by a change of the commodity which enters the definition of the given own-rate of interest.

II.5 A conclusive remark

This note seems to offer two rival conclusions, if we compare the arguments of sections II.2 and II.4. In fact, the dependence of an equilibrium on the choice of the numéraire is admitted in the case of price-making firms (equations 4) and is rejected in the case of an intertemporal equilibrium subject to the assumption of a fixed interest rate. Such a discrepancy is often encountered and is even more articulated in different fields of analysis, as the present author has illustrated by means of a collection of passages quoted from the works of a variety of economists (Parrinello, 2011, pp. 328-329, appendix pp. 352 -354). Granted that nobody has a monopoly of the meaning of "change of the numéraire", two alternative settlements can be envisaged in the face of such a puzzling range of positions. One way would be to adopt a unified meaning of that expression and reduce the claim of numéraire dependence to the case in which a specific standard of value is changed, still keeping the same general numéraire. The other alternative would be to share the pluralism of the *status quo*, where the expression "change of the numéraire" is used in different meanings, depending on and clarified by the theoretical context. In particular, in the next section "numéraire dependence" has the meaning attributed to such term in the context of the theory of price-making firms represented by the equations [4], section II.2.

III. A sketch of agenda for the theory of normal prices

The method adopted in the theory of normal prices does not require the assumption of pricetaking firms, but in such a context the hypothesis of zero net profits is sufficient to avoid the numéraire dependence in the presence of price making firms. On this ground, a long period equilibrium under free competition is similar to Chamberlin's theory of monopolistic competition in the long period. However, a more extensive notion of long period equilibrium can hardly avoid the problem of numéraire dependence in the absence of the institution of a monetary standard, as suggested in section II.3. Two arguments can be adduced to support such claim. Firstly, random normal shocks can be conceived as an inherent characterization of a long period position.¹⁵ Resuming Arrow's (1959) provocative query "if all are price takers, who makes the prices?", a problem of numéraire dependence would re-emerge repeatedly during an

¹⁵ Cf. Parrinello (1990).

ongoing adjustment process deprived of a fictitious auctioneer. In fact, temporarily but repeatedly, some firms would become price-makers who receive positive or negative net profits. Secondly and more importantly for applied general equilibrium modelling, the approach of long period equilibrium should relax the condition of an overall uniform rate of profit and explore models of economies in which the industries under free competition interact with non-competitive sectors, characterized by price-making firms and positive net profits. It goes without saying that the approach based on the forms [5], [6] presupposes the modelling of a monetary economy. This is a theoretical field which has remained rather peripheral to the formal developments of the theory of normal prices, despite the solitary, but non accidental, recall of the money rate of interest in Sraffa's book. Furthermore, it should be noticed that the avoidance of indeterminate equilibrium, due to the arbitrary choice of the numeraire, does not protect the theory of long period equilibrium of the industry from the possible existence of badly-behaved excess demand functions¹⁶.

We finally return to a specific problem of interpretation which was mentioned only in passing in Note I. One of the two alternative closures of the model with exhaustible resources rests on the assumption of a given royalty and on the transformation of the canonical Sraffian model with two distributive variables – the rate of profit and the real wage rate – into a model in which the royalty is added as a third one. If we look for an economic interpretation of this third given, we should take into account the economic-institutional structure of the economy. The assumption at issue might be explained like the classical assumption of a given real wage and traced back to the so called "institutional factors" and/or by relying on the Marxian vision of class struggle. A lively debate is still ongoing around this issue¹⁷ and it cannot even barely touched here. In particular we do not take a position among the advocates and the critics of a relation between the Marxian theory of the absolute rent and the assumption of a given royalty. We only stress a single point by exploiting the main argument of this note. If we attribute a price-making power to the agents engaged in the sector of the exhaustible resource (say, the oil sector once more), surrounded by the free competition in the rest of the economy, a profit maximization, combined with persistent extra profits in the resource sector, would bring about the possibility of numéraire dependence for the long period equilibrium of the economy as a whole. From this point of view, an explicit introduction of money into the theory of normal prices would become a pertinent generalization.

¹⁶ This lack of protection is proved by Opocher and Steedman (2015), who purposely assume a full industry equilibrium characterized by *zero net profits*.

¹⁷ Cf. Fratini (2016).

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