Towards a 'Classical-Keynesian' analysis of effective demand in the long period

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> [I]t is in the 'present' that the 'normal' rate of profits has always been firmly located. … [B]ecause this is the rate of profits which is being realised *in the present* …, it is also the rate of profits which that present experience will lead entrepreneurs in general to expect *in the future* from their current investment. (Garegnani, 1979, original emphases)

1. Introduction

The 'generalisation' of *The General Theory*—the extension of its short-period concern to the long-period matters—was an immediate task that Keynes's disciples such as Joan Robinson felt is their duty to tackle (Robinson, 1937, 1956). It was conceived, in particular, that Keynes's principle of effective demand—the proposition that investment is autonomous from the saving that ensues from the 'normal' capacity utilisation, or equivalently that it is investment that generates saving of the same volume, not the other way around—could be applied not only to the short-period determination of income but also to the long-period analysis of economic growth and capital accumulation (and, therewith, the long-period distribution of income).

It is well known that the profession has since witnessed two main developments along such lines. One is what can be called the 'Cambridge Keynesian' approach. In the long period, the autonomy of investment reveals itself in the form of the autonomy of the share of investment in income (or, as has become more customary, the autonomy of the rate of accumulation). Combined with the differential propensities to save across the income categories (wages and profits, as in Kaldor, 1955-56), the social classes (workers and capitalists, as in Pasinetti, 1962), or the economic institutions (households and corporations, as in Kaldor, 1966), this autonomy brings about variations in the distribution of income (the real wage rate and the 'normal' rate of profits) such a way that the volume of saving ensuing from income distribution matches the volume of autonomous investment. The other, which has appeared relatively recently and apparently has become the dominant approach among the 'Post-Keynesians', is what is usually called the 'Kaleckian' approach. The main route of the generation of saving matching autonomous investment is the varying level of income, as in the short period. But its analysis is in the framework of long-period analysis. The long-period autonomy of investment is revealed, as in the Cambridge Keynesian approach, in the form of the autonomy of the rate of accumulation. However, different rates of accumulation result in different 'long-period equilibrium' degrees of utilisation of productive capacity, from which different levels of income ensues.

Amidst significant differences between the two approaches, they share a common feature. Both focus on the *steady state* of the economy. The object of analysis of the Cambridge Keynesian approach is the state of the economy which grows at a constant rate over time (at a uniform rate across the industries in a multi-industry economy), with productive capacity being utilised at the 'normal' level continuously over time (across the industries). The Kaleckian approach, whilst differing from the former by allowing productive capacity to be utilised at a level other than the 'normal' one in long-period equilibrium, shares its focus on the steady state.

Both approaches betray their own claims on the Keynesian pedigree, however. As for the Cambridge Keynesian approach, continuous normal utilisation over time contradicts the autonomy of investment (Garegnani, 1982, 1992; Ciccone, 1984; Palumbo and Trezzini, 2003). At the beginning of each period the economy is equipped with a given configuration of capital equipment. Given also the saving behaviour of the economy, the normal utilisation of that productive capacity dictates the volume of saving (normal capacity saving) that will ensue from such utilisation. The macroeconomic equilibrium condition of equality between saving and investment, then, requires that investment should follow suit after normal capacity saving.[1] The Kaleckian approach is no better in this regard. The steady state obtained in the Kaleckian models is characterised, not only by a constant rate of accumulation, but also by a *constant* 'equilibrium' degree of utilisation (even if it usually deviates from the normal level). Given the size of productive capacity in a period (and the saving behaviour of the economy), then, the volume of saving which corresponds to that degree of utilisation in that period is pre-determined, without any reference to the volume of investment. It follows that investment should be brought into line with that pre-determined saving (even if that saving is not necessarily at the 'normal' level) *if the steady state is to be maintained*. The Kaleckian claim on the Keynesian pedigree is based on the result that different (autonomous) rates of accumulation generate different steady-state paths of the economy, along which different 'equilibrium' degrees of utilisation prevail. However, the continuing stay on a given steady-state path can only be achieved by the violation of the principle of effective demand.[2]

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[1] This criticism may be too harsh, though, when the steady state is considered a normative state the economy (in particular, the government) should try to achieve (as in Pasinetti), or merely a decoy intended to show that such a state is almost an impossibility in reality (as in Joan Robinson). The criticism is more applicable to the case in which such the steady state with full employment is considered a normal lot of the economy (as in the early Kaldor).

[2] A better-known criticism of the Kaleckian approach is that the deviation of the 'equilibrium' from the 'normal' degrees of utilisation over the long period contradicts the economic rationality of entrepreneurs (e.g. Committeri, 1986; Ciccone, 1986; Palumbo and Trezzini, 2003). Faced with this difficult-to-refute criticism, some Kaleckian models make the 'normal' and the 'equilibrium' degrees adjust each other and thus eventually coincide. On the one hand, this may be taken as a stronger case for the principle of effective demand in the long period, for different rates of accumulation generate different 'normal' states of the economy (and thus

Whether the case is the Cambridge Keynesian or the Kaleckian approach, therefore, the ground for the claim on the Keynesian pedigree collapses along the steady state path.

There is a third, less known, approach to effective demand in the long period. The approach, sharing some aspects with the two better-known approaches, has not been clearly differentiated from them and up to now failed to develop to its full potential. The present paper will christen it the 'Classical-Keynesian' approach.[3] The denomination 'Classical' is based on the fact that its perspective regarding the long-period state of the economy is that of Classical economics—revived and refined in the modern form by Sraffa (1960). The long-period state of the economy is the 'normal' state where all economic variables—relative prices, the quantities of output and the distributive variables (the real wage rate and the rate of profits)—are at their respective 'normal' levels. In particular, the requirement that the rate of profits prevailing in the long-period state should be at the 'normal' level implies that the

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different corresponding volumes of 'normal' capacity saving). On the other, however, the contradiction with the principle of effective demand *along a given steady-state path* is more salient, for now investment should be adjusted continuously to 'normal' capacity saving.

[3] One of the motives for the Classical-Keynesian approach is the fact that Sraffa (1960) takes the quantities of output as 'given' when considering the determination of prices. He gives little hint at how the quantities of output have come to be what they are, thus leaving open the determination of the quantities of output. This is, it is conceived, where Keynes's principle of effective demand can enter: the given quantities of output in the Sraffa system can be interpreted as those determined in reference to the state of effective demand (see e.g. Garegnani, 1984, 1990; Kurz, 1990, 1992, 1994; Cesaratto, 1995).

The initiative for the approach was taken by Garegnani's 1962 work (Garegnani, 1962), which has been followed by a series of his own elaborations (Garegnani, 1982, 1983, 1992). Some important work along these lines was done in the 1980s, in some cases arguing that *The General Theory* itself presents a long-period analysis (e.g. Milgate, 1982; Eatwell, 1983; papers in Eatwell and Milgate, 1983) and in others exhibiting high hopes for some kind of formalisations to deal with the topic (e.g. papers in Bharadwaj and Schefold, 1990). Efforts of positive construction, however, seem to have been increasingly distracted by the theoretical brawls with the Kaleckian approach, regarding the long-period deviation in the latter between the equilibrium and the normal degrees of utilisation. Meanwhile Serrano's (1995) idea of the 'Sraffian supermultiplier' was an important constructive contribution, if it subsequently came in for criticisms within the Sraffian camp (Trezzini, 1995, 1998, Park, 2000). In contrast to various attempts to 'formalise' the determination of output in the long period, Palumbo (1996), Garegnani and Palumbo (1998), and Palumbo and Trezzini (2003) take a critical stance to such attempts, arguing that the actual process of capital accumulation and output growth is too complex to be dealt with in a formalised way. Recently Garegnani and Trezzini (2010) and Trezzini (2011) pursue some particular Sraffian-Keynesian lines of research. That there is continuing interest in the approach is witnessed by a collection of essays published recently (which were originally discussed in a 1998 conference) (Ciccone *et al*, 2011, Parts III and IV).

long-period state of the economy is characterised with ‘full adjustment’ between output and productive equipment so that the latter is utilised at the ‘normal’ level to produce the former. In addition, the distribution of income is ‘exogenous’ in the sense that one of the distributive variables at its ‘normal’ level is conceived determined in reference to the factors which lie beyond the purely economic circumstances (Bharadwaj, 1963).

The nomenclature ‘Classical-Keynesian’ may sound oxymoronic to some readers, for it is a conventional conception that Classical economics, with its association with Say’s Law, is incompatible with the principle of effective demand: the principle is the very antithesis of the law.[4] It apparently seems so, indeed. Given the size of capital stock in a period and the saving behaviour of the economy, the ‘normal’ utilisation required for the long-period state dictates the volume of ‘normal’ capacity saving, prior to the determination of investment. Then, it would seem, it is investment that should be aligned to that volume of saving in the long-period state (this is precisely what we have seen in relation to the Cambridge Keynesian approach).

The task of the present paper is to show how this seemingly obvious conclusion is not a necessity and to indicate the way (or, one of the ways) how the combination of the Classical economics perspective (in its modern, Sraffian, version) and Keynes’s principle of effective demand could instead yield a fruitful framework for an analysis of effective demand in the long period. (The long introductory discussion above hints at one of the key ideas necessary for such an attempt: the distancing away from the steady-state analysis.) Discussion in the present paper is presented in reference to the aggregate economy, but the case can be generalised to the economy explicitly featuring a multiple number of industries; in the latter case, some of the simplistic quantitative results obtained for the aggregate economy are replaced by more complicated and interesting ones.[5]

The paper proceeds as follows. Section 2 explains the concept of the ‘long-period position capital equipment’, one of the key concepts to be used in our attempt. It is also proposed that the long-period analysis consists of two parts: one dealing with the long-period position ‘existing in the present’ and another with the movement of the economy over time, in which the ‘present-existing’ long-period position plays a central role. Section 3 provides three systems of equations for the first part of the long-period analysis. These systems are respectively intended to describe three states of the economy related to the long-period position of the economy; it will be proposed that a proper long-period analysis should

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[4] However, Garegnani (1978, 1979a) argues that Say’s Law—expressed in the form of equality between normal capacity saving and investment—is not an integral part of Classical economics: it is a straightforward assumption, without any mechanism which brings about the equality (in contrast, such a mechanism, that is, variations in the rate of interest, is an integral part of neoclassical economics); hence that Say’s Law can be dropped out of Classical economics and replaced by Keynes’s principle of effective demand.

[5] A companion paper dealing with the multi-industry economy can be obtained on request from the author. The ideas underlying the present paper have been advanced, if in a much less refined form, in the second half of Park (2011).

consider all these systems of equation. Section 4 discusses the short-period position of the economy, to be distinguished from its long-period position; the key difference is to be found in the real wage rate—the real wage rate contemplated to be realised in the long-period position is the 'normal' real wage rate whilst that in the short-period position need not. Section 5 looks at one possible way of the determination of the normal real wage rate and the relation to it of the short-period real wage rate. The penultimate section touches briefly upon the movement of the economy over time. The conclusion section is for summary.

2. The long-period position capital equipment

At the beginning of a period, the economy is equipped with the capital equipment of a given configuration, $\overline{K}$.[6] Should this capital equipment be utilised at the normal degree, output would be at the normal capacity level, Y^*. Given the saving behaviour of the economy, the saving ensuing from Y^* is normal capacity saving, S^*. Then, the macroeconomic equilibrium condition of equality between saving and investment requires that investment should be at the level equal to S^*: investment should follow suit after normal capacity saving. Such volume of investment, I^*, would be warranted, should the given capital equipment be utilised at the normal level; hence, it shall be called the 'warranted investment' corresponding to the given capital equipment.

The principle of effective demand means that actual investment, I, is equal to warranted investment only by chance. Corresponding to actual investment is the level of output, Y, which accordingly deviates from the normal capacity level. That level of output is such that the volume of actual saving ensuing from it is equal to I. As actual output deviates from the normal capacity level, the ratio between them, that is, the degree of utilisation, $u \equiv Y/Y^*$, will be at a level different from unity.

We now pose the following question. *Given actual investment, what configuration of capital equipment would have had that investment as the warranted investment associated with it?* We denote such hypothetical capital equipment by $\widetilde{K}$. Had the existing capital equipment been $\widetilde{K}$, actual investment I would have been the warranted investment corresponding to $\widetilde{K}$, and this capital equipment would have been utilised at the normal level.[7]

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[6] By the 'configuration' of capital equipment, we mean both the level of and the proportions among the various means of production constituting the capital equipment. Discussion in terms of the aggregate economy exempts us from explicitly considering the proportion aspect; such exemption is, of course, not allowable when considering the economy explicitly disaggregated into a multiple number of industries.

[7] The concept of $\widetilde{K}$ was first proposed in Park (1994), with inspiration from Harrod (1939, 1948). The use of the concept in that work is, however, slightly different from that in the present paper. That work proposes an interpretation of Keynes's theory of employment as a 'short-period analysis in the long-period framework': corresponding to a given 'state of long-term expectation' held by entrepreneurs exists a long-period position, which, characterised with normal utilisation of productive equipment, is achievable (or set as an objective to

Figure 1 illustrates the case. If actual investment is smaller than the warranted one that would utilise the existing capital equipment at the normal level ($I < I^*$) so that actual output is below the normal capacity level ($Y < Y^*$), the capital equipment which would have produced the actual output at the normal utilisation must be smaller than the existing capital equipment ($\widetilde{K} < \overline{K}$). We shall call $\widetilde{K}$ the 'long-period position (LP) capital equipment' associated with autonomous investment *I*.

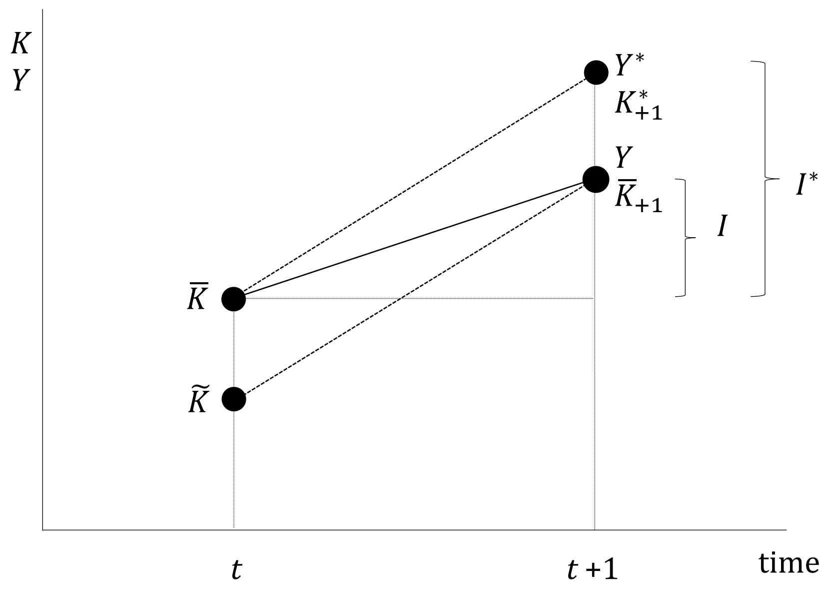

Figure 1. The long-period position capital equipment

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achieve) *in the future*; the economy tries to reach that long-period position through the appropriate utilisation of productive equipment in the short period; the principle of effective demand means that the long-period position is different from the state that would result from continuous normal utilisation; hence, over the periods up to the period when the long-period position is achieved (if only theoretically), productive equipment will be observed to be utilised at levels, in general, different form the normal one. That is, the short-period utilisation of productive equipment (with its associated production of output and labour employment in the short period) is geared to the achievement of the long-period position. $\widetilde{K}$ is obtained in each period corresponding to the short-period utilisation of the existing capital equipment; however, it only plays the role of guiding the economy towards *a long-period position which exists in the future*. By contrast, in the current paper, the state of the economy expressed by $\widetilde{K}$ is itself considered to be a 'long-period position': the long-period position 'exists in the present' (see below). Park (2000), in the discussion of 'Sraffian supermultiplier', utilises the concept of $\widetilde{K}$ in the meaning nearer to that developed in the present paper.

To understand why we nominate such capital equipment for the status of 'long-period', we need to first understand the concept of a 'fully-adjusted position'. The state of the economy is a fully-adjusted position when the relation between productive equipment and the quantity of produced output is such that the following two conditions are satisfied: (*i*) productive equipment is utilised at the 'normal' level to produce that quantity of output, and (*ii*) the quantity of produced output is at the level exactly matching its total use (that is, use for the replacement of used-up capital equipment, new investment and consumption in the economy as a whole).

The state of the economy represented by $\{\bar{K}, Y^*, I^*\}$ is a fully-adjusted position: Y^* is obtained in such a way that not only is $\bar{K}$ utilised at the normal level but also Y^* is exhausted for the replacement of used-up part of $\bar{K}$, net addition to $\bar{K}$ (which is warranted net investment I^*), and consumption. Such a state of economy can be called, in the spirit of Harrod (1939), the 'warranted growth' (WG) state. What is crucial for our objective is that there is also full adjustment between $\tilde{K}$ and Y. These latter two magnitudes will be determined in such a way as satisfies the two conditions of a fully-adjusted position. As much being a fully-adjusted position as the WG state but, differently from it, *configured under the constraint of effective demand*, the state of the economy represented by $\{I, Y, \tilde{K}\}$ shall be given the name of a 'long-period position' (LP), in the place of a possible full name, the 'effective-demand-constrained fully-adjusted position'.

The conferment of 'long-period' to such a state of the economy is grounded on the following reasoning. The long period is characterised with the free movement of (financial) capital in pursuit of a higher rate of return and thus implies changes in the configuration of capital equipment. In this pursuit, entrepreneurs endeavour to adjust the configuration of their capital equipment to the demand for their products so that the installed capital equipment can be utilised at the normal level (and the return on their capital can be at the normal level). In our present case where the current state of effective demand is actual investment being made at the level of I, that configuration of capital equipment is $\tilde{K}$.

This reasoning leads to the next. It is obvious that, in a growing economy in particular, $\tilde{K}$ (understood as its particular level) should not be the final goal which entrepreneurs would wish eventually to achieve: as far as net investment is positive, the capital stock keeps growing; then so does actual investment (unless the proportion which net investment bears to the capital stock shrinks faster than the growing capital stock). This means that $\tilde{K}$ (in its general meaning) will also keep growing. In this context, $\tilde{K}$ (in its general meaning) is better conceived simply as a configuration that entrepreneurs regard as 'normal' with regard to the current state of effective demand. Then, precisely for that reason, it will act as a 'reference point' that entrepreneurs take into account when they make decisions regarding investment next period.[8] In Figure 1, actual investment having been *I* in period *t*, the economy begins

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[8] If the much-used expression 'a centre of gravity' does not connote the eventual convergence to a situation of an *unchanging* configuration, one may also use that expression

period t +1 with capital equipment $\overline{K}_{+1}$. In the new period, various factors will enter into the determination of actual investment, among which is, undoubtedly as one of the main determinants, the gap between $\overline{K}$ and $\widetilde{K}$. Though $\widetilde{K}$ (in its general meaning) is a non-observed configuration of capital equipment, it is expected to exert observable influence on the investment behaviour of entrepreneurs in the coming periods.

Thus, we understand the long-period position not exclusively as some state of the economy that will be reached in a distant future (conventionally as a stable, steady-growth, state), but foremost as a state of the economy that 'exists in the present' as a fully-adjusted position and, being considered 'normal' in relation to the currently given state of effective demand, directs the movement of the economy in the coming periods. This understanding also indicates our conception of the long-period analysis. We take the long-period analysis as consisting of two parts. One part is concerned with how a 'present-existing' LP is configured corresponding to the current state of effective demand. The concern of the other part is how the LP evolves from period to period, in interaction with the state of effective demand; actual investment being the main determinant of an LP, this implies the need for investigations into the determination of actual investment.[9] The first part of the long-period analysis has been left in the cold whilst the second part—foremost in the form of analysis of steady-state growth, hence unwittingly in contradiction with the principle of effective demand—has been attracting researchers' attention out of proportion.[10]

Equipped with this understanding, we now turn to a formalised representation of the long-period position and the long-period analysis.

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instead of our term which has a much weaker connotation.

[9] It turns out that this understanding of the long-period position and the long-period analysis fits nicely into a paraphrasing of Garegnani's (1979b) expression, quoted as the epigraph of the present paper: it is in the 'present' that the long period position is firmly located; because this is the state of the economy which is being regarded as 'normal' with reference to the state of effective demand *in the present*, it is also the state of the economy that present experience will lead entrepreneurs in general to take into account when they make decisions on their investment *in the future*.

[10] Milgate (1982), Eatwell (1983), and Eatwell and Milgate (1983) have argued that Keynes's *General Theory* is in fact an example of the long-period analysis, despite the fact that analysis in the book is carried out with the elements conventionally considered as defining a short-period state of the economy such as the existing capital equipment. Our framework for the first part of the long-period analysis may be considered as opening a welcome gate to such an interpretation; or, at least, while avoiding getting embroiled in intricate interpretative matters, as showing how consideration of conventional short-period elements is not incompatible with—but rather necessary to—the long-period analysis. Indeed, the present paper is a highly sophisticated development, in a formalized framework, of Park (1994), which attempts to interpret Keynes's analysis in *The General Theory* as a 'short-period analysis in the long-period framework'.

3. Three systems of equations for the long-period analysis

This section presents the first-half part of the long-period analysis of effective demand: the characterisation of the long-period position associated with a given current state of effective demand.

The framework we propose for that purpose consists of three systems of equations. The first system is to formalise the WG state. The second is to represent what we shall call the 'effective long-period position' (*e*LP). Through this system the LP capital equipment is obtained.[11] We need to consider the WG state before the *e*LP because the former will serve as the baseline in comparison with which the latter is specified. These two systems share a common characteristic in that their respective objects of analysis—the WG state and the *e*LP—are both fully-adjusted positions. The third system is to describe what we shall call the 'realised long-period position' (*r*LP), which results from the consideration of the *e*LP against the backdrop of the existing capital equipment. The *r*LP is not a fully-adjusted position except by chance. Consideration of all these three systems of equations is, it will be suggested, necessary for a comprehensive analysis of the *long-period* configuration of the economy.[12]

The dominant technique in operation in the economy is represented by the capital-output ratio (v) and the labour-output ratio (l), both engineering-specified in reference to the normal utilisation of capital equipment; whilst the realised capital-output ratio changes in response to the degree of utilisation of capital equipment, the realised labour-output ratio is assumed to remain the same as the engineering-specified ratio regardless of the degree of utilisation. Capital equipment depreciates at the rate of δ^* per period when it is utilised at the normal level.[13] The triplet $\{v, l, \delta^*\}$ represents the given technical parameters.[14] There are two social classes in the economy, workers and capitalists: workers do not save and capitalists save a fraction s of their income (profits). In accordance with the Classical-Sraffian perspective, the real wage rate w is assumed exogenously given (or, essentially the same

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[11] In the case of the multi-industry economy, the *e*LP system will also determine the configuration of long-period relative prices ('normal prices', the prices of production), which will be used in valuing commodities in various kinds of long-period positions of the economy; hence, the qualifier 'effective'.

[12] When we move from the system of equations for the *e*LP to one for the *r*LP, we shall consider another system which intends to describe what is to be called the 'notional long-period position' (*n*LP). The *n*LP system can eventually be excluded from our framework for the long-period analysis; it merely serves as a jumping stone for an explanation of the *r*LP.

[13] The Sraffians will not agree with this setting of the 'radioactive' depreciation rate; however, excuse can be granted for the expositional purpose. The depreciation rate may be affected by the degree of utilisation; this aspect of depreciation takes non-negligible importance when we specify the third system of equations.

[14] In fact, δ^* is a particular revelation, to be observed when the degree of utilization is normal, of a general engineering relation that is to hold between the rate of depreciation and the degree of utilisation of capital equipment; see below.

argument can be made assuming the normal rate of profits r^* is exogenously given).

The existing capital equipment at period *t* is $\bar{K}$. The WG state of the economy corresponding to this capital equipment is one where it is utilised at the normal level and the resulting quantity of output Y^*, which is normal capacity output, exactly matches its use in the economy, that is, it is exhausted for the replacement of used part of the capital equipment, net investment and consumption. Thus, the following relations represent the WG state:

(WG-1) $Y^* = \bar{K}/v$

(WG-2) $Y^* = (\delta^* + g^*)\bar{K} + C^*$

(WG-3) $Y^* = (\delta^* + r^*)\bar{K} + wlY^*$

(WG-4) $Y^* - C^* \equiv S_G^* = sr^*\bar{K} + \delta^*\bar{K}$

where g^* is the rate of net accumulation which is to be observed under the stated conditions and hence called the 'warranted rate of (net) accumulation'. The first equation specifies the condition of the normal utilisation of the existing capital stock (hence, the rate of depreciation and the rate of profits appearing in the other equations are their respective normal levels). The second is the 'quantity equation', which describes produced output being exhausted as replacement, net investment and consumption. The third equation is the 'price equation': the value of output is divided into depreciation, profits and wages. The fourth equation is the (gross) saving function. The satisfaction of the first two equations means that the state of the economy described by the system of equations is a fully-adjusted position. The WG state, to be described by $\{Y^*, g^*, r^*, C^* \mid w, \bar{K}\}$,[15] is a fully-adjusted position holding for *the existing capital equipment*. (For discussion in later parts of the present paper, and also as a pointer for an extension to the multi-industry economy, it should be mentioned here that in the WG state the unit price of physical output, to be denoted by P, is at the 'normal' level $\tilde{P}$. The 'normal price' of output may be considered to be unity until we specify the money wage rate W; in this latter case, because $W = w\tilde{P}$, the 'normal price' of output can be calculated for given values of w and W. In the system above, thus, all variables are in 'real' terms.)

The resulting Cambridge equation, $g^* = sr^*$, represents equality between (net) investment and (net) saving. With $\bar{K}$ exogenously given and r^* determined solely in reference to the exogenously given variables ($\bar{K}$ and the technical variables), the resulting saving ($sr^*\bar{K}$) is the normal capacity saving corresponding to the given capital equipment and is determined independently of the volume of investment. It is the case, thus, that investment should be brought into line with the normal capacity saving. This investment, $I^* \equiv g^*\bar{K}$, is the volume of net investment which is *required* in order to ensure the normal utilisation of the capital equipment; hence dubbed, in the spirit of Harrod (1939) again, the 'warranted (net) investment'. The warranted investment is not autonomous.

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[15] Variables on the left side of the vertical separator are endogenous variables whilst those on the right side are exogenous variables (though not featuring the technical variables and the saving behavior parameters).

Part of the task of specifying the system of equations for a long-period position involves discussion on the way of expressing the autonomy of investment formulaically so that it can be made to stand in the system of equations designed for the determination of a long-period position. For that purpose, we shall express the volume of actual *gross* investment in the current period, denoted by J, as a fraction (or a multiple) z of its warranted counterpart, J^*:[16]

$$J = zJ^* \equiv z(\delta^* + g^*)\bar{K}$$

Then, the autonomy of investment can be expressed by the proposition that $z = 1$ is not necessarily the case. This way of formularising actual investment reflects an important aspect of the long-period analysis. The long-period analysis pays attention to both of the two effects of investment: the effect of constituting effective demand in the current period and the effect of generating the productive capacity of future periods. An increase in productive capacity is always accompanied, for the given parameters, with an increase in warranted investment. Thus, in the above formulisation of the determination of actual investment, the part J^* reflects the capacity-generating effect of (past) actual investment and the part z stands for its (present) effective-demand-constituting effect (and, with $z = 1$ not necessarily the case, the autonomy of investment).

The following set of relations is a formalised representation of the 'effective long-period position' (*e*LP) associated with gross investment autonomously given at the volume of $J = zJ^*$:

(*e*LP-1) $\tilde{Y} = \tilde{K}/v$

(*e*LP-2) $\tilde{Y} = (\delta^* + \tilde{g})\tilde{K} + \tilde{C}$

(*e*LP-3) $\tilde{Y} = (\delta^* + \tilde{r})\tilde{K} + wl\tilde{Y}$

(*e*LP-4) $\tilde{Y} - \tilde{C} \equiv \tilde{S}_G = s\tilde{r}\bar{K} + \delta^*\bar{K}$

(*e*LP-5) $(\delta^* + \tilde{g})\tilde{K} \equiv J = z(\delta^* + g^*)\bar{K}$

The first equation expresses the first condition for a fully-adjusted position: output $\tilde{Y}$ is produced by the normal utilisation of capital equipment $\tilde{K}$. The second equation—the quantity equation—incorporates the second condition for a fully-adjusted position ('output = its total use'). $\tilde{K}$ being utilised at the normal level, the depreciation rate is at the level corresponding to the normal utilisation; $\tilde{g}$ stands for the ratio that net investment bears to LP capital equipment $\tilde{K}$.[17] The third (the price equation) now has the rate of profits at $\tilde{r}$, which

[16] The preference to consider in terms of *gross* investment is based on two reasons, one analytical and another economic; see footnote 18 and 22.

[17] A warning is in order at this junction: $\tilde{g}$ is not a measure of how the LP capital equipment accumulates over time (the 'rate of accumulation' of the LP capital equipment). This is because it is not that J is actually grossly added to $\tilde{K}$ to change the size of LP capital

should reflect the difference from the WG state in the level of output and the size of capital equipment. The fourth is the corresponding saving function. The fifth expresses the idea that actual gross investment is given at the level of J. The state of the economy, to be described by $\{\tilde{Y}, \widetilde{K}, \tilde{g}, \tilde{r}, \tilde{C} \mid w, J\}$ is a fully-adjusted position under the constraint of effective demand, that is, in short, a long-period position.[18]

It turns out that $\tilde{r} = r^*$; also, the ratio of net investment to capital equipment in this state, $\tilde{g} \equiv (J/\widetilde{K}) - \delta^*$, is equal to its WG counterpart, g^*; thus, the Cambridge equation derived for the LP, $\tilde{g} = s\tilde{r}$, is nothing but a copycat of its counterpart for the WG state, $g^* = sr^*$.

Differences exist, however, between the LP and its WG counterpart. An obvious, but the most important, qualitative, difference is that in the former an autonomously given volume of investment determines output and its corresponding fully-adjusted capital stock, whilst in the latter the existing capital stock dictates the volume of investment that will bring about its utilisation at the normal level. There are also quantitative differences. The comparison of the two states of the economy reveals:

$$\frac{\widetilde{K}}{\overline{\overline{K}}} = \frac{\tilde{Y}}{Y^*} (\equiv \tilde{u}) = z$$

The *e*LP is the uniformly scaled-down (or scaled-up) version of its corresponding WG state: the factor that determines the scale is z.[19] The degree of utilisation being, for example, less than unity means that $\widetilde{K}$ is smaller than the existing capital equipment; that is, capital equipment should have been $\widetilde{K}$, smaller than the existing one, if there was to be the normal utilisation of capital equipment given the current state of effective demand, which is represented by actual gross investment J, smaller than its warranted counterpart. This judgment of entrepreneurs will set in train investment decision in the next period. Though the LP capital equipment is a hypothetical magnitude, unobservable in the actual economy, it ‘exists’, so to speak, beneath the surface of the actual economy and gives impetus to it.

The hypothetical character of the LP capital equipment, however, carries some unsettling implications. The state of the economy described by $\{\tilde{Y}, \widetilde{K}, \tilde{g}, \tilde{r}, \tilde{C} \mid w, J\}$ is not observable, for $\widetilde{K}$ foremost, being hypothetical, is not observable (at least, not directly). The capital

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equipment but it is that $\widetilde{K}$ is *determined* in reference to J; as $\widetilde{K}_{+1}$ is determined in reference to J_{+1}, the evolution of the LP capital equipment ($\widetilde{K}_{+1}/\widetilde{K}$) shadows the evolution of actual investment (J_{+1}/J); see below.

[18] This system of equations makes clear one of the reasons, an analytical one, why we take gross rather than net investment when representing the autonomy of investment. If we took net investment, the actual net investment would be bound to be nil when the warranted net investment was nil; then, the last equation in the system would lose its operational significance (though, with the fourth equation gone, one should and could make z_t, the key variable in our framework, appear in some of the other equations).

[19] This simplistic result does not necessarily hold in a multi-sector framework.

equipment to be observed in the actual economy is $\bar{K}$, the capital equipment existing in the period under consideration. This implies that, even if the quantity of output is at its *e*LP level $\tilde{Y}$ in the actual economy, the *observed* values of the rate of accumulation and the rate of profits, *which are to be measured in relation to the existing capital equipment*, will not be $\tilde{g}$ and $\tilde{r}$, respectively. The *e*LP, where output stands at $\tilde{Y}$ corresponding to the given current state of effective demand J,[20] should appear, when described against the backdrop of the existing capital equipment, in a different shape from that obtained in the second system of equations.

The description in question is given by the following set of equations:

(*n*LP-1) $\tilde{Y} = (\bar{K}/v)\tilde{u}$

(*n*LP-2) $\tilde{Y} = (\delta + g)\bar{K} + C$

(*n*LP-3) $\tilde{Y} = (\delta + r)\bar{K} + wl\tilde{Y}$

(*n*LP-4) $\tilde{Y} - C \equiv S_G = sr\bar{K} + \delta\bar{K}$

(*n*LP-5) $(\delta + g)\bar{K} = J = z(\delta^* + g^*)\bar{K}$

In this system of equations, capital equipment is the existing capital equipment $\bar{K}$; autonomous gross investment is, as always, given at the volume of $J = z(\delta^* + g^*)\bar{K}$, which now reveals itself in the form of $(\delta + g)\bar{K}$, as is expressed in the fourth equation;[21] and output is at the *e*LP level $\tilde{Y}$ corresponding to that gross investment. The first equation merely repeats the fact that, when output is $\tilde{Y}$, the existing capital equipment is to be observed as utilised at the degree of $\tilde{u} = \tilde{Y}/(\bar{K}/v) = \tilde{Y}/Y^*$. The resulting state, $\{\tilde{u}, g, r, \delta, C \mid w, J, \bar{K}, \tilde{Y}\}$, is the configuration of the *e*LP *described against the backdrop of the existing capital stock.*

However, this state thus described has a peculiar characteristic: the rate of depreciation δ is entirely endogenously determined. This cannot be the case. When capital equipment is utilised at the normal level, its depreciation rate is δ^*. It may well be expected, however, that the rate of depreciation of capital equipment is affected by its degree of utilisation: how intensively a machine is utilised in a given period may have some impact on the extent of its

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[20] As has been mentioned above, the *e*LP system involves the determination of long-period *relative* prices ('normal prices') as well as the LP capital equipment in each industry and its associated long-period level of output. The current focus on the aggregate economy leaves no room, of course, for discussion on relative prices; here, one can only talk—if a need arises for such a talk—about the 'normal price level' ($\tilde{P}$): usually one sets $\tilde{P}$ at unity, but when the money wage rate $\bar{W}$ is explicitly given, one can obtain $\tilde{P} = \bar{W}/w$.

[21] Thus, one always has $\delta + g = z(\delta^* + g^*)$. This implies that considering the autonomy of investment in terms of taking the volume of autonomous investment as a fraction z of the volume of gross warranted investment, $J = z(\delta^* + g^*)\bar{K}$, is equivalent to considering it in terms of taking the gross 'realised' rate of accumulation $(\delta + g)$ autonomously as a fraction z of the gross warranted rate of accumulation.

wear and tear over the same period (for that matter, the case in which no such impact exists, though extreme, is possible).[22] The relation between the rate of depreciation and the degree of utilisation must be understood as a matter of engineering, not a matter to be determined entirely endogenously. The engineering specification in question is given by[23]

$$\delta = \delta(u)$$

An immediate result of this is that, with the degree of utilisation given at $\tilde{u} = \tilde{Y}/Y^*$, there is generally a gap between the entirely endogenously determined depreciation rate and the engineering-determined one. When the engineering relation is added to the system of equations consisting of (nLP-1) to (nLP-5), the system is over-determined. The gap between the two rates of depreciation is equivalent to the deviation between gross investment (which is given at J) and the gross saving ensuing when the depreciation rate observes the engineering-specified rule.[24] With $\bar{K}, J, \tilde{Y}$ *and* $\delta(u)$ given, the economy will not be in equilibrium. In this sense, the state of the economy described by the above system of equations is a long-period position only notionally; hence, the name of the '*notional* LP' (nLP).

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[22] This is another reason, an economic one, why we take gross, not net, investment when considering autonomous investment. The degree of utilisation is endogenously determined and so will depreciation be if depreciation is a function of the degree of utilisation. In this situation, it seems economically more realistic to suppose that entrepreneurs make plans on the sum of depreciation and net increase of the capital equipment, the two elements not distinguished, and leave the partition between them endogenously determined, rather than fix the net increase first whilst bearing later the burden of depreciation endogenously determined as a result of that net increase.

[23] Over the domain $0 < u < u^{max}$ (with u^{max} usually larger than unity—the 'full' utilisation is higher than the 'normal' utilisation), $d\delta_t/du_t$ can be positive or negative or in alternate signs (each, linearly or at varying rates of change). One can consider two extremely simplified cases. One is where the depreciation rate is a function of time only, 'the angel's share', so that depreciation proceeds at a certain fixed rate (including the full-force rate, δ^*) regardless of how capital equipment is utilised: $\delta(u) = \bar{\delta}$. Another is where the depreciation rate is 'linear' with respect to the degree of utilisation: $\delta(u) = u\delta^*$.

[24] Given $\tilde{Y}$, the first equation determines the degree of utilisation at $\tilde{u}$; the engineering relation then determines the rate of depreciation at $\tilde{\delta}$; given $\tilde{\delta}$ and J together with $\bar{K}$, the rate of accumulation g is obtained in the fourth equation; also, given $\tilde{\delta}$ and $\tilde{Y}$, the value accounting relation—the price equation—determines the rate of profits. Another rate of profits can be calculated, however, from the quantity equation and the saving function. There is no guarantee that the two rates of profits thus obtained, denoted respectively r_1 and r_2, should always be equal. The difference between them reflects the difference between investment and saving, as it can be shown that $r_1 \gtreqless r_2$ as $J \gtreqless sr_1\bar{K} + \delta(\tilde{u})\bar{K}$.

The principle of effective demand comes into play: diversion between investment and saving will bring about changes in income in such a way that saving equals investment. In fact, these changes in income entirely take the form of changes in the quantity of output, without changes in its price (which has been set at unity). This is because we are contemplating the state of the economy where *the real wage rate is exogenously given* (at the 'normal' level; see below for more). A constant level of the real wage rate means that the money wage rate and the price level change, if at all, proportionately;[25] however, then, the proportionate change in the two magnitudes implies that the impact of any change in income will fall entirely on the quantity of output.

The state of the economy where this adjustment of output has been completed to equalise investment and saving is described by the following system of equations:

(rLP-1) $Y' = (\bar{K}/v)u'$
(rLP-2) $Y' = (\delta' + g')\bar{K} + C'$
(rLP-3) $Y' = (\delta' + r')\bar{K} + wlY'$
(rLP-4) $Y' - C' \equiv S'_G = s_c r'\bar{K} + \delta'\bar{K}$
(rLP-5) $(\delta' + g')\bar{K} = J = z(\delta^* + g^*)\bar{K}$
(rLP-6) $\delta' = \delta(u')$

The state of the economy that this system of equations, the third for our long-period analysis, describes is the '*realised* LP' (rLP). The reason for this christening is as follows. One of the crucial variables which define a long-period position is the 'given' real wage rate. Since our concern is the long-period state of the economy, in fact, the present paper has up to now been implicitly assuming that this real wage rate is what can be called the 'normal' real wage rate

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[25] This is easily seen in the aggregate economy. The case of the multi-industry economy is not that straightforward but the conclusion is essentially the same. The real wage rate is defined as $W/\mathbf{dp}$ where W = the money wage rate; $\mathbf{d} \equiv (d_1, d_2, \cdots, d_n) =$ the exogenously given unit wage basket; $\mathbf{p} \equiv (p_1, p_2, \cdots, p_n)^T =$ commodity prices. We start from the situation where the current money wage rate is W° and current relative prices are $\mathbf{p}^\circ$, hence the current real wage rate being $W^\circ/\mathbf{dp}^\circ = w^\circ$. Now suppose that W and $\mathbf{p}$ respectively change to $W' = \theta W^\circ$ and $\mathbf{p}' \neq \theta\mathbf{p}^\circ$, where θ is a non-zero scaler, whilst the real wage rate has remained at w°. This will be the case if and only if $\mathbf{dp}' = \theta\mathbf{dp}^\circ$. There are infinitely many combinations of p_i''s that satisfy this condition. However, *as far as there is no inherent mechanism which will lead p_i''s to satisfy the condition*, it is entirely by a fluke that the realized constancy of the real wage rate is due indeed to prices changing in that particular way. Thus, if $\mathbf{p}' \neq \theta\mathbf{p}^\circ$, the general case is $\mathbf{dp}' \neq \theta\mathbf{dp}^\circ$, hence the real wage rate not being maintained at w°. This in turn leads to the proposition that, in general, the continuing prevalence of w° under a new money wage rate $W' = \theta W$ means that new normal prices are $\mathbf{p}' = \theta\mathbf{p}$, that is, no change in the proportions among prices whilst their scale changes in proportion with the change in the money wage rate.

in the sense adopted in Classical economics, whether exogenously given at the 'subsistence' level or endogenously determined.[26] Now, given the real wage rate and gross investment, the *e*LP system determines the *e*LP quantity of output $\tilde{Y}$ and implicitly its 'normal price' $\tilde{P}$, this latter considered to be determined in relation to the given real wage rate and the prevailing money wage rate. Maintaining the given 'normal' real wage rate with the prevailing money wage rate implies, of course, maintaining the 'normal price' of output.[27] The *e*LP system and the *r*LP system share the same 'normal' real wage rate and hence the same 'normal price' of output. This is the very sense in which we understand the *r*LP system as referring to the *long-period* state of the economy despite the quantity of output differing from $\tilde{Y}$.

On the other hand, it is precisely this adjustment of output being carried out when we move from the *e*LP to the *r*LP system that entitles us to confer the characteristic of '*realisation*' to the state of the economy described by the latter. The existing capital equipment, which is the magnitude present physically in the actual economy, may suffer different degrees of wear and tear in accordance with varying degrees of utilisation. Unless the rate of depreciation and the degree of utilisation observe a particular relation (see below), the quantity of output cannot be maintained at the level calculated in the *e*LP system. For this latter level of output is obtained outright on the basis of normal utilisation (the LP capital equipment is endogenously determined at such size as to be utilised at the normal level), whilst the actual capital equipment is being utilised at an other-than-normal level. Adjustment of the quantity of output ensues and continues until the given volume of autonomous investment generates saving of the same volume *in the actual economy*. The variables determined in the *r*LP system are those which are to be *observed in the actual economy* when the economy is in equilibrium under the stipulated constraints. The economy will be observed to be in the shape of $\{Y', u', g', r', \delta', C' \mid w, \bar{K}, J\}$.[28]

The *r*LP is not a fully-adjusted position: the existing capital equipment is not utilised at the normal level. Despite that fact, however, the *r*LP *is* a long-period state in the sense given above: the economy, starting with the existing capital equipment ($\bar{K}$) and operating under the constraint of effective demand (J), has established the 'normal price' of output and consequently realised the given 'normal' wage rate (w)—at the same time, satisfying the macroeconomic equilibrium condition of the saving-investment equality in the actual

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[26] Section 5 touches upon a possible endogenous determination of the normal wage rate in the Classical way.

[27] This is, in general, the case also in the multi-industry economy, as has been argued in footnote 25.

[28] In the case where the depreciation rate is a function of time only, the realised rate of net profits is $r' = z(\delta^* + r^*) - \bar{\delta}$ and the realised rate of net accumulation is $g' = z(\delta^* + g^*) - \bar{\delta}$. From this, incidentally, one can notice that our framework allows for the shrinking of the economy, even if the WG state depicts a growing economy: it may be the case that z is so small that $z(\delta^* + g^*) - \bar{\delta} < 0$ even with $g^* > 0$. The case of the 'linear' depreciation rate has $r' = zr^*$ and $g' = zg^*$.

economy. The rLP is the shape of the economy as it will appear in reality when it settles in the long-period state under the constraint of effective demand.[29]

Figure 2 illustrates the argument in this section.

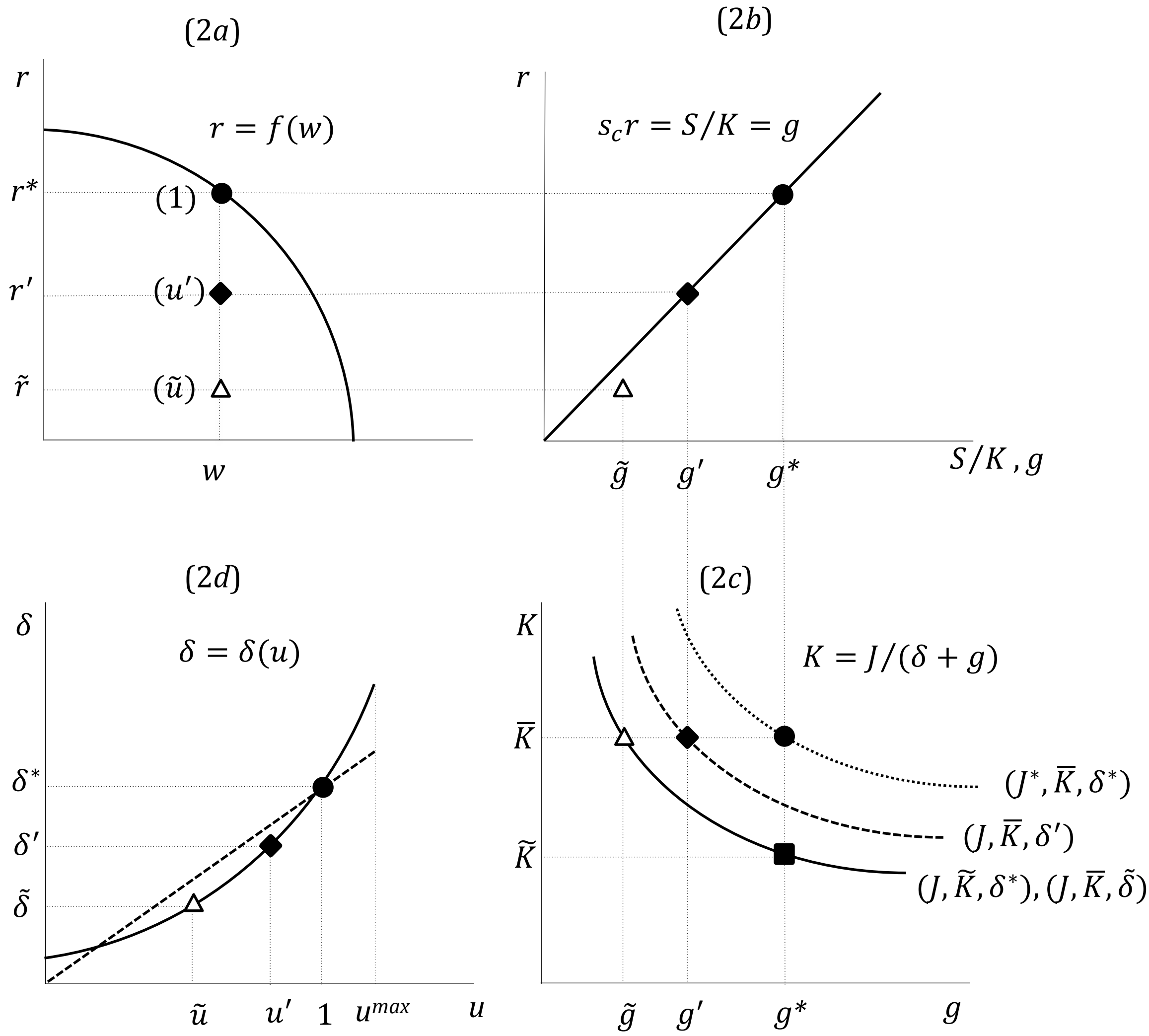

Figure 2. The states of the economy

- ● the warranted growth (WG) state
- ■ the effective long-period position (eLP)
- △ the notional long-period position (nLP)
- ◆ the realised long-period position (rLP)

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[29] The rLP coincides with the nLP if the rate of depreciation is linear in relation to the degree of utilisation: $\delta' = u\delta^*$ (this relation is obtained as the solution of the system for the nLP). Therefore, if the engineering relation is a linear map, $\delta(u) = u\delta^*$ for all u, then the rLP never exists in a different form from the nLP. In this case the rLP is a scaled-down (or scaled-up if $z > 1$) version of the WG state, with z as the scaling factor.

The curve in Panel (2*a*) is the relation between the rate of profits and the real wage rate when capital equipment is utilised at the normal level: the 'w-*r* curve'. For a given real wage rate, a lower degree of utilisation is associated with a lower rate of profits (in the illustration, $1 > u' > \tilde{u}$). Panel (2*b*) describes equality between saving and investment: the line associates the rate of profits with the net saving per unit of capital ensuing from it and thus, when the equality in question obtains, with the rate of net accumulation. (The line is usually taken to represent (a version of) the Cambridge equation; however, the line in the panel does not incorporate the particular causality constituting the Cambridge equation—causality will be in one or reverse direction depending on the states of the economy under consideration.) The curves in Panel (2*c*) are 'iso-gross-investment' curves, depicting those relations between the size of capital equipment (K) and the rate of net accumulation (g), for a given rate of depreciation (δ), which give rise to constant volumes of gross investment. Panel (2*d*) is the space for the engineering relation between the degree of utilisation and the rate of depreciation: the solid curve represents the case where depreciation accelerates with utilisation (and is in force at a certain positive rate even without being utilised); the dotted line is for the case of 'linear depreciation': $\delta = u\delta^*$. We assume the solid curve is in effect.

The story begins from Panel (2*a*). The real wage rate is given at w. The normal rate of profits is obtained as r^* on the *w*-*r* curve. Correspondingly the line in Panel (2*b*) gives net normal capacity saving per unit of capital equipment; owing to equality between saving and investment, the matching investment per unit of capital equipment is the net warranted rate of accumulation g^*. In Panel (2*c*) the lightly dotted curve, notated with $(J^*, \bar{K}, \delta^*)$, passes through the meeting point, marked by a filled circle, of g^* and the existing capital equipment, given at $\bar{K}$. This means that the volume of investment J^* associated with the curve is the gross warranted investment for $\bar{K}$. Since the existing capital equipment would be utilised at the normal level (u = 1), the rate of depreciation is δ^*, as is seen in Panel (2*d*). Hence, the notation $(J^*, \bar{K}, \delta^*)$ for the curve. This is the part of the story for the 'warranted growth' (WG) state, with the point marked by a filled circle in the panels representing the WG state corresponding to w and $\bar{K}$.

The autonomy of investment reveals itself as actual investment different from (usually smaller than) warranted investment; the volume of such autonomous gross investment, given at J, is represented by the heavily dotted curve, notated with $(J, \tilde{K}, \delta^*)$, in Panel (2*c*). The task now is to find the configuration of capital equipment that would be utilised at the normal level for the given real wage rate w and the given investment J. Such capital equipment, named the LP capital equipment, is found as $\tilde{K}$ when g^* is matched with J. Since this capital equipment is to be utilised at the normal level, Panel (2*d*) shows that its rate of depreciation is δ^*. The notation $(J, \tilde{K}, \delta^*)$ for the iso-gross-investment curve under consideration crystallises the idea behind this part of the story. The point on the curve, marked by a filled square, represents the 'effective long-period position' (*e*LP) corresponding to w and J. In the panels other than (2*c*), the point for the *e*LP overlaps with the one for the WG state (hence, the mark for it is not given an appearance there). The overlapping reflects the fact that both the WG state and the *e*LP are fully-adjusted positions.

The LP capital equipment $\widetilde{K}$ is not observable, however. The capital equipment to be observed in the actual economy is the existing capital equipment $\overline{K}$. The normal capacity output ensuing from $\widetilde{K}$ is to be observed in the shape of the output ensuing from the utilisation of $\overline{K}$ at the degree of $\tilde{u}$. It follows from Panel (2*d*) that the rate of depreciation of $\overline{K}$ is $\delta(\tilde{u}) = \tilde{\delta}$ and accordingly from Panel (2c) that the rate of accumulation over $\overline{K}$ is to be obtained as $\tilde{g} = (J/\overline{K}) - \tilde{\delta}$. All this is in fact a straightforward translation, against the backdrop of $\overline{K}$, of what is happening in the *e*LP; thus, the point representing the *e*LP, marked by a filled square on the curve $(J, \widetilde{K}, \delta^{*})$ in Panel (2*c*), simply moves along the curve up to the point where capital equipment is $\overline{K}$ (the terminal point is marked by an unfilled triangle). This means that the curve previously notated with $(J, \widetilde{K}, \delta^{*})$ can be equivalently notated with $(J, \overline{K}, \tilde{\delta})$, as is shown in Panel (2*c*). Panel (2*a*) shows that, measured against $\overline{K}$ and ensuing from its utilisation at the degree of $\tilde{u}$, the observed rate of net profits is $\tilde{r}$.

A problem arises at this junction. With the rate of profits and the rate of accumulation standing respectively at $\tilde{r}$ and $\tilde{g}$, it may turn out that $\tilde{g} = s_c\tilde{r}$ is not always the case: saving and investment may deviate from each other. The illustrated case in Panel (2*b*) is where saving falls short of investment: the meeting point of $\tilde{r}$ and $\tilde{g}$, marked by an unfilled triangle, lies below the line. The reason is that, given the engineering-determined relation between u and δ represented by the solid curve in Panel (2*d*), an increase in the utilisation of $\overline{K}$ at the lower range of the degree of utilisation causes depreciation at a lower rate than it generates profits; the result is that, at the degree of utilisation $\tilde{u}$, the levels of profits (hence, net saving) and of depreciation (hence, net investment) are not such as to bring about equality between net saving and net investment. The state of the economy marked by an unfilled triangle in the panels is the ‘notional long-period position’ (*n*LP).

The gap between saving and investment sets in train the adjustment of output. In the present case, where investment is larger than saving, output increases. Deprecation gets accelerated as the degree of utilisation gets higher, lowering net investment at a faster rate than before. Meanwhile, profits continue to increase at an unchanged rate with respect to utilisation. This means that at some degree of utilisation the rate of increase of deprecation exceeds that of profits and that some more increase in utilisation will be enough to make net saving catch up with net investment: this happens at the degree of utilisation u', higher than $\tilde{u}$. The corresponding rate of profits is r' (Panel (2*a*)) and the volume of net saving per unit of capital equipment, $s_c r'$ (Panel (2*b*)). Panel (2*d*) shows that depreciation is $\delta(u') = \delta'$, higher than $\tilde{\delta}$. The resulting volume of net investment, $J - \delta'\overline{K}$, is now equal to that of net saving, $s_c r'\overline{K}$. With the higher rate of depreciation, the given volume of gross investment J will now be represented in Panel (2*c*) by the curve notated with $(J, \overline{K}, \delta')$, its location being higher than the curve notated with $(J, \overline{K}, \tilde{\delta})$ reflecting the higher rate of depreciation. This is how the ‘realised long-period position’ (*r*LP), marked by a filled rhombus in the panels, is brought about.

(Alternative endings of the story exist. The material for variation is the engineering relation between u and δ. If the relation is a strictly concave function, the principle of effective demand works in the way of lowering the degree of utilisation, so that $u' < \tilde{u}$ and the iso-gross-investment curve for the *r*LP lies lower than the one for *n*LP. Make linear depreciation the case, the engineering relation being a linear map. Then, equality between saving and investment is ensured at any $\tilde{u}$; in other words, the *n*LP and the *r*LP always coincide—hence, no case arises for considering an iso-gross-investment curve for the *r*LP separate from the one for the *n*LP. The function in question may be set to be strictly concave and strictly convex in different parts over the possible domain of the degree of utilisation. The direction of output adjustment will then depend upon which part of the function the *n*LP degree of utilisation $\tilde{u}$ falls on.)

4. The short-period state of the economy

It may seem that the state of the economy obtained as an *r*LP is indistinguishable from the conventionally contemplated state of short-period equilibrium. In both cases, all technical variables—the capital-output ratio, the labour-output ratio and the engineering relation between the rate of depreciation and the degree of utilisation—are given; the existing capital equipment is given; the volume of investment is autonomously given and accordingly the degree of utilisation is endogenously determined, not necessarily at the normal level. In addition, the given real wage in the *r*LP system can be incorporated into a conventional system of short-period equilibrium through a given level of money wage rate and an appropriately given price level.[30] It would appear, thus, that no substantial difference existed also in the aspect of the real wage rate. But the understanding of the 'given' real wage rate is one of the crucial places where one should find the demarcation line between the *r*LP and a short-period equilibrium state—or, a 'short-period position' (SP).

The 'given' real wage rate in our long-period analysis is, following the perspective of Classical economics, the 'normal' (or 'natural') wage rate. It may be 'fixed' at the 'subsistence' level, reflecting not only physiological but also institutional, social and cultural needs of workers; or, it may be determined by the balance of power between employers and employees, with the ratio between the number of employed workers and the number of available workers (in the modern parlance, the 'employment rate') being one of the elements which affect the balance of power.[31] We understand the long-period position as the configuration of the economy where the normal wage rate, which is given for a given period, is realised: the relation between the money wage rate and the price level should be such as to give rise to the normal wage rate. In contrast, no such realisation is contemplated in the short-period position: the money wage rate and the price level need not be so aligned to each other as to produce the normal real wage rate—the real wage rate in the short-period analysis is whatever it turns out as the result of the short-period money wage rate and the short-period

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[30] For the multi-industry economy, the counterpart of an 'appropriately given price level' is an 'appropriately given scale of normal prices'.

[31] See Section 5.

price level.

There is another element that should serve as part of the demarcation line between the long-period position and the short-period position—the element that becomes visible only in the analysis of the multi-industry economy. The *e*LP system, when constructed for the multi-industry economy, determines not only the configurations of LP capital equipment (and their associated quantities of output) in the respective industries but also long-period relative prices, also to be called prices of production or normal prices. In the long-period position, whether the *e*LP, the *n*LP or *r*LP, commodities are valued in terms of normal prices. By contrast, the SP does not require the establishment of normal prices. Short-period prices are simply what they are (until one finds a mechanism that determines them); or, as in the case of 'market prices' in Classical economics, they may be determined under the influence of the size of the quantity of output 'brought to the market' relative to the quantity of output that ensues in correspondence with normal prices ('effectual demand').

Having pointed out this element of demarcation, however, we may still say that the characterisation of the long-period position simply with the realisation of the normal real wage rate can serve the purpose. Given the unit basket of wage goods, the realisation of the normal prices which correspond to a given normal real wage rate implies the realisation of that real wage rate. The reverse—the realisation of the normal real wage rate implying the realisation of normal prices—is not always true, for a fluke combination of changes in prices may leave the value of the wage basket intact. But, in general and at least in the current case of the aggregate economy, the two can be considered equivalent.[32]

The following system of equations describes the short-period position associated with gross investment J:

(SP-1) $Y'' = (\bar{K}/v)u''$

(SP-2) $Y'' = (\delta'' + g'')\bar{K} + C''$

(SP-3) $Y'' = (\delta'' + r'')\bar{K} + w''lY''$

(SP-4) $Y'' - C'' \equiv S_G'' = s_c r''\bar{K} + \delta''\bar{K}$

(SP-5) $(\delta'' + g'')\bar{K} = J = z(\delta^* + g^*)\bar{K}$

(SP-6) $\delta'' = \delta(u'')$

(SP-7) $w'' = \bar{W}/\bar{P}$

We need explain only equations (SP-3) and (SP-7): now, the real wage rate is not given at the normal level but determined at w'' as the ratio between a given money wage rate ($\bar{W}$) and a given price level ($\bar{P}$). As we are concerned with a given period, we may take the given money wage rate for granted; then, the distinction between the long-period position (in its many incarnations) and the SP is to be found in the possibility that the short-period price level may differ from the normal level, the latter having been set at unity. We shall assume that the short-period price level is whatever it is, without discussing any possible theory of its

[32] See footnote 25.

determination.[33] The configuration of the economy described by $\{Y'', u'', g'', r'', \delta'', C'', w'' \mid J, \bar{K}, \bar{W}, \bar{P}\}$ is the SP that will arise in correspondence with the given parameters.

In Figure 3, each curve plots the relation between the short-period price level and the short-period quantity of output for a given money wage rate, a given normal wage rate (and given normal prices in the multi-industry economy), a given volume of gross investment and the existing capital equipment.

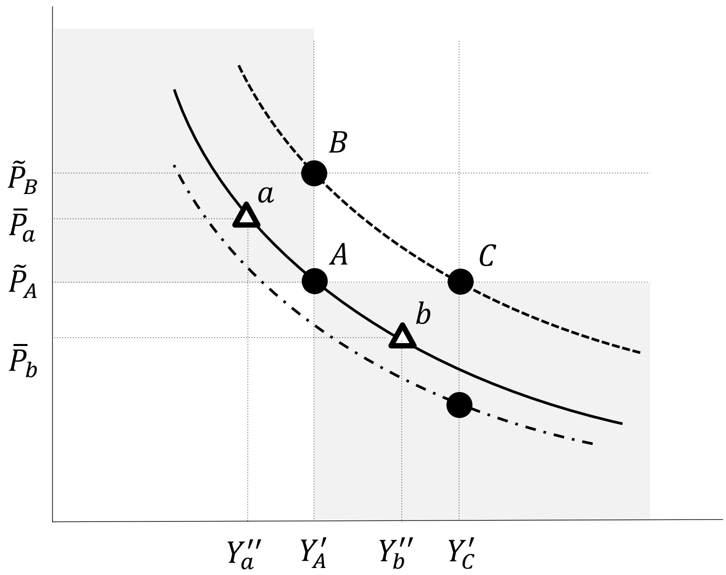

Figure 3. The *P*-*Y* curve

It is important to understand that each curve—to be called simply a '*P*-*Y* curve'—is defined in reference to an *r*LP. Point A, which the solid curve passes through, is an *r*LP where output Y'_A is the *r*LP output which ensues given the existing capital equipment, the normal wage rate and the volume of gross investment, and price level $\tilde{P}_A$ is the normal price level obtained for a given money wage rate $\bar{W}$. Given these variables, all the other points on the solid curve represents individual SPs. Point *a* is an SP where the price level is $\bar{P}_a$ (and, hence, the real

[33] One may think of 'mark-up' theory. As an alternative to taking the short-period price level as given exogenously, one may take, in a typically Classical way, the quantity of output (Y'') as given (the quantity 'brought to the market') and obtain the price of output as its 'market price' (but the SP system does not admit, as determinants of the 'market price', any 'accidental', non-quantifiable, influences that Classical economists tended to remark on). The same argument can apply to the SP of the multi-industry economy.

wage rate is $w''_a = \bar{W}/\bar{P}_a$) and output is Y''_a. Similarly point *b* is another SP with its associated price level (and thus real wage rate) and output.

If the money wage rate increases *ceteris paribus*, the *r*LP moves up to point *B*. The only result for the *r*LP of this change is a rise in the normal price level to $\tilde{P}_B$, with no impact on the *r*LP output. A *ceteris paribus* increase in the volume of investment moves the *r*LP out to the right to a point such as *C*, with the *r*LP output increasing to Y'_C; as far as there is no change in the money wage rate, there is no change in the normal price level. Should the normal wage rate increase whilst the money wage rate was held constant, the *r*LP should be dislocated to a point such as *D*: the normal price level should be lowered to bring about the higher real wage rate. But a more general case, where both the money wage rate and the price level increase with the result that the real wage rate increases (decreases), a new *r*LP will be found to the North-East (South-West) of point *A*.

An inverse relation between the price level and output, each of the *P-Y* curve in Figure 3 might be considered to be another version of the conventional ‘aggregate demand function’ (AD curve). But the *P-Y* curve is to be sharply distinguished from it. What the *P-Y* curve is saying is, in fact, not that the price level moves inversely with the quantity of output. What it describes in essence is an inverse relation between the *direction* of the gap of the short-period price level from the normal level ($\bar{P} - \tilde{P}$) and the *direction* of the gap of the quantity of short-period output from the quantity of the *r*LP output ($Y'' - Y'$): the former is positive (negative) if and only if the latter is negative (positive). The SP must be understood as a *deviation from* a given *r*LP; each *P-Y* curve is defined in reference to a particular *r*LP, which acts as the centre around which the short-period sets of price and output find their place. The place of an SP is either to the North-West or the South-East of the *r*LP in reference to which the SP is defined (the two shaded areas when the *r*LP is point *A*). The *P-Y* curves illustrated in Figure 3, while exhibiting the characteristic just mentioned, represent particular cases in the sense that they also exhibit inverse relations between the price *level* and the *quantity* of output. But this feature is not a necessary one and the following shape of the *P-Y* curve is possible, if extreme.

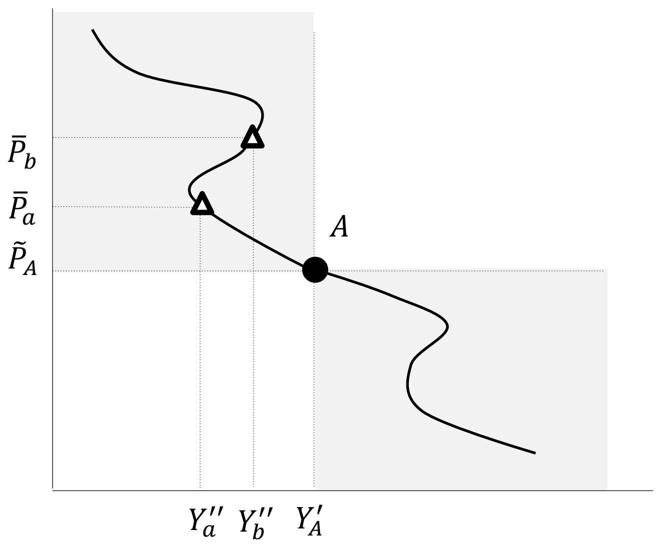

Figure 4. The P-Y curve: a possible case

In this case, as one compares an SP a with another b for example, one observes a *positive* relation between the price level and the quantity of output.[34] The culprit for this is the engineering relation $\delta = \delta(u)$. Over the possible domain of u, the technique may be such that δ responds to u in a complicated way, increasing with u in some parts and decreasing with u in other parts, and that very slowly in some parts and very fast in others.

If the AD curve has a part exhibiting a positive relation between price and output, the analysis based on such an AD curve will be faced with a grave theoretical problem: the stability of an equilibrium state cannot always be guaranteed. This problem arises because, along the AD curve, any set of the price level and output is a candidate for an equilibrium state (and, to make a theoretical sense, the equilibrium state must be stable): among the sets on the AD curve, any set which satisfies the 'aggregate supply' condition will be selected as an equilibrium state. That is not the case with the *P-Y* curve. *Not all* points on the *P-Y* curve are candidates for a resting state. The reason is to be found in the meaning of the SP: the SP is defined as a deviation of an *r*LP. Each *P-Y* curve passes through an *r*LP, which is determined for a given normal wage rate and a given state of effective demand. All the other points on that *P-Y* curve, representing individual SPs, are determined for short-period wage rates. Now, in Classical economics, the short-period real wage rate is not meant to stay where it is now but to tend to 'gravitate' towards the normal real wage rate. If, and as long as, the normal real wage rate acts as the 'centre of gravity' for the short-period real wage rate, the *r*LP corresponding to that normal real wage will also act as the 'centre of gravity' for the SP. Thus there is *only one* point on the *P-Y* curve that can be a resting point—the *r*LP corresponding to a given normal real wage and a given state of effective demand. (The argument of the 'centre of gravity', put forward in terms of the absolute size of the existing capital equipment and the absolute volume of effective demand, may be difficult to swallow with ease. Then, consider the matter in terms of u and z, instead of Y, J and $\bar{K}$. The *P-Y* curve can be converted one-to-one into the '*P-u*' curve whose position is determined by the normal real wage rate w and the state of effective demand z. As long as w and z is given, and as long as w acts as the centre of gravity for w'', the short-period position (P'', u'') on the *P-u* curve is only a

[34] The curve has been drawn with the price level as an independent variable, with the result that the mapping from the price level to the quantity of output is a function but the inverse mapping is not. If one takes the quantity of output ('brought to the market') as an independent variable, the curve will be drawn in such a way as to have each quantity of output matched with one price level whilst there are some price levels each of which is associated a multiple-number of quantities of output.

temporary situation on the way to the rLP $(\tilde{P}, u')$ which lies on the same curve.)

5. The normal real wage rate

A task left for us is, then, to show how the normal real wage rate is determined and how it acts as the centre of gravity for the short-period real wage rate. The determination of the real wage rate in Classical economics is one of the most controversial issues in the history of economic thought. Two interpretations are well known: the 'New View', associated with Samuelson (1978), Casarosa (1978) and Hicks & Hollander (1977) among others, and the 'Fix Wage' interpretations, held notably by the Cambridge economists such as Kaldor (1955-56) and Pasinetti (1959-60). Stirati (1994, 2011) criticises both of them on the ground that both rely on the wage fund theory and hence eventually on the concept of the down-sloping labour demand curve. She proposes her own 'Alternative' interpretation (which is in turn owed to Garegnani, 1984, 1990), where, in a given period, the normal real wage rate is determined by 'two sets of circumstances'. The first is 'sedimented historical circumstances, such as the customary standard living of workers, which determines the subsistence floor'. The other is 'current situations, such as economic factors affecting the bargaining position of the parties', among which prominence is given to 'the ratio of the quantity of labour demand and its supply' (Stirati, 2011, p. 352). We shall follow her interpretation in order to consider the determination of the normal real wage rate and its characteristics.

Stirati's interpretation can be given a formalised representation such as follows:

$$w = w(e) \equiv \begin{cases} \bar{w} \ > \ 0 & \text{if } 0 < e \leq \bar{e} \\ \bar{w} + f(e) > \ \bar{w}, \ \ \frac{df}{de} > 0 & \text{if } 1 \geq e > \bar{e} \end{cases}$$

where $\bar{w}$ is the subsistence level of wages, e the 'employment rate' (the ratio of the quantity of labour demanded to the quantity of labour available for employment), and $\bar{e}$ the threshold of the employment rate beyond which the normal wage rate starts to increase with the employment rate. The value of $\bar{w}$ reflects not only physiological but also institutional, social and cultural needs of workers; it is given in a given period though it may change over time for various reasons ('sedimented historical circumstances'). Similarly, in a given period, the threshold $\bar{e}$ is given though capable of changing over time as the 'bargaining position' of the two parties involved (workers and capitalists) changes. The bargaining position also reveals itself in the shape of function $f(e)$: if circumstances are more in workers' favour, the function is steeper. Function $w(e)$ shall be called the 'wage bargaining function'.

In a given period, the quantity of available labour is also given; denote it by L_s. The quantity of labour demanded in the rLP, denoted by L'_d, is:

$$L'_d = lY'$$

Then one can obtain the employment rate as

$$e = L_d'/L_s = e(w;\ \overline{K}, v, l, \delta(u), J, L_s)$$

Function $e(w;\cdot)$ shall be called the '(rLP) employment function';[35] it expresses the relation between the employment rate to be realised in the rLP and the real wage rate, in reference to the existing capital equipment, the technique, the state of effective demand and the existing quantity of available labour.

In a given period, where the productive technique, the configuration of capital equipment in use and the quantity of available labour are given, the normal wage rate is determined by the interaction between the wage bargaining function and the employment function. Figure 5 illustrates the case.

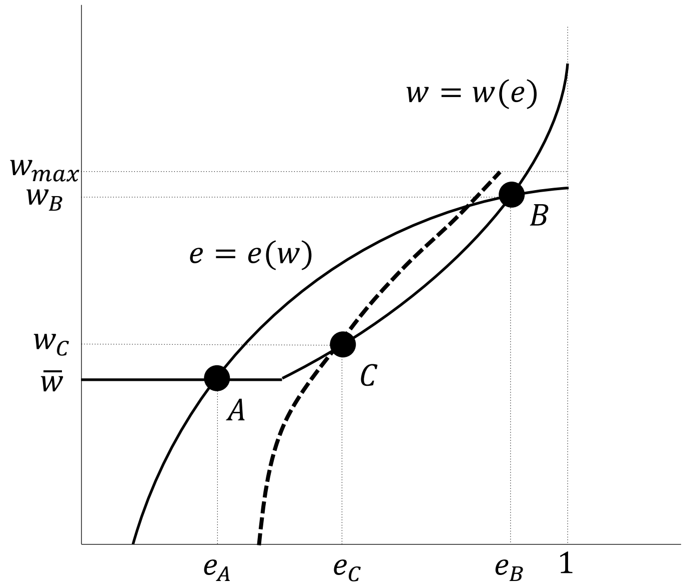

Figure 5. The determination of the normal real wage rate

Along the employment function, the employment rate increases with the real wage rate; labour employment is of a positive quantity even if the real wage rate is zero; there is a finite maximum real wage rate which corresponds to $r' = 0$ (or, as is theoretically possible, $r' = -1$). The employment function represented by the solid curve in the figure is drawn such that it is a strictly convex function (which is the case if, for example, $\delta(u) = u\delta^*$) and

[35] We shall shortly need to distinguish between the rLP employment function and the SP employment function. The inverse function of $e(w)$ may not exist. It can be shown that $\frac{de}{dw} > 0$ and $\frac{d^2e}{dw^2} > 0$ as long as $\frac{d\delta}{du} > 0$.

reaches the 'full employment' level ($e = 1$) before the real wage rate reaches its maximum (these features, of course, are not inherent ones). The wage bargaining function illustrated in the figure is such that the normal wage rate being tugged from both sides involved increases rapidly as the employment rate approaches the 'full employment' level.

Figure 5 illustrates the case where the two functions meet at two points, *A* and *B*. Only one of them represents a stable state: point *A*. It is in *two* meanings that the stability of the real wage rate represented by point *A* is to be understood. One is in regard to the other real wage rate represented by point *B*, and the other is in regard to the short-period real wage.

Let us discuss the first meaning first. The employment function drawn in the figure traces the relation between the employment rate and the candidates for the normal real wage rate, for the function is derived from the *r*LP system. If a 'candidate' normal real wage rate lies between $\bar{w}$ and w_B, the resulting employment rate, which lies between e_A and e_B, is not high enough for workers to maintain that real wage rate; the real wage rate keeps lowered until it reaches $\bar{w}$. By contrast, a 'candidate' normal real wage rate, if lower than $\bar{w}$, will lead to the employment rate at which the balance of power or the social norm regarding the living conditions of workers exerts influences favourable to workers—resulting in a higher real wage rate.[36] The real wage rate $\bar{w}$ is a stable normal real wage rate, and *this*—not w_B, which is an unstable normal real wage rate—is the normal wage rate which enters into the systems (*e*LP, *n*LP and *r*LP) for determining the long-period position. The (stable) normal real wage rate need not necessarily be at the 'subsistence' level $\bar{w}$, or 'fixed' at that level. If the technique in use is such that the employment function is represented by the dotted curve, the resulting normal real wage rate is w_C, higher than the subsistence level.

The second meaning of the stability of the real wage rate is in relation to the SP system *corresponding to that real wage rate*. Given the money wage rate at $\bar{W}$ in a given period, the short-period real wage rate is determined by the short-period price level P''; the short-period-ness of P'' is identified by its deviating from the normal price level, which is, for the normal wage rate at $\bar{w}$ (or w_C) given in a period, $\tilde{P} = \bar{W}/\bar{w}$ (or $\bar{W}/w_C$). Corresponding to the resulting short-period real wage rate, $w'' = \bar{W}/P''$, the SP system yields the SP level of labour employment: $L''_d = lY''$, and the 'SP employment function': $e = L''_d/L_s = \varepsilon(w'';\ \bar{K}, v, l, \delta(u), J, \bar{W}, P'', L_s)$. It is imperative to understand that the SP employment function has the characteristic that it always passes through a point such as *A* or *C*. This is because the SP which gives rise to that SP employment function is defined in very reference to the *r*LP represented by point *A* or *C*. It follows that such points, even when conceived as lying on the SP employment function, represent situations where the normal price level (normal prices, in the multi-industry economy) is prevailing.[37] Indeed, in the present

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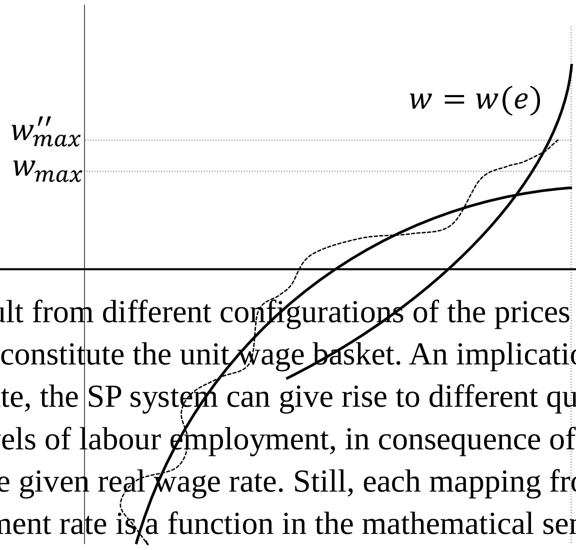
[36] If $e(w)$, with $\frac{de}{dw} > 0$, cuts $w(e)$ from below, the resulting normal real wage rate is stable. As $e(0) > 0$ and $\lim_{e\to 0} w(e) = \bar{w} > 0$, at least one normal real wage rate is stable if $w(e)$ and $e(w)$, with $\frac{de}{dw} > 0$, cross each other at all.

[37] It has been noted in footnote 25 that, in the multi-industry economy, the same real wage

aggregate economy model, the employment functions resulting from the *r*LP and the SP respectively are exactly the same (though the difference between them should be found in the meaning of the price level that corresponds to each real wage rate). This understood, the argument that the normal real wage rate at point *A* or *C* will act as the 'centre of gravity' for the short-period wage rate can proceed in a similar way as in the previous paragraph. In the multi-industry economy, the two employment functions usually deviate from each other, as is illustrated in Figure 6,[38] but the argument for the stability of the normal real wage rate with respect to its short-period counterpart will not be affected at all.



rate can result from different configurations of the prices of wage goods which in fixed proportions constitute the unit wage basket. An implication of this is that, even for a given real wage rate, the SP system can give rise to different quantities of output and therefore different levels of labour employment, in consequence of different configurations of prices matching the given real wage rate. Still, each mapping from the short-period real wage rate to the employment rate is a function in the mathematical sense, for each SP employment function is obtained for given configurations of prices that give rise to each given real wage rate (see the next footnote). What the implication above means is that the inverse mapping may not be a function. Nor does the implication have any impact on the characteristic just mentioned of the SP employment function: the SP employment function must contain a point, such as *A* or *C*, that represents the *r*LP in reference to which the SP is defined.

[38] The nature of the SP employment function remarked on in the previous footnote means that one cannot contemplate a determinate shape and location of it, apart from such points as *A*, until one knows the complete configurations of short-period prices which give rise to each of the possible real wage rate (for a given money wag rate). Hence, all the points, except point *A*, on the SP employment function in Figure 6 are arbitrary up to the condition that short-period prices corresponding to a given real wage rate w° be $\mathbf{p}^{\circ}$ such that $\mathbf{dp}^{\circ} = \overline{W}/w^{\circ}$: it is in reference to one, arbitrarily chosen, of the configurations of $\mathbf{p}^{\circ}$, which are infinitely many, that the SP quantity of output and therefore the SP level of labour employment corresponding to w° are determined so as to give rise to a particular SP employment function. With this caveat, the SP employment function in Figure 6 is drawn for the normal wage rate of $\overline{w}$; it is also assumed (for purely mind-fluttering purpose) that the maximum real wage rate (w''_{max}) corresponding to zero (or minus one) rate of profits in the SP system is larger than its counterpart in the *r*LP system and that, even at that maximum real wage rate, not all available labour is employed.

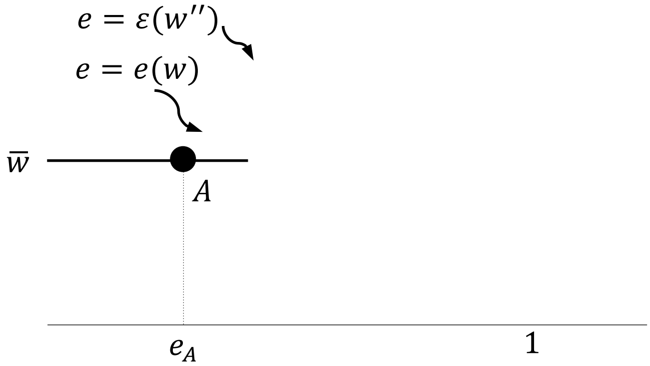


Figure 6. The stability of the normal wage with respect to the short-period wage (in the multi-industry economy)

The stability of the normal real wage rate, such as $\bar{w}$ and w_C, has an important implication. Note that at this level of real wage rate the level of employment in the long-period position is below the full employment level. But as long as there is no response from the given parameters, nor will the real wage rate change in response to this situation. More specifically, it does not decrease, in the neoclassical way, to increase the demand for labour. Should the real wage rate accidentally respond by going down, the quantity of labour demanded would fall therewith; then, at the resulting employment rate, the balance of power between the two parties concerned or the social norm regarding wages is such as to make the current real wage rate deemed 'too low'. The final result is the real wage rate going back, through wage bargaining, to the previous level. The less-than-full employment of labour can be a stable, long-period position (in the 'present-existing' sense), feature of the economy.

Back to Figure 5, we can carry out some 'comparative statics' exercises. If some factors other than the employment rate turn the bargaining position more into workers' favour, the wage bargaining function will move upwards (and the threshold $\bar{e}$ may also get smaller). The effect on the normal real wage rate is always to increase the stable real wage rate, as is naturally expected (whilst to decrease the unstable one). An increase in the volume of gross investment will move the employment function to the right. The result is, whether the employment function is convex or concave, an increase in the stable normal real wage rate (unless the economy is stuck in the subsistence wage interval); this is because the increase in labour employment ensuing from a higher volume of investment will enable workers to settle wage bargaining at that higher level of real wage rate. (Note that, by contrast, the effect on the unstable real wage rate of the same increase in investment is negative.)[39]

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[39] The number of stable real wage rate may be more than one, depending on the shapes of both the employment function and the wage bargaining function: for example, possibly in the case of the employment function ensuing from the *P*-*Y* curve given in Figure 4. Which one among them is realised in the actual economy will depend on where the economy has been in

6. The movement over time

We are now finally in the position to move on to the second part of the long-period analysis: the analysis of the movement of the economy over time. It is here that the role of the long-period position as a 'reference point' comes to the fore. The autonomy of investment does not necessarily mean that the entire part of z is given exogenously. The ratio of actual to warranted investment can be conceived to evolve over time. This evolution will be represented by a recurrence relation for z, and in this recurrence relation the configuration of the long-period position takes a central place.

However, now that we have distinguished between the *e*LP and the *r*LP, the problem arises which of them should serve as the reference point for investment next period. The *e*LP is the configuration of the economy which is considered as fully adjusted for a given state of effective demand. The *r*LP is the configuration where, against the background of the existing capital stock, the saving-investment equality brings about the adjustment of the quantity of output to the given state of effective demand, and, depending on the engineering relation regarding depreciation, this quantity of output may be different from the quantity ensuing from the *e*LP. It seems that no inherent priority can be given to either of them for the role. To make the matter simple, we shall in this section assume that the engineering relation regarding depreciation is that of linear depreciation. Then, the *n*LP and the *r*LP coincide for all levels of utilisation, and the *e*LP quantity of output is equal to its *r*LP counterpart.

The recurrence relation for z may be represented by either of the following:

$$z_{+1} = \bar{z} + \alpha[(\widetilde{K}/\overline{K}) - 1]\, z, \text{ with } 0 < \alpha < 1$$
$$z_{+1} = \bar{z} + \alpha[(Y'/Y^{*}) - 1]\, z, \text{ with } 0 < \alpha < 1$$

In the first, the 'reference point' role is given to the LP capital equipment. If the LP capital equipment is smaller than the actual stock, which means that the capital stock should have been smaller than the actual one if there was to have been normal utilisation, entrepreneurs are so discouraged in their investment behaviour that they set the ratio of the actual to the warranted investment in the next period below the level $\bar{z}$ (whose meaning will be discussed shortly). The second recurrence relation gives the role to the *r*LP configuration. The *r*LP quantity of output is compared with the WG quantity of output, and if the resulting degree of utilisation is lower than the normal level, investment next period is so much discouraged. But if we assume linear depreciation, these two ratios are equivalent.

Actual gross rate of accumulation $G(\equiv J/\overline{K}_t)$ in period *t* is related to warranted gross rate $G^{*}(\equiv J^{*}/\overline{K} = \delta^{*} + g^{*})$ through z:

$$G = zJ^{*}/\overline{K} = zG^{*}$$

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the recent periods: in general, the point which is nearer to the recent one will be realised; however, in a period of great turmoil, the other point may be realised.

Using the recurrence relation for z above and noting that $\widetilde{K}/\overline{K} = (J/G^*)/\overline{K} = G/G^*$, one gets[40]

$$G_{+1} = z_{+1}G^* = \bar{z}G^* + \alpha[(G/G^*) - 1]\, G$$

With this expression it may become possible to interpret the part $\bar{z}G^*$ as standing for the *expected rate of output growth*: entrepreneurs have in mind a certain expectation regarding the future growth of output and, taking it as the baseline of their investment decisions, adjust the rate of accumulation period by period around that expected rate of output growth in the light of the realised degree of utilisation (which reflect the degree of adjustment between the productive capacity and the actual output). One may also say that $\bar{z}G^*$ (or $\bar{z}$ alone, as G^* is determined by the given fundamental parameters) represents the state of 'animal spirits': the value of $\bar{z}$ which lies between zero and the unity $(0 < \bar{z} < 1)$ stands for a state of animal spirits 'below the par' ('low' animal spirits) and that which is larger than unity $(\bar{z} > 1)$ for a state of animal spirits 'above the par' ('high' animal spirits).

The long-period position capital stock $\widetilde{K}$, being determined in accordance with the actual investment, evolves over time in step with the latter. It turns out that the evolution of the rate of net accumulation along the path of long-period positions, $\widetilde{K}_{+1}/\widetilde{K}$, is precisely the same, if one period behind, as that of the actual rate of net accumulation: that is, $\widetilde{K}_{+1}/\widetilde{K} = G_{+1}$.

It goes without saying that the above formulation of the evolution of the rate of actual accumulation is not the only possible one; some other formulations have been in use in the literature.[41] However, the above formulation proves to facilitate discussion regarding some

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[40] Note that, from our discussion in Section 5 of the determination of the normal real wage rate, the evolution of z may be accompanied by the corresponding evolution of the normal wage rate (including the case in which the rate remains at the subsistence level); the evolution of the normal wage rate depends additionally on the movement of labour supply. In turn, the recurrence relation itself, expressed in terms of G, is affected by the evolution of the normal wage rate, for the LP capital equipment is determined in reference to the (now possibly changing) normal wage rate.

[41] They are:

(1) $z = 1$ for all *t*. This is the usual case of steady-state growth (the path of warranted growth): the capital stock is utilised continuously at the normal level. The case is better interpreted as the *required* condition for continuous normal utilisation and thus is in contradiction to the autonomy of investment.

(2) $z = \bar{z}$ for all *t*. The 'Kaleckian' approach (Rowthorn, 1981 and Dutt, 1990, to name just two from among the vast literature) focuses on a steady-state growth (at the gross rate of $\bar{z}G^*$) with the possibility of the degree of utilisation being permanently different from the normal level.

important points related to our conceptualisation of the effective-demand-constrained long-period analysis. The economy described by the above recurrence relation converges to the state of steady growth where the gross rate of accumulation is, if the economy is a growing one, different from the warranted rate.[42] It follows that the economy settles down at a state where the capital equipment is *not* utilised at the normal level.

This 'long-period' result—the steady state with constant utilisation of productive capacity at a level different from the normal one—is subject to at least two criticisms. Staying on the steady-state growth path contemplated here requires the violation of both the economic rationality of entrepreneurs and the principle of effective demand, as has been argued at the very beginning of the present paper.

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(3) $z_{+1} = z + \alpha[(\widetilde{K}/\bar{K}) - 1]$. The baseline is the current ratio (the current rate of accumulation) and an adjustment around the baseline is made in *no* reference to the current ratio (the current rate of accumulation). One stationary point exists, which is unstable: once $\widetilde{K}$ is ever different from $\bar{K}$, the actual rate of accumulation explodes in either direction over time. This obviously approximates Harrod's (1939, 1948) position.
(4) $z_{+1} = 1 + [1 - (1/z)]\bar{z}$, with $\bar{K} = \widetilde{K}$ for all *t*. This is an interpolation from Serrano's (1995) 'Sraffian supermultiplier'. Serrano considers the 'long-period' path of output where the capital stock is utilised always at the normal level (hence, $\bar{K} = \widetilde{K}$ for all *t*); the driving force of economic growth is non-induced investment ('autonomous investment'), which grows at an exogenously given rate ($\bar{G} \equiv \bar{z}G^*$). The recurrence relation shows the evolution of the rate of net accumulation which is *required* in order to ensure the normal utilisation of the capital stock period by period; the rate evolves over time as if the baseline was the warranted rate ($1 \times G^*$), pre-determined in accordance with the given fundamental parameters, and the variation from it was a fraction of the exogenously given rate of growth of non-induced investment, the fraction reflecting the difference between the warranted rate and the rate of accumulation of the previous period. As Trezzini (1995, 1998) and Palumbo and Trezzini (2003) aptly point out, this attempt contradicts the Keynesian principle of effective demand by stipulating that the capital stock be utilised always at the normal level (even if it does not follow the path of steady-state growth).
(5) $z_{+1} = 1 + \alpha[(\widetilde{K}/\bar{K}) - 1]$. This can be considered a variation over the Harrodian case, surprisingly with the opposite result. It is different from Harrod's case by taking the warranted rate itself as the baseline, with the adjustment continuing to be made with no reference to the current rate. There is a unique stationary point, which is globally stable, where the pre-determined warranted rate prevails.
(6) Palumbo (1996), Garegnani and Palumbo (1998), and Palumbo and Trezzini (2003) take a critical stance to attempts to understand the evolution of investment in a formulaic way such as above, arguing that the actual process of capital accumulation and output growth is too complex to be subject to such formalisation.
[42] The recurrence relation converges to a value between δ and $\bar{z}G^*$ (exclusively) if $G^* > \delta$, and equals G^* if $G^* = \delta$.

This must induce one to take the above formulation of investment behaviour over time as nothing but a heuristic tool. The important point, which the recurrence relation under consideration intends to convey as a heuristic tool, is that during most of the time the economy is being operated at a degree of utilisation that is different from the normal level, and this is the direct implication of the principle of effective demand. Over a multiple number of periods, this makes significant difference in the configuration of productive capacity of the economy and thus the capacity to employ labour. At the same time, as the relation also serves to convey, it cannot be denied that entrepreneurs continually make attempts to achieve full adjustment between the capital stock and output. There is a tendency in the real economy for the adjustment, if only approximate, between the configuration of the capital equipment and that of output so that the degree of utilisation of the productive equipment actually installed tends to approach the normal level. But this adjustment, even if fully achieved, should be taken to be the result of '[t]he elasticity [of] the capitalist economy … in reacting to incentives for a more rapid growth by bringing about additional productive capacity, or, symmetrically, by … erasing the visible traces of the losses in output due to a low such incentive' (Garegnani, 1992, p. 53). The tendency of the adjustment, whether full or only approximate, between the capital equipment and output should not be taken as an evidence of the latter following the former (supply-led growth). The truth is the opposite: the adjustment is a decisive evidence of the productive capacity being determined in relation to output (demand-led growth).

6. Conclusion

We argue that the analysis of effective demand in the long period consist of two parts. The first part deals with the configuration of a 'long-period position existing in the present'—that is, the configuration that will be generated in reference with the technique in use, the normal real wage rate and the state of effective demand, all prevalent in the current period. We take the 'long-period position' not as some state of the economy that will be reached in a distant future, usually as a stable, steady-growth state, but as a state of the economy that 'exists in the present' and, being considered 'normal' for the currently given state of effective demand, directs the movement of the economy over time. The second part is concerned with the movement of the economy by the configuration of the long-period position in the current period.

The present paper focuses on the first part of the long-period analysis of effective demand. It is an endeavour that analyses primarily how the 'present-existing' long-period positions are configured corresponding to the current state of effective demand *period by period*. For such an analysis, we have proposed to consider three systems of equations, respectively describing what we have named the warranted growth (WG) state, the effective long-period position (*e*LP) and the realised long-period position (*r*LP). The WG system provides the framework to determine the warranted investment corresponding to the existing capital equipment. The *e*LP system considers the fully-adjusted position of the economy under the constraint of effective demand, with the state of effective demand represented in relative terms to the WG state. The *r*LP system describes the long-period state in which all the relevant magnitudes—including the quantity of output, the rate of accumulation, the rate of profits and the depreciation rate of the existing capital equipment—are adjusted, against the backdrop of the existing capital equipment, to realise the given normal wage rate (and the 'normal price' of output) and to ensure that the volume of autonomously given investment generates saving of the same volume.

The evolution of long-period positions over time may be conceived by way of some formalised relation that intends to describe that evolution, but this is, in a sense, not absolutely necessary for the long-period analysis; there may be many possible alternative formalisations; or, as some insist, any attempt at such formalisations may be misleading. Whatever the stance, however, the crucial point should be that that evolution shadows the evolution of effective demand whilst at the same time guiding it.

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