Incomes from Capital in Alternative Economic Theories

Saverio M. Fratini

Incomes from Capital in Alternative Economic Theories

Saverio M. Fratini[*]

[*]*Roma Tre University*

Abstract

This paper addresses the theoretical explanation of profit and interest within three different approaches: i) classical/Marxian; ii) marginalist; and iii) Arrow-Debreu. In the classical/Marxian approach, incomes from capital are understood as a surplus whose amount depends on the class conflict. In the marginalist theory, the rate of interest is the price firms pay to households for the use of a factor of production named 'capital'. In the Arrow-Debreu equilibrium theory, interest rates and firm profit are still present, but, as we shall try to show, they can hardly be seen as income from capital.

Keywords: Capital profit; interest.

JEL codes: B51; D24; D51.

1. Introduction[1]

Empirical studies claim that, in the most industrialized countries, the share of capital in the national incomes is in the interval 25-30% (see Piketty 2014, p. 222, figure 6.5).[2] The nature of these incomes is quite evident in practice: they are equity dividends, capital gains, interest on loans, and managerial incentives. Much more complex is their analysis from the point of view of economic theory. In particular, as is clear, this analysis is closely linked to the notion of capital adopted.

As is well known, as far as income distribution is concerned, there are two alternative views in economic theory (see, in particular, Garegnani 1984, pp. 291-292). The first is the classical/Marxian approach, according to which income distribution is a social

---

[1] For comments and suggestions, thanks are due to Roberto Ciccone and Christian Gehrke. As for remaining errors and omissions, the usual disclaimer applies.

[2] According to Rognlie (2016), the decadal average of the net capital share of private domestic value added is above 30% in Canada and Italy (cf. Rognlie 2016, p. 16, table 2).

phenomenon: it depends on the conflict among social classes. The second is the neo-classical theory, which understands income distribution as a market phenomenon regulated by the equilibrium between supply and demand.[3]

Here we shall try to address the theoretical explanation of incomes from capital from both standpoints. We shall start, in section 2, with the classical/Marxian approach, in which – as we shall see – capital is understood as the amount of value that allows producers to pay costs in advance, before revenues are obtained, and profit is a residual – a surplus value. As for the neo-classical approach, we shall consider two different versions. In section 3, we shall deal with the traditional marginalist theory, in which capital is seen as a 'factor of production' and the rate of interest as the price firms pay for its use. Then, in section 4, we shall discuss the Arrow-Debreu model, in which capital is neither a 'factor of production', nor an amount of value that allows firms to pay costs in advance. Some conclusions are drawn in section 5.

2. Capital and profit in the classical/Marxian approach

In order to introduce the conception of capital within the classical/Marxian approach, we need to start with a quick reconstruction of the basic features of the capitalist mode of production. In particular, we can list the following important characteristics of this social organization:

    i) Products are commodities. Production is intended for the market and not (directly) for consumption.

    ii) Labour is performed by wage-earning workers. Workers are forced to sell their labour-power due to their separation from the means of production. Labour-power is the commodity workers sell in exchange for wages.

    iii) Natural resources (lands, mines, oilfields, etc.) are of private property. Every natural resource has an owner, and the class of landowners is distinct from the class of capitalists.[4]

---

[3] As is well-known, Veblen introduced the term 'neo-classical' with reference to Marshall's theory. With the aim of avoiding possible confusions, we can stress that here we consider as 'neo-classical' a certain representation of the working of the economic system. According to the neo-classical view, there are two groups of agents in the economy: households and firms. Households want to maximize their utility; firms their profit. Supply and demand functions arise from these maximizations. Prices and produced quantities are set so as to balance supply and demand on each market. For sure, both the traditional marginalist theory and the Arrow-Debreu theory refer to this view and, accordingly, they can be considered as two versions of the neo-classical approach.

  The neo-classical description of the working of the economic system is decidedly different from the one provided by the classical economists and Marx. In the latter, the key element is the organization of the society in classes with opposed interests. Economic activities mainly concern the conflict between different social classes and the competition among the members of each class. Although there is a 'core' of regularities whose study can be handled with purely logical (or even mathematical) tools, the working of an economic system cannot be understood without reference to its social and institutional characteristics.

[4] In the classical political economy, the distinction between capital and natural resources, as well as between capitalists and landowners, plays an important role. In Marx's analysis, in particular, landed

iv) Production processes are organized (directly or indirectly) by capitalists. They buy the inputs and sell the outputs.

The inputs of the production processes are: commodities (raw materials, tools, machines, etc.); labour-power performed by workers with different skills; and the productive uses of natural resources. Points i-iii above imply that these inputs must be purchased on the market. Assuming, for the sake of simplicity, that they must be paid *ex-ante*, at the beginning of the production process, capital is necessary in order to finance the expenses of production, and this is the reason why capitalists are the only possible organizers of the production processes (point iv).

In fact, since production takes time, in every single process, the employment of inputs must precede the production of outputs. Accordingly, inputs are generally purchased and employed before outputs are sold. Therefore, costs and revenues of the same process are not simultaneous, as the former generally precede the latter. As a result, the costs of a certain process cannot be financed by the revenues of the same process. Hence, capital is the amount of purchasing power that is required, for each process, to finance the costs.

The amount of capital invested is then recovered out of revenues when the outputs are sold. Moreover, revenues leave, normally, a surplus over and above costs. This surplus value is profit.[5] The profit per unit of capital invested (for a single process) is the rate of profit.

2.1. An example

Let us assume there are N commodities, A different kinds of labour services, and B different sorts of natural resources. The production process of a generic commodity n, with $n = 1, 2, \ldots, N$, which starts in a certain period t, employs a vector of commodities $\mathbf{X}_t^n \in \mathbb{R}_+^N$, a vector of labour services $\mathbf{L}_t^n \in \mathbb{R}_+^A$, and a vector of productive uses of natural resources $\Lambda_t^n \in \mathbb{R}_+^B$ in order to get an output C_{t+1}^n in the period $t + 1$.

Referring to the economy as a whole, the employment of inputs in period t is $\mathbf{X}_t = \sum_n \mathbf{X}_t^n$, $\mathbf{L}_t = \sum_n \mathbf{L}_t^n$ and $\Lambda_t = \sum_n \Lambda_t^n$ and the output of period $t + 1$ is a vector $\mathbf{C}_{t+1} = [C_{t+1}^1, C_{t+1}^2, \ldots, C_{t+1}^N]$. Let $\mathbf{p}_t \in R_+^N$, $\mathbf{w}_t \in \mathbb{R}_+^A$ and $\rho_t \in \mathbb{R}_+^B$ be the (row) vectors of commodity prices, wage rates, and rent rates in period t, if wages and rents are paid *ex-ante* − namely, at the beginning of the process, then the total investment of capital in period t is:

$$K_t = \mathbf{p}_t \cdot \mathbf{X}_t + \mathbf{w}_t \cdot \mathbf{L}_t + \rho_t \cdot \Lambda_t \tag{1}$$

The capital advanced is then recovered by revenues in period $t+1$. However, under normal conditions, $\mathbf{p}_{t+1} \cdot \mathbf{C}_{t+1} > K_t$. This means that the revenues leave a profit over and above the costs of production:

$$\Pi_{t+1} = \mathbf{p}_{t+1} \cdot \mathbf{C}_{t+1} - K_t \tag{2}$$

The rate of profit is the amount of profit obtained for each unit of capital invested. Accordingly:

$$\pi_{t,t+1} = \frac{\Pi_{t+1}}{K_t} \tag{3}$$

From this example, it should be clear that:

i) The amount of capital K_t is not the quantity of an input. The quantities of inputs employed are $\mathbf{X}_t$, $\mathbf{L}_t$ and $\mathbf{\Lambda}_t$.

ii) The rate of profit is not the price of capital. The amount of profit Π_{t+1} is a residuum and not the result of a price-times-quantity multiplication.

2.2. The ordinary rate of profit

Assuming free competition among producers, this approach focusses the attention on a theoretical (persistent) position characterized by the uniformity of the rate of profit on the capital invested in different sectors of economic activity. Following Smith, we can call this uniform rate the 'ordinary rate of profit' and the relative prices associated with it the 'natural prices.'[6]

Thanks to Sraffa's theory (1960), we know that commodities' natural prices and the ordinary rate of profit corresponding to a given wage rate must be determined simultaneously as the solution of a system of equations. In particular, starting from the same example considered in the previous sub-section, for the sake of simplicity, we can set aside natural resources and assume that there is just one kind of labour service.[7] In this case, if capital must get the same rate of profit π independently of the sector in which it is invested, then the following conditions must hold:[8]

$$p^n C_{t+1}^n = (\mathbf{p} \cdot \mathbf{X}_t^n + w L_t^n)(1 + \pi) \quad \forall\, n = 1, 2, \ldots, N \tag{4}$$

Adopting the national net output as numéraire commodity – as Sraffa does – then:

$$\mathbf{p} \cdot (\mathbf{C}_{t+1} - \mathbf{X}_t) = 1 \tag{5}$$

For a given level of the wage rate w, equations (4) and (5) are able to determine the vector of natural prices $\mathbf{p}$ and the ordinary rate of profit π.

Finally, equations (4) allow us to define the price vector as a function of the distribution variables: $\mathbf{p} = \mathbf{p}(w, \pi)$. Substituting this function within equation (5), we get:

---

[6] In Marx's analysis, the same concepts correspond to the 'general rate of profit' and the 'prices of production'.

[7] As Garegnani argues, within the classical approach, the possibility of referring to homogeneous labour and a single wage rate rests on the hypothesis of a given structure of wage rates for workers with different skills (for details, see Garegnani 1984, p. 293, footnote 5).

[8] Since we are here referring to a persistent position of the economy, the time index of the quantities is only used to stress that the inputs are used before obtaining the outputs.

$$\mathbf{p}(w, \pi) \cdot (\mathbf{C}_{t+1} - \mathbf{X}_t) = 1 \qquad (6)$$

Equation (6) is what Garegnani (1984) calls the 'surplus equation'. It is particularly useful for a comparative statics exercise. It shows how the rate of profit that solves the system of equations (4)-(5) varies as the wage rate level changes. In particular, equations (6) tells us that there is an inverse relation between w and π. Therefore – under normal conditions and *ceteris paribus* – incomes from capital tend to be high when the wage rate level is low. This fact provides the theoretical ground for the conflict between social classes that can be observed in the real world.

3. Capital as a factor of production

Within the classical/Marxian approach, the organization of the society in three classes – workers, landowners, and capitalists – entails the division of the national income into three parts: wages, rents, and profit. Within the marginalist theory, these three different incomes are understood as what firms pay to households for the employment of three 'factors of production': labour, land and capital.[9]

Once this standpoint is adopted, the existence of the social classes becomes inessential. The working of the system depends on the decisions taken by two different sorts of economic agents: households and firms.

Firms organize the production processes. They demand factors of production and supply commodities. Factor prices – wage rate, rent rate, and interest rate – are included into the price vectors. Given a price vector, each firm decides its production plan in order to maximize its (net) profit, namely, the difference between revenues and the costs for the employment of the production factors.

Households supply production factors – which are available in given quantities[10] – and demand the commodities produced.[11] Given a price vector, each household decides its consumption plan in order to maximize its utility subject to the budget constraint.

In this framework, workers, landowners, and capitalists – provided that they can be distinguished by some special assumption – are on the same side: that of households. They are providers of factors of production. Then, wages, rents, and interest paid by firms

---

[9] Actually, the idea of distribution variables as the prices paid for the use of the factors of production preceded the development of the marginalist theory. The embryonic form of this idea was already present in the 'vulgar economists' towards whom Marx addresses numerous criticisms. In particular, according to Marx, these factors of production are the result of the transformation of social classes into things. See Marx (1909, p. 966).

[10] As is clear, although households are endowed with given quantities of factors of production, this does not mean that they are inelastically supplied. It may well be assumed that factor services are also desired by households for consumption. In this case, the quantity supplied to firms may change as the price system varies.

[11] In most versions of the theory, households demand consumption goods and services. Nonetheless, there are also models in which households demand capital goods too. Specifically, in Walras' theory, households demand capital goods with their saving in order to sell their productive services to firms. See Walras ([1926] 1977), p. 267.

to households have exactly the same nature. They are incomes for households and costs for firms.

3.1. The equilibrium rate of interest

Once the distribution variables are understood as factor prices, they must be determined at the same time and by the same mechanism as all the other prices, namely, by a system of general equilibrium conditions.

Focussing on the capital market, the rate of interest is thought to fall whenever the demand for capital by firms is smaller than the quantity of this factor made available by households, and to rise in the opposite case. The variation of the rate of interest with respect to the other factor prices should entail a change in the methods of production in use. In particular, a fall in the rate of interest should bring about the adoption of more capital-intensive methods so that, *ceteris paribus*, the demand for capital increases, leading to a reduction of the initial excess supply.

The fundamental idea behind this mechanism of substitutability between factors – or better, between their productive services – is well known. The first-order conditions for firms' (net) profit maximization impose the equality between factor prices (expressed in terms of produced commodity) and their respective marginal productivity. Hence, the principle of diminishing marginal productivity implies that, if the rate of interest falls and the other factor prices are unchanged, the adoption of a more capital-intensive production plan is needed in order to restore the equality between the marginal product of capital and the rate of interest.

Therefore, the equilibrium rate of interest is typically conceived as the rate that makes demand for capital by firms equal to the supply provided by households – which in turn depends on their present and past saving decisions.[12] In Marshall's words:

> interest, being the price paid for the use of capital in any market, tends towards an equilibrium level such that the aggregate demand for capital in that market, at that rate of interest, is equal to the aggregate stock forthcoming there at that rate (Marshall 1920, p. 534).

3.2. Interest and profit

In the marginalist approach, incomes from capital are what firms pay to households for the use of a factor of production. They are understood as the result of a price-times-quantity multiplication and enter into firms' costs. Hence, incomes from capital lose their residual nature. They are not the difference between revenues and costs.

Let us denote by L^n, Λ^n K^n the quantities of the factors labour, land, and capital employed in the production of a certain commodity *n*, with *n* = 1, 2, … *N*. Given the technical conditions of production, the amount of commodity *n* obtained is expressed as a certain (differentiable) function of the quantities of the factors of production used:

$$C^n = F^n(L^n, \Lambda^n, K^n) \tag{7}$$

---

[12] As for the supply of capital in the marginalist framework, we refer the reader to Fratini (2019b).

Accordingly, let w, ρ, and i be the wage rate, the rent rate, and the interest rate, respectively, the total costs of production of a final output C^n are: $wL^n + \rho\Lambda^n + (1 + i)K^n$.

For a given price p^n of commodity n, the amount of profit earned from the production of a quantity C^n is:

$$\Pi^n = p^n C^n - [wL^n + \rho\Lambda^n + (1 + i)K^n] \tag{8}$$

Within this theory, the amount of profit Π^n is not part of the incomes from capital. Here, the net income earned because of the employment of capital is the amount of interest iK^n that is included in the costs. The profit Π^n is understood as income earned by firms. In fact, as noted at the beginning of this section, in the marginalist theory, firms decide on their production plans in order to maximize their profit, namely, the difference between revenues and the costs of the employment of the production factors.

Once it is clear that they are not understood as incomes from capital, the nature of firm profits is not very clear. It is not clear what their counterpart in the real world can be. However, this is not a serious problem, because these profits disappear with their maximization.

In fact, substituting equation (7) into equation (8), the first-order conditions for the maximization of the amount of profit Π^n are:

$$\begin{cases} p^n F_L^n(L^n, \Lambda^n, K^n) - w = 0 \\ p^n F_\Lambda^n(L^n, \Lambda^n, K^n) - \rho = 0 \\ p^n F_K^n(L^n, \Lambda^n, K^n) - (1 + i) = 0 \end{cases} \tag{9}$$

where $F_J^n(\cdot)$ is the partial derivative of $F^n(\cdot)$ with respect to factor J, with $J = L, \Lambda, K$. Therefore, if the production function $F^n(\cdot)$ exhibits constant returns to scale – i.e. it is homogeneous of degree one – and firms are using the factors in optimal quantities, then $\Pi^n = 0$.[13,14]

3.3. Capital and the value of capital goods

It should be clear that the idea of the interest rate as a price represents the other side of the coin with respect to the idea of capital as a factor of production – that is, as something that firms materially employ in production, together with labour and land. Capital must,

---

[13] According to Euler's formula for homogeneous functions, if the production function $F^n(\cdot)$ is homogeneous of degree one, then: $C^n = F_l^n(L^n, \Lambda^n, K^n) \cdot L^n + F_\Lambda^n(L^n, \Lambda^n, K^n) \cdot \Lambda^n + F_K^n(L^n, \Lambda^n, K^n) \cdot K^n$. Therefore, equation (8) can be written as follows: $\Pi^n = [p^n F_l^n(L^n, \Lambda^n, K^n) - w]L^n + [p^n F_\Lambda^n(L^n, \Lambda^n, K^n) - \rho]\Lambda^n + [p^n F_K^n(L^n, \Lambda^n, K^n) - (1 + i)]K^n$. It is now clear that the first-order conditions (9) entail $\Pi^n = 0$.

[14] Wicksteed – the 'purist' of the marginalist theory – writes that the existence of a profit or surplus above the costs is nothing more than a fantasy. He maintains that anyone who understands the facts cannot claim that, once the remuneration of factors corresponds to their marginal product, there remains any residuum to be divided or appropriated. He concludes – clearly as a criticism of the classical/Marxian approach – that 'the vague and fervid visions of this unappropriated reserve, ruling upward as we recede from the marginal distribution, must be banished for ever to the limbo of ghostly fancies' (Wicksteed, 1910, p. 573).

in fact, be substitutable for other factors and therefore must play the same role and must satisfy the same need.

The conception of capital as a factor of production represents, no doubt, the biggest difficulty met by the marginalist theory of distribution. The idea of capital as something that is materially used in the production process is associated with its identification with the set of capital goods (tools, raw material, machines, etc.) employed. Hence, the theory tries to present capital as an amalgam of capital goods.[15] That is the reason why the problems encountered by this theory with reference to capital as a factor of production are often erroneously believed to be problems of 'aggregation' – i.e. the transformation of a vector of quantities of heterogeneous commodities into a single magnitude. Actually, no real problem exists for the aggregation of the capital goods into the corresponding amount of value by means of their prices. The problems arise if we want to regard this amount of value as the quantity of a factor of production and the rate of interest as the price of its use.

At the cost of a drastic simplification, we can say that the basic idea is that of the existence of a special substance, a sort of 'jelly', of which all capital goods are made and from which their productivity derives. If this jelly existed, the form of the various types of capital goods that it actually takes would be a secondary aspect. In fact, first, to every possible set of heterogeneous capital goods, one could associate the corresponding quantity of jelly. Secondly, sets of capital goods containing a greater quantity of jelly would make it possible to obtain a larger product, all other things being equal.

Despite several attempts, the last of which was Samuelson's (1962), a substance or jelly with these extraordinary properties has never been found. Thus, in its absence, the employment of capital was generally identified with the value of capital goods used in production. It was therefore tried to create a hybrid between the (right) idea of capital as a value fund that advances costs and the (wrong) idea of capital as an amalgam of capital goods. However, as was already pointed out by Wicksell (1934, p. 149), the value of capital goods employed cannot be considered as the quantity of a factor of production.[16] Doing that can in fact lead to paradoxical results.

---

[15] The double presence of capital and capital goods generates a sort of schizophrenia in the marginalist theory. Taking Solow as the personification of this theory, Samuelson wrote that '[o]ne might almost say that there are two Solows': (a) 'the orthodox priest of the MIT school', who regards capital as 'a great variety of heterogeneous physical capital goods'; and (b) 'the busman on a holiday who operates brilliantly and without inhibitions in the rough-and-ready realm of empirical heuristics' and makes use of a 'Clark-like concept of aggregate "capital"' (Samuelson, 1962, p. 193).

[16] In the marginalist theory, the amount of output obtained depends on the productive services provided by the factors of production in use. Therefore, the quantity of each factor must be representative of the quantity of productive services that that factor is able to provide. It must be expressed in technical units. To give an example, we could try to measure the employment of labour in meters (adding up workers' heights) instead of hours (i.e. adding up the hours worked by the labourers). In this case, since taller workers do not generally provide more productive services than shorter workers, an increase in the amount of work would not lead with certainty to the production of a greater output. The relationship between the quantity of labour employed, measured in meters, and the quantity produced would become ambiguous, so that we could no longer represent it by a mathematical function of general validity. Hence, labour must be measured in technical units, that is, in hours of work.

First, as Samuelson (1966, p. 582) wrote, there is, in general, no unambiguous way to say that a process of production is more capital-intensive than another – namely, that it entails a greater employment of capital per unit of labour. Since the prices of capital goods change as the interest rate varies, if we take two methods of production of the same commodity, it is possible that the method that is more capital-intensive for a certain level of interest rate becomes the method that is less capital-intensive for a different level. The ranking of methods on the basis of capital intensity cannot be done independently of the level of interest rate.

Second, given a certain interest rate, the method that employs more capital per unit of labour is not necessarily the one that enables the highest output per unit of labour to be obtained. This point is essentially an implication of the previous one. Suppose that, for a certain level of the rate of interest, method α is more capital-intensive than method β and also provides a greater output per unit of labour. If, for a different level of the interest rate, method β has become the most capital-intensive, then, for the same amount of labour employed, the method that employs the greatest capital is the one that results in the lowest output.

Third, when an increase in the interest rate entails a change in the production method in use – i.e. the one that minimizes the unit cost of production – the method that comes into use does not necessarily employ less capital per unit of labour than the method previously used. In other words, an increase in the interest rate may not make (relatively) more expensive those methods that require more capital. This result clearly casts into doubt the idea that the interest rate can be seen as the price that firms pay for the use of the factor 'capital'.

The point is that, since there is no convincing way to conceive the productive factor 'capital', the rate of interest cannot be thought of as its price. This emerged during the Cambridge capital theory debate[17] and was, in fact, explicitly stated by Bliss in his authoritative book on capital theory:

> The value which accrues from a sale is the product of price and quantity sold. Hence if the rate of interest is the price of capital, the quantity of capital must be the wealth on which an interest yield is calculated. It will be shown shortly why this view is incorrect, but to cut a long story short, the conclusion may be announced at once. The rate of interest is not the price of capital (Bliss 1975, pp. 6-7).

However, as Wicksell stresses, value is not a technical unit of measure of capital goods since '[t]he productive contribution of a piece of technical capital, such as a steam engine, is determined not by its cost but by the horse-power which it develops' (Wicksell, 1934, p. 149). Therefore, although there is no problem in aggregating capital goods in terms of value, this amount of value cannot be understood as representative of the technical contribution of the set of capital goods. It cannot be understood as the quantity of a factor of production.

[17] A number of surveys of the Cambridge capital theory debate are available. In particular, the readers may refer to Harcourt (1969 and 1972).

4. Interest and profit in the Arrow-Debreu theory

As shown in section 2, in the classical/Marxian approach, incomes from capital are a profit, namely, the difference between revenues and the costs of production – which were totally or partially advanced by capital. By contrast, in the marginalist theory, they are the interest firms pay on the capital borrowed from households, and hence they are included into the costs. In this framework, the difference between revenues and costs represents the profit of firms, and vanishes in equilibrium.

We shall now consider the Arrow-Debreu theory of value, in which prices are determined by a market-clearing equilibrium, as in the marginalist approach. However, in contrast to the latter, production processes are not assumed to employ 'factors of production', but instead, are assumed to employ Arrow-Debreu commodities, namely, goods and services with a specific place and date of delivery.

Actually, the Arrow-Debreu model is a very peculiar one. On the one hand, no form of capital seems present in this framework: neither the classical (an amount of value that allows the payment of costs at the beginning of the process), nor the marginalist (a factor of production). On the other hand, profit and interest – at least in name – are still there, but they cannot correspond to the incomes from capital discussed in sections 2 and 3. Therefore, as will be clear at the end of this section, Arrow-Debreu theory provides no support for the marginalist explanation of incomes from capital.[18]

4.1. Own-rates of interest

In the Arrow-Debreu framework, as previously mentioned, production processes do not employ factors of production. Accordingly, the rate of interest is not understood as the price firms pay for the use of capital.

In fact, in the Arrow-Debreu models, instead of one rate of interest there are many 'own-rates of interest'.[19] Let p_t^n and p_{t+1}^n be the prices of commodity n (with $n = 1, 2, \ldots,$ N) delivered in period t and $t+1$ respectively, the own-rate of interest of commodity n between the two periods $r_{t,t+1}^n$ is defined by the equation:

$$\frac{p_t^n}{p_{t+1}^n} \equiv 1 + r_{t,t+1}^n \tag{10}$$

It is clear from equation (10), that this rate (or factor) of interest in neither the price of a factor of production, nor a source of income. It is just a relative price: $(1 + r_{t,t+1}^n)$ is the quantity of commodity n delivered in period $t+1$ that an agent must pay in order to have

---

[18] Some scholars seem to believe that Arrow-Debreu theory is the rigorous, but formally complicated, version of the old marginalist theory. Actually, it is not so. Notwithstanding these theories are both neo-classical as far as the essential features of the representation of the economic system are concerned (see footnote 2), their views about income distribution are utterly different. The marginalist idea according to which production processes employ services provided by factors of production, so that distributive variable are the prices of these services, is completely absent in Arrow-Debreu theory. Specifically, advocates of the Arrow-Debreu approach are opposed to the interest rate understood as the price for the use of capital (see Koopmans, 1957, pp. 113-115, and Bliss, 1975, pp. 10 and 346).

[19] On the notion of own rate of interest, see in particular: Debreu (1959, pp. 33-34) and Bliss (1975, pp. 51-55).

a unit of commodity *n* delivered in period *t*. Besides, since this quantity can be less than 1, the own rate of interest $r_{t,t+1}^n$ can be negative (but not smaller than -1). Hence, as is clear, the own-rates of interest are essentially useless for the explanation of incomes from capital.

4.2. Profits in a private ownership economy

As far as profit is concerned, the Arrow-Debreu theory takes up the marginalist idea that the difference between revenues and costs forms the profit of firms. Hence, in this theory, in contrast to the classical/Marxian approach, profit has no linkage with the investment of capital.

If there are *N* different goods and services with *T* possible dates of delivery, putting aside the possibility of different places of delivery,[20] there are $L = N{\times}T$ Arrow-Debreu commodities. Let $\mathbf{p} \in \mathbb{R}_+^L$ be a price vector and $\mathbf{y}^f \in \mathbb{R}^L$ the production plan of firm f – that is, a list of quantities of inputs, with negative sign, and outputs, with positive sign – then $\pi^f = \mathbf{p} \cdot \mathbf{y}^f$ is the firm's profit.[21] In a 'private ownership economy' (Debreu 1959, pp. 78-80), this profit – which can be gains or losses – is divided amongst households, and entered into their budget constraints, in accordance with some exogenously given shares.

These shares cannot reflect the investment of capital (savings) households made for the very simple reason that saving and investment are inconceivable within this framework. As is known, in the Arrow-Debreu model, the *L* commodities, or rather, the promises of their delivery, are traded simultaneously in a single instant, the initial moment of the first period. This assumption has a number of relevant implications, one of which will be focused on here, namely, the impossibility of finding a role for the investment of capital.[22]

On the producers' side, if all the markets are open for one single instant only, then every firm can trade both inputs and outputs simultaneously, in the single instant in which markets are open. In other words, revenues and costs, in this model, are necessarily simultaneous, and this makes the investment of capital impossible. In fact, as we have seen, capital is invested in order to finance the costs in advance, before revenues are obtained. For instance, as is known, if wages are paid in the same moment that output is sold, no capital is involved in the payment. In the Arrow-Debreu framework, this applies not only to wages, but also to expenditure on all the inputs (including capital goods),

---

[20] The existence of just one possible place of delivery and one stream of events can be assumed here for simplicity.

[21] In other words, let $\mathbf{y}^f = [y_1^f, y_2^f, \ldots, y_L^f] \in \mathbb{R}^L$ be the production plan of a firm f, and then $\mathbf{y}^f$ is a vector of net supplies of commodities. This means that if $y_n^f < 0$, then it (taken in terms of absolute value) is the quantity of commodity n employed as input by firm f. If instead $y_n^f > 0$, then it is the quantity of commodity n obtained as output by that firm. As a result, $\mathbf{p} \cdot \mathbf{y}^f$ directly expresses the difference between revenues and costs.

[22] For a survey of the literature on saving and investment in intertemporal equilibrium models, the reader is referred to Fratini (2019a).

which must necessarily take place in the same instant as the outputs obtained with them are sold.

On the consumers' side, households cannot and do not need to move their purchasing capacity across time. Households' wealth arises and is entirely spent in the one instant in which the markets are open. Saving in order to transfer purchasing power to some future date would thus actually be impossible in the Arrow–Debreu model, as no further trade can take place after the initial instant of the first period.[23] As Currie and Steedman (1990, p. 147) have pointed out, the idea of transferring wealth over time has no real meaning in this framework.[24] It is clear that capital cannot exist in this theory, and that, therefore, no form of income from capital, be it profit or interest, can exist either.[25]

Finally, we can stress that, independently of any consideration about its nature and the principle adopted for its distribution, the profit maximized by firms is not a real source of households' wealth – at least if constant returns to scale are assumed. In fact, let $\mathbf{p}^*$ and $\mathbf{y}^*$ be equilibrium price vector and aggregate production plan, it can be easily proved[26] that $\pi^* = \mathbf{p}^* \cdot \mathbf{y}^* = 0$. This means that, in equilibrium, households' purchasing capacity depends on the value of their endowments of commodities only, as there is no income coming from firms' profit.

---

[23] Some authors claim that the assumption that markets do not reopen after the initial instant is innocuous because even 'if markets were reopened at later dates for the same Arrow-Debreu commodities, then no additional trade would take place anyway' (Geanakoplos 1987, p. 122). This is not so. The question is not whether further trade will take place when the markets reopen, but rather, whether agents will behave differently in the initial instant in the knowledge that markets will open again. In particular, the assumption that markets will reopen gives rise to all the problems connected with expectations and speculative trade, which are completely avoided in the Arrow-Debreu framework.

[24] In each period there can be – and typically there are – existing stocks of commodities (produced or inherited from the past) that exceed the quantities consumed in that period. Is this saving? The point is complex, and an answer cannot be given here. However, we want to warn the readers about a possible confusion between consumption understood as the activity of consuming and consumption as an expenditure. It is this latter concept that is generally related to saving and capital accumulation. On the point, see also Fratini (2020).

[25] With the aim of avoiding possible misunderstandings, it should be stressed that this statement does not refer to the neo-Walrasian theory in general, but to the Arrow-Debreu equilibrium theory. As already pointed out, the latter is a very peculiar version of the neo-Walrasian approach in which a number of complications are ruled out thanks to ad hoc assumptions. By contrast, the phenomena related to the investment of capital can very well be found in neo-Walrasian models in which trades take place sequentially.

[26] Let $\mathbf{z} : \mathbb{R}_+^L \to \mathbb{R}^L$ be the market excess-demand function from households. Because of the market clearing condition, we know that $\mathbf{z}(\mathbf{p}^*) = \mathbf{y}^*$. Since the Walras' Law entails $\mathbf{p}^* \cdot \mathbf{z}(\mathbf{p}^*) = 0$, then $\mathbf{p}^* \cdot \mathbf{y}^* = 0$. See the proof of the proposition 17.F.1 in Mas-Colell, Whinston and Green (1995), p. 607.

This means that there can be no profit in aggregate, but it seems possible, at first glance, that one firm can have strictly positive profit if another has losses. However, if the set of feasible production plans includes the possibility of inactivity (the null vector), then no firm, in equilibrium, will adopt a production plan that entails losses. As a result, in equilibrium, each firm must have zero profit.

5. Conclusions

Three different approaches are considered in this paper. In the classical/Marxian approach (section 2), incomes from capital are understood as a surplus whose amount depends on the class conflict. At the beginning of the process of production, a certain amount of capital M is invested in order to advance the costs for the inputs employed. The inputs are transformed into a bundle of commodities C. When these commodities are sold on the market, an amount of revenues M' is obtained. It is the capitalist circuit M-C-M' and capitalists' incomes arise from the difference $M' - M$. This difference or surplus value is the profit, that is, the residual part of revenues over and above the costs of production.

By contrast, in the marginalist theory (section 3), the distribution variables – i.e. wage rate, rent rate, and interest rate – are understood as the prices of three factors of production: 'labour', 'land', and 'capital'. According to this view, incomes from capital are what firms pay to households for the employment of the factor of production called 'capital'. In particular, the rate of interest – seen as the price for the use of capital – is determined by means of a supply-and-demand equilibrium.

Thanks to the capital theory debates of the 1960s, it became clear that the idea of capital as a factor of production and the rate of interest as the price of its use is untenable. This was explicitly admitted by authoritative neoclassical economists, such as Samuelson (1966), Hahn (1982), and Bliss (1975). However, at the same time, these scholars maintained that the modern general equilibrium theory is not affected by those problems, since it does not rest on the idea that production processes employ factors of production, but instead, employ Arrow-Debreu commodities.

As a matter of fact, in the Arrow-Debreu general equilibrium model (section 4), there is neither capital understood as the amount of value invested at the beginning of each process, nor capital as a factor of production. The problem is that, in this model, there is no idea or role for capital. As a result, incomes from capital can hardly find an explanation within the Arrow-Debreu theory.

As capital, in the Arrow-Debreu model, is not understood as a factor of production, the traditional conception of the rate of interest is no longer there. In its place, we find many commodity own-rates of interest, but they are essentially useless in the explanation of income distribution. Profit – intended as what firms maximize – is still there, but: i) it is distributed across households on the basis of shares arbitrarily assigned; and ii) its equilibrium amount must be zero.

Therefore, on the basis of the analysis developed in this paper, we can conclude that neither the marginalist theory, nor the Arrow-Debreu theory, can provide us with a convincing theoretical explanation of incomes from capital.[27] Thus, the old theory of the

---

[27] The result achieved here is much more general than that proposed by Eatwell in a recent paper, in which he claims that 'there is no neo-classical theory of the rate of profit' (Eatwell 2019, p. 10). More precisely, Eatwell maintains that there is no way to use Debreu's theory in order to solve Adam Smith's problem concerning the determination of 'natural prices.' Consequently, it cannot arrive at the determination of the level of the ordinary/normal rate of profit (in this respect, see section 2.2 here). Although Eatwell's conclusion is certainly agreeable, it seems to be just a corollary of the point already raised by Garegnani (1976 and 2012) about the neo-Walrasian approach and its abandonment of the 'normal position' method.

classical economists – submerged and forgotten because of the advent of the neoclassical approach – seems to be the only possible way to proceed.

References

Bliss, C.J. (1975), *Capital Theory and Distribution of Income*, Amsterdam/New York, North-Holland/Elsevier.

Debreu, G. (1959), *Theory of Value*, New York, Wiley.

Eatwell, J. (2019), '"Cost of production" and the theory of the rate of profit', *Contributions to Political Economy*, 38(1): 31-58.

Fratini, S.M. (2019a). 'On the second stage of the Cambridge capital controversy' *Journal of Economic Surveys*, 33(4): 1073-1093.

Fratini, S.M. (2019b), 'Neoclassical theories of stationary relative prices and the supply of capital', *Metroeconomica*, 70(4): 723-737.

Fratini, S.M. (2020), 'Interest, profit and saving in Arrow-Debreu equilibrium models', *Bulletin of Political Economy*, forthcoming.

Garegnani, P. (1976), 'On a change in the notion of equilibrium in recent work on value: a comment on Samuelson', in Brown, M., Sato, K. and Zarembka, P. (eds), *Essays in Modern Capital Theory*, Amsterdam, North-Holland.

Garegnani, P. (1984), 'Value and Distribution in the Classical Economists and Marx', *Oxford Economic Papers*, 36(2): 291-325.

Garegnani, P. (2012), 'On the present state of the capital controversy', *Cambridge Journal of Economics*, 36(6): 1417-1432.

Geanakoplos, J. (1987), 'Arrow-Debreu model of general equilibrium', in Eatwell, J., Milgate, M. and Newman, P. (eds.), *The New Palgrave Dictionary of Economics*, Vol. I, London, Macmillan: 116-124.

Hahn, F.H. (1982), 'The neo-Ricardians', *Cambridge Journal of Economics*, 6(4): 353-362.

Harcourt, G.C. (1969), 'Some Cambridge controversies in the theory of capital', *Journal of Economic Literature*, 7(2): 369-405.

Harcourt, G.C. (1972), *Cambridge Controversies in the Theory of Capital*, Cambridge, Cambridge University Press.

Koopmans, T.C. (1957), *Three Essays on the State of Economic Science*, New York/London/Toronto, McGraw-Hill.

Marx, K. (1909), *Capital*, Vol. III, Chicago, C.H. Kerr & Co.

Mas-Colell, A., Whinston, M.D. and Green, J.R. (1995), *Microeconomic Theory*, Oxford, Oxford University Press.

Piketty, T. (2014). *Capital in the Twenty-First Century*. Cambridge (Mass.) and London: Harvard University Press.

Rognlie, M. (2016), 'Deciphering the fall and rise in the net capital share: Accumulation or scarcity?', *Brookings Papers on Economic Activity*, 2015(1): 1-69.

Samuelson, P.A. (1962), Parable and realism in capital theory: the surrogate production function, *Review of Economic Studies* 29(3): 193-206.

Samuelson, P.A. (1966), 'A summing up', *Quarterly Journal of Economics*, 80(4): 568-583.

Walras, L. ([1926] 1977), *Elements of Pure Economics or the Theory of Social Wealth*, New York, Augustus M. Kelley Publishers.

Wicksell, K. ([1911] 1934), *Lectures on Political Economy*, Vol. I, London, Routledge.

Wicksteed, P.H. (1910), *The Common Sense of Political Economy*, London, Macmillan.

Author contact information:

Saverio M. Fratini
Department of Economics, Roma Tre University.
00145 – Rome (Italy)
saveriomaria.fratini@uniroma3.it