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Capacity Utilization, Obsolete Machines and Effective Demand

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Abstract
This paper is concerned with certain conditions under which an autonomous intended change in demand becomes effective. A simple model describes an economy, which initially is assumed in a steady state characterized by a given conventional wage rate, a uniform rate of profit and the existence of obsolete machines which are still used and receive quasi-rents, although not produced anymore. Small changes in effective demand, allowed by variable capacity utilization of fixed capital at given prices, are distinguished from large changes which affect the relative prices of commodities and the distribution of income. In both cases the steady state and the adjustment process towards a new steady state are compatible with unemployment; but large effective changes in demand require a higher flexibility of capacity utilization, compared with small changes. This occurs through a deviation of prices and income distribution from their normal values and a change in quasi rents, which make profitable a change in the types and the amounts of the obsolete machines in use. The distinction stressed in the paper is preliminary to the further distinction between impulse-wise and persistent changes in the rates of growth of demand, that is left as a research agenda.

Keywords: growth, Keynesian analysis, capacity utilization, obsolete capital goods, long and short period analysis.

JEL codes: B50, E22, O40

1. Introduction

This paper is related to the efforts which aim to extend Keynes's theory of effective demand from the short to the long period and to the theory of economic growth.[2]

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Starting from a state of the economy characterized by a uniform rate of profit and an unlimited supply of labour at a fixed conventional wage, an autonomous change in demand for the social product becomes *effective* through a regime of flexible *quasi-rents*. This claim, which will be argued in the next sections, suggests that a certain flexibility in the distribution of income should be admitted to cope with the assumption of variable capacity utilization within investment-constrained growth models.

A useful description of a normal state of a growing capitalist economy should not be confined to a pure steady growth, despite the fact that such a state can be used as an ideal configuration for some preliminary analytical purpose. In particular, a normal state should include the co-existence of obsolete and non obsolete machines. A production economy in long-period equilibrium with obsolete machines has been analyzed rather cursorily in Sraffa (1960, ch. XI, section 91) and extensively in Schefold (1989, part II, section B) and Kurz-Salvadori (1995, ch. 12) in the context of the theory of joint production. The latter authors have used this case to stress some limitations of the classical method of long-period equilibrium. A distinction should be made within types of obsolete machines: a machine can be said obsolete in a *strong sense* if it cannot be produced *and* used anymore in any profitable process. In a *weak sense* a machine is obsolete if it is not produced anymore but can still be used in a profitable way. The profitability is relative to existing technology and the corresponding range of possible prices and income distribution. In this paper *obsolete* machines mean obsolete in a weak sense: machines that can be used and may receive non-negative Marshallian *quasi-rents*, but are not produced anymore. From now onwards, we shall use the word '*rent*' instead of '*quasi-rent*', since the absence of Ricardian land and pure rent from the present analysis does not require such a qualification.

Obsolete machines are ubiquitous and their amount is an important component of the capital stock of any capitalist economy, in comparison with the available non obsolete machines. We shall argue that obsolete capital goods should play a substantial role in modeling a growing economy where changes in capacity utilization depend on driving deviations of demand and income distribution from normal values. In the next section a simple model of variable capacity utilization will introduce a more complex model – the topic of the rest of the paper – in which the state of capacity is related to the existence of obsolete machines and is consistent with a cost-minimizing choice of techniques. Only a few properties of the model will be analyzed to provide some examples of possible capacity adjustments. The main analytical result of this paper conforms to common intuition. In the presence of obsolete machines, the margins of changes in capacity utilization, driven by changes in effective demand, are wider if prices and distribution are allowed to deviate from their long period values associated with free competition and a given conventional wage rate. Some related questions remain analytically unsettled and are left to speculative reasoning. Different types of changes in effective demand (a once-for-all impulse versus a persistent shift in the absolute level or in the rate of growth of demand) can have different qualitative

[2] In Parrinello (2014) the interested reader may consult a bibliography of the debate about the extensions of Keynes's theory of effective demand to the theory of growth with variable capacity utilization.

consequences in the long period after a transition phase: a convergence either toward the initial income distribution or to a different one associated with a different conventional wage rate. Furthermore, different speeds of adjustment may characterize the prices and quantity proportions of non obsolete machines compared with those of obsolete machines.

2. A simple treatment of variable capacity utilization.

A simple approach to variable capacity utilization has been adopted in slightly different variants either explicitly[3] or implicitly in many works which aim to extend Keynes's theory of effective demand to the long period and to a growth theory.

Let us assume a one-commodity model, where a commodity is produced by means of labour and the same commodity. Suppose that only one technique is available and is subjected to constant returns to scale and to a variable degree of capacity utilization defined as follows. Let us assume that a change in capacity utilization affects the productivity of labour and capital in the same proportion, u, and define the critical ranges of capacity utilization

$u = 1$: normal capacity,
$0 < u < 1$: excess capacity,
$1 < u < u_m$: capacity overutilization, where u_m is a technical maximum.

Let λ, μ denote technical coefficients: respectively the amount of labour and physical capital per unit of output when productive capacity is used at the *normal* rate $u = 1$. Define

$Y \equiv \frac{K}{\mu}$ normal output of a given stock of capital K,
uY actual output,
$u_m Y$ full capacity output.

Let us denote the nominal values:

w: wage rate;
p: price of commodity C;
ϱ_c: rent on capital good C
The nominal price equation: $w\lambda + \varrho_c \mu = pu$.

The relative price equation with commodity C chosen as the *numéraire* ($p = 1$):

$$w\lambda + \varrho_c \mu = u \qquad [1]$$

where wages are supposed to be paid *post factum*. Given a conventional real wage rate, $w = \overline{w}$, equation [1] with $u = 1$ determines the *normal* rent ϱ_c. Depending on the assumption about the lifetime of the capital good and its efficiency performance,

[3] Cfr. Foley and Michl (2000, ch. 11), Parrinello (2014).

combined with equation [1], it is possible derive the normal rate of profit r. For simplicity assume that the capital good lasts n periods with constant efficiency[4]. Next r must be a solution to the following equation derived from the equality between the price (p = 1) of the currently produced commodity and the present value of a constant annuity ϱ_c paid for n periods and discounted at a constant rate of interest r :

$$\varrho_c = \frac{r(1+r)^n}{(1+r)^n - 1}$$

with the important special cases: $\varrho_c = 1 + r$ if $n = 1$ (circulating capital); $\varrho_c \to r$ if $n \to \infty$ (land).

This simple notion of variable capacity utilization can be used in preliminary demand-led growth models, but it should be revised for at least two reasons.[5] Firstly, it can be argued that a single number (u) is an excessively simplified measure of the degree of capacity utilization even in a one-commodity model, considering that what is at issue in many industrial experiences (e.g. electric power supply) are *distributions* of utilization over time, falling in a certain range of normality which preserves the long term expectations, versus abnormal distributions outside that range which lead to revised expectations and investment plans. Secondly and more importantly, variable u is used in those models as an adjustment variable between aggregate capacity savings and investments, but its "equilibrium" value may not be consistent with a cost-minimizing choice of techniques. A change in u above or below the norm u = 1 may not leave unaffected the unit costs of production. If the change in aggregate demand is persistent, it should be explained why producers do not pursue the technical efficiency which is implicit in the usual analytical definition of a production process. This paper is focused on the second shortcoming, leaving aside the first one. It aims to offer a model of production where a variable degree of capacity utilization is admitted and can make *effective* a change in demand, but at the same time is consistent with a cost-minimizing choice of techniques.

3. An economy with obsolete machines under two regimes

Let us assume an economy with labour L and three commodities: a produced commodity C used as a means of production (machine of type C) and for consumption; and two types of obsolete machines M_1, M_2 . Three processes are available for the production of C, and described by vectors of positive (output) and negative (input) *absolute* quantities:

[4] For a general treatment of fixed capital and its depreciation without the restriction of constant efficiency, see Sraffa (1960, ch. X).
[5] Cfr. Parrinello (2014).

$$m_0 \equiv \begin{bmatrix} C^{(0)} \\ -L_0 \\ -C_0 \\ 0 \\ 0 \end{bmatrix}; \; m_1 \equiv \begin{bmatrix} C^{(1)} \\ -L_1 \\ -C_1 \\ -M_1 \\ 0 \end{bmatrix}; \; m_2 \equiv \begin{bmatrix} C^{(2)} \\ -L_2 \\ -C_2 \\ 0 \\ -M_2 \end{bmatrix}$$

Inputs of labour and commodity C intervene in each process; instead machines of types M_1, M_2 are used only in processes m_1, m_2 respectively. Quantities M_1, M_2 are given; the total product $C^{(0)} + C^{(1)} + C^{(2)}$ is not constrained by a limited supply of labour and is determined by a given effective demand D.

Let us denote the total quantities

D: demand for C, equal to the total product $C^{(0)} + C^{(1)} + C^{(2)}$;
$K = C_0 + C_1 + C_2$: total quantity of machines C in use;
$L = L_0 + L_1 + L_2$: total employment of labour;

and the *nominal* values

w: wage rate;
p: price of commodity C;
ϱ_c, ϱ_1, ϱ_2 : rents on machines C, M_1, M_2.

For the sake of argument *we assume* that, despite the fact that both obsolete machines M_1, M_2 by definition are not produced anymore, because their *own* cost of production is too high, there is a partial ranking of cheapness among m_0, m_1, m_2 in terms of increasing costs of production of C, *net* of rents ϱ_1, ϱ_2, such that, as the effective demand D hypothetically increases, the processes can be activated in the partial order[6]: m_1, m_0 first; m_2 second. Let us start from a state of reproduction in which the level of demand D is such that, at the given wage rate, process m_2 is not operated ($C^{(2)} = 0$) and D is allocated to method m_1 up to the full capacity of M_1, whereas the residual quantity of D is supplied by method m_0. This state can be interpreted as a quasi-long period equilibrium and will be called a *steady state*. Let us write the price and quantity equations corresponding to the assumptions $C^{(2)} = 0$; $\varrho_c, \varrho_1 > 0$. In conditions of free competition, the price equations in the unknowns w, ϱ_c, ϱ_1 are

$$\left.\begin{aligned} wL_0 + \varrho_c C_0 &= pC^{(0)} \\ wL_1 + \varrho_c C_1 + \varrho_1 M_1 &= pC^{(1)} \end{aligned}\right\} \qquad [2]$$

subject to the productivity constraint $0 \leq C_j/C^{(j)} < 1$, $j = 0, 1$. We assume constant returns to scale and choose commodity C as the numéraire ($p = 1$). The price equations in terms of technical coefficients:

[6] It might seem strange that m_1, a process with an obsolete machine, can be cheaper or equal-profitable than another process, m_0, with a non obsolete machine. However, according to the definition of *weak* obsolescence adopted at the beginning, the source of the distinction between the two categories of capital goods is upstream of their productive efficiency and attains their own cost of production.

$$\left.\begin{array}{l} w\,\lambda_0 + \varrho_c\, c_0 \;\; = 1 \\ w\,\lambda_1 + \varrho_c c_1 + \varrho_1\,\mu_1 = 1 \end{array}\right\} \qquad [3]$$

where $\lambda_j = L_j/\ C^{(j)}$, $c_j = C_j/\ C^{(j)}$, $_{j\ =\ 0,\ 1,}$; $\mu_1 = M_1/\ C^{(1)}$. The quantity equations in the unknowns $C^{(0)}$, $C^{(1)}$, K, L:

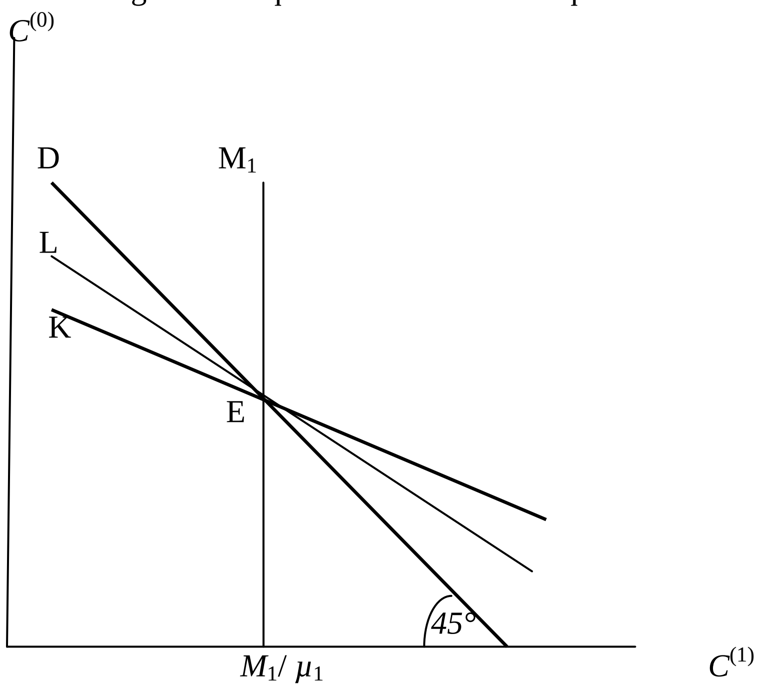
$$\left.\begin{array}{l} C^{(0)} + C^{(1)} = D \\ c_0\, C^{(0)} + c_1\, C^{(1)} = K \\ \lambda_0\, C^{(0)} + \lambda_1 C^{(1)} = L \\ \mu_1\, C^{(1)} = M_1. \end{array}\right\} \qquad [4]$$

Let us consider two distributive regimes and closures of the model.

A *surplus regime with rents*

Suppose that the wage rate is fixed at a conventional level $w = \overline{w}$, so that equations [3] determine the rents ϱ_c , ϱ_1 sequentially; then the rate of profit follows from the value of ϱ_c and the formula of the present value of a constant annuity (see section 2). The unknowns of system [3], [4,] are w, ϱ_c, ϱ_1, $C^{(0)}$, $C^{(1)}$, K, L, with $w = \overline{w}$. In figure 1 the D, K, L, M lines represent equations [4] and the point of intersection E describes a steady state on the quantity side.

Figure 1 - Equilibrium with more equations than variables



A pure rent regime

By contrast, suppose that the economy has reached a full employment equilibrium represented by point E, through an adjustment process where the demand is endogenous and the assumption of unlimited supply of labour with a fixed conventional wage is replaced by that of a limited supply with a flexible wage. The amounts K, L, M_1 are *given* but not arbitrarily given; E is the common intersection point of the resource constraints and can be interpreted as a *degenerate*[7] solution to a linear programming (LP) problem in which D is the maximand, subject to K, L, M linear constraints. We know from the duality theorem of LP that, if the primal problem is degenerate, then the solutions to its dual problem are infinite. In our case prices w, ϱ_c, ϱ_m, conceived as shadow-prices of the dual LP problem, would have one degree of freedom.[8] This degree apparently can be filled by setting a conventional wage rate and therefore the two (surplus and pure rent) regimes seem to have been re-conciliated. However this is illusory. A marginal change in the given amounts of K, L, M_1 can make only two resources binding and, as a consequence, the LP problem on the quantity side would become non-degenerate and no room would be left to a conventional wage rate in the price solution to the dual problem.

4. Deviations from steady states due to an autonomous change in D

Let us distinguish two alternative assumptions (a, b) on the change in D and specify the assumption (c) of degree of capacity utilization.

Assumption **a.** The change in D is temporary and normal, although the exact time of its occurrence may not be foreseen with certainty

Assumption **b.** The rate of change in D is persistent

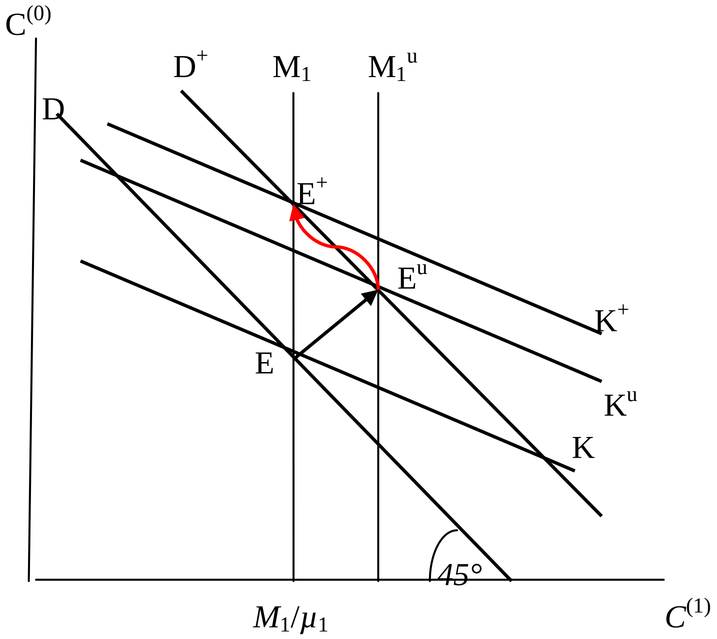
Assumption **c.** The degree of utilization of machines can be variable according to two possibilities: a continuously variable and uniform degree of utilization of each machine or a binary degree of utilization (either active or idle) of each machine, such that a variable number of machines of each type can be operative and the rest is kept idle. The two options may not be indifferent in terms of costs, but for simplicity we suppose that they are and therefore an idle machine *in demand* should receive the same rent paid for an active machine.

Let us assume that the quantity side of an initial steady state of the surplus regime is represented by point E in figure 2. An autonomous increase in demand is described by a shift of the D line, say up to D^+ line. *How can the new demand become effective (actual) instead of remaining potential, if i) quantities K, M* are given, *and ii) only techniques m_0, m_1 are available?*

[7] The degenerate feature of the solution derives from the fact that the number of constraints which bind the optimal solution is greater than the number of the choice variables.

[8] This interpretation of a fully adjusted equilibrium in the rent regime as a degenerate problem of L.P. and its dual with infinite solutions is not new. As far as I know, beside my own early persuasion, I remember that the same property has been stressed by Harvey Gram in his presentation at a conference held in the 1990s.

Figure 2 - Transition between steady states driven by a ‘small’ increase of demand

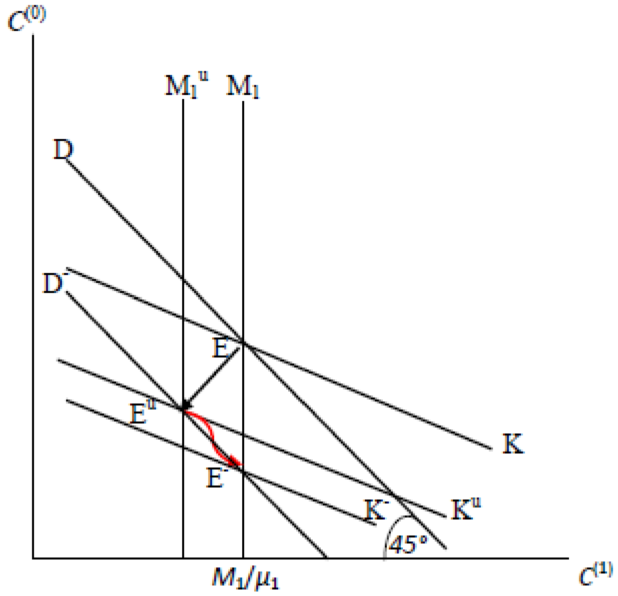


A possible adjustment sequence is described in figure 2 by points E, E^u , E^+ and two oriented connectors. For simplicity we assume, like in section 2, a single variable *u* which defines a *uniform* degree of capacity utilization. The revised resource constraints are:

$$\left.\begin{array}{c} c_0\, C^{(0)} + c_1\, C^{(1)} = Ku \\ \lambda_0\, C^{(0)} + \lambda_1 C^{(1)} = Lu \\ \mu_1\, C^{(1)} = M_1 u \\ 0 \leq u \leq u_{\max} \end{array}\right\} \quad [5]$$

At point E both types of machines, C, M_1, are used at the normal rate u = 1 and this condition underlies the K and M_1 lines. Instead K^u and M^u lines correspond to the same number of machines used at a degree of capacity which is higher than normal: $1<u\leq u_{max}$ and the K^+ line describes a production constraint where the number of machines of type C is fully adjusted to the higher demand and used at the normal rate u = 1. Point E^u represents an intermediate equilibrium along a route towards a new steady state E^+. The normal prices and the conventional wage of the initial equilibrium E can be preserved along the path from E to E^u . A similar argument can be applied, *mutatis mutandis*, to the case of a fall in effective demand represented in figure 3. The adjustment path is described by the succession of points E, E^u , E^-. The quantity $C^{(0)}$ may even fall to zero and $\varrho_1 = 0$, if D falls even more.

Figure 3 - Transition between steady states driven by a ‘small’ fall of demand



A different adjustment to a new steady state may turn out if demand D increases to such an extent that it cannot be satisfied by the given amounts of machines of type C and M_1 used at their maximum capacity $u = u_{max}$. The following case describes this possible adjustment of capacity utilization.

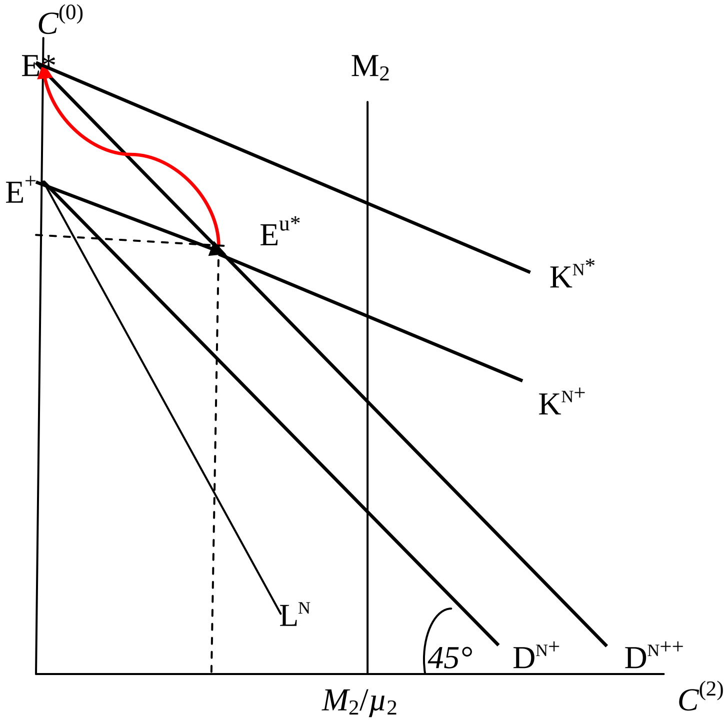
By assumption process m_2, which uses obsolete machines of type M_2, is not profitable at the normal prices prevailing in E ($\varrho_2 < 0$). However, a change from the surplus regime to a rent regime can make process m_2 profitable and $C^{(2)}$ positive. Such a regime is not a ‘pure’ rent regime because the corresponding equilibrium wage rate, despite the fact that it may deviate from the conventional level, does not clear the labour market and is consistent with the assumption of an unlimited labour supply. The following price equations, where λ_2, c_2, μ_2 are the technical coefficients of process m_2, determine the rent ϱ_c and the wage w which allow machines M_2 to be used at the margin and receive a rent equal to zero, provided that $C^{(2)} \leq M_2/\mu_2$

$$\left.\begin{array}{c} w\,\lambda_0 + \varrho_c\, c_0 = 1 \\ w\,\lambda_2 + \varrho_c c_2 + \varrho_2\, \mu_2 = 1 \\ \text{with } \varrho_2 = 0 \text{ and } w,\ \varrho_c \geq 0. \end{array}\right\} \qquad [6]$$

A positive solution to [6] may not exist and we shall recall this possibility in the final section.

By assumption the order of cheapness among processes m_0, m_1, m_2 is such that $C^{(1)}$ in a steady state is equal to the normal capacity output of machines M_1, that is $\bar{C}^{(1)} = M_1/\mu_1$. Suppose that m_2 is more labour intensive and less capital intensive compared with process m_0: $\lambda_2/c_2 > \lambda_0/c_0$. Therefore the wage rate will have to decrease below the conventional level, the rate of profit implicit in ϱ_c increase and part of K will be reallocated from process m_0 to process m_2 allowing for a positive product $C^{(2)}$, a further decline of $C^{(0)}$ and a higher employment. Figure 4 describes a quantity adjustment in two dimensions ($C^{(0)}$, $C^{(2)}$), due to a higher increase of demand. It starts from steady state E^+, where $C^{(0)} = \bar{C}^{(0)}$, $C^{(1)} = \bar{C}^{(1)}$, $C^{(2)} = 0$, and passes through the intermediate state E^{u+}, in which part of the total amount M_2 is used, up to the new steady state E^*, where machines of type M_2 are idle again. The lines D^N, K^N, L^N are isoquants of net quantities $D^N \equiv D - \bar{C}^{(1)}$, $K^N \equiv K - c_1\bar{C}^{(1)}$, $L^N \equiv L - \lambda_1\bar{C}^{(1)}$.

Figure 4 - Transition between steady states driven by a 'large' increase of demand



This argument suggests that a wider range of excess capacity exists in the economy with obsolete machines, if we admit the possibility of a deviation of relative prices from their normal values; in particular a deviation of the wage rate from its conventional level. Only through a process of induced accumulation, which increases the number of non obsolete machines C, it is possible to reach a new steady state represented by points E^+ in figure 3 or E^* in figure 4 and possibly recover the original income distribution associated with the conventional wage rate $\bar{w}$. The adjustment process requires that effective demand persists at the higher level D^*. This would not be the case if the *rate of growth* of the demand should rise, instead of a constant level of higher demand D^*, and become persistent. This change in the rate of growth cannot be accommodated only through a change in capacity utilization, which eventually restores the initial income distribution.[9] Normal prices corresponding to the terminal point E^*

[9] A similar view is advanced by Nell (1998 pp. 492-493).

may not be equal to the prices of the initial steady state E. Our argument does not imply full employment neither in a steady state nor along the adjustment path with a rent regime. In any case it cannot be ruled out the possibility that the change in demand is abnormal to such an extent that it will remain potential instead of actual.

A similar argument applies to the case of a 'large' fall in demand, where the *mutatis mutandis* clause includes the activation of another type of obsolete machine used at the margin with its own null rent and by a more capital intensive and less labour intensive process compared with process m_0.

5. Modeling an intermediate-short period

Different fragments of modeling presented in the previous sections can be merged into a complete, albeit not general, model of capacity adjustment. The steady state model is represented by equations [3] and [4] with $D = \bar{D}$; $M_1 = \bar{M}_1$; $M_2 = \bar{M}_2$ and the unknowns are w, ϱ_c, ϱ_1, $C^{(0)}$, $C^{(1)}$, K , L. A solution to the equations is characterized by a quantity $K = K^*$ and process m_0 in use. The adoption of m_0 can be interpreted as the result of a cost-minimizing choice among many available techniques which produce commodity C without using obsolete machines. It implies the achievement of *a maximum* uniform rate of profit, given the conventional wage rate, the level of demand and the amounts of obsolete machines. A short period model, confined for simplicity to a uniform normal degree of capacity utilization, where u = 1 and $K=K^*$, can exemplify one among possible transition equilibrium states towards a new long period position with a different level of effective demand D^+ ; $D^+ \neq D$. The formalized model:

$$\left.\begin{aligned} w\lambda_0 + \varrho_c c_0 &= 1 \\ w\lambda_2 + \varrho_c c_2 + \varrho_2 \mu_2 &= 1 , \quad \varrho_2 = 0 \end{aligned}\right\} [7_a]$$

$$\left.\begin{aligned} w\lambda_1 + \varrho_c c_1 + \varrho_1 \mu_1 &= 1 \\ \varrho_c &= \frac{r(1+r)^n}{(1+r)^n - 1} \\ p_{m_1} &= \sum_{t=1}^{n_1} \frac{\varrho_1}{(1+r)^t} \\ p_{m_2} &= \sum_{t=1}^{n_2} \frac{\varrho_2}{(1+r)^t} \end{aligned}\right\} [7_b]$$

$$\left.\begin{aligned} C^{(0)} + C^{(1)} + C^{(2)} &= D^+ \\ c_0 C^{(0)} + c_1 C^{(1)} + c_2 C^{(2)} &= K^* \\ \mu_1 C^{(1)} &= \bar{M}_1 \\ \lambda_0 C^{(0)} + \lambda_1 C^{(1)} + \lambda_2 C^{(2)} &\equiv L \\ \mu_2 C^{(2)} &\leq \bar{M}_2 \\ w, \varrho_c, \varrho_1, C^{(0)}, C^{(1)}, C^{(2)} &\geq 0 \end{aligned}\right\} [7c]$$

where p_{m_1}, p_{m_2} denote the price of machines of type M_1, M_2 and n_1, n_2 their finite lifetime under the assumption of constant efficiency.[10] The unknowns of system [7_a], [7_b], [7_c] are w, ϱ_c, ϱ_1, r, p_{m_1}, p_{m_2}, $C^{(0)}$, $C^{(1)}$, $C^{(2)}$, L; the givens include n, n_1, n_2, D^+, K^*, $\bar{M}_1$, $\bar{M}_2$. Notice that a hierarchy exists in the determination of values, since w and ϱ_c can be determined before the other prices through equations [7_a]. Furthermore, the values of obsolete machines are determined *ex-post* as equal to a constant rent capitalized at the profit rate r, and therefore a uniform rate of return rules on the price of all capital goods used in the economy.

A solution to equations [7_a], [7_b], [7_c] can be interpreted as an *effective deviation* from an initial normal state, driven by a downwards or upwards shift of aggregate demand. In particular, the wage rate determined by [7_a] can be conceived as a deviation from its long period conventional level. *Effectiveness* in such a context means that the deviation can be actual and may trigger a change in the economy.

6. Some unsettled questions

The mechanism of variable capacity utilization exemplified so far is not heterodox, although the assumption of a finite number of techniques may lead, in the face of a change in demand D, to discrete changes in prices and distribution and to a kinked curve of labour employment L plotted against D. Other results can be derived without subverting the basis of the Keynesian theory. In particular, the Kahn-Keynes multipliers of employment and national income can be reframed in our context of flexible prices, assuming that the demand for consumption is positively related to national income, which includes the rents on obsolete machines. Also the so-called Cambridge equation[11] can be reinterpreted assuming that savings are proportional to the sum total of profits and rents. The accumulation of capital would consist only of non obsolete machines (commodity C in our model), since investment in obsolete machines is excluded. Still, the previous analysis suggests a reflection on some questions which have not been answered here, but cannot be avoided if the theory of the effective demand has to be applied to a theory of economic growth and cycles.

First question

In a more general model with heterogeneous capital goods, we encounter well known "paradoxes" of capital theory (re-switching of techniques and capital intensity reversal) and, as a consequence, a change in demand may bring about non orthodox effects through a change in income distribution. However, even outside such cases, a problem should be noticed in the previous examples. The equations [6], which reappear in system [7], may not have a positive solution. In this case processes m_0 and m_2 cannot co-exist under the condition $\varrho_2 = 0$ and a different choice of methods of production should cope with the assumed change in D and the deviation of prices from their long period values.

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[10] Notice that from $\varrho_2 = 0$ it follows $p_{m2} = 0$.

[11] The Cambridge equation is $g = s_\pi r$, where the symbols denote: g the rate of accumulation, s_π the propensity to save out of profits and r the rate of profit.

Second question

We have argued that a transitory change between two regimes, the surplus and a (non pure) rent regime, can bring about a higher flexibility in capacity utilization and accommodate a change of demand in a sort of two-way avenue: from the surplus regime to the rent regime and back to the surplus regime. However it is not said how, to use a metaphor, the shuttle can return from the space (the rent regime) to the earth (the surplus regime) if, during the travel in the space, the typical social connotations of the surplus regime (like a conventional wage rate on which the normal prices depend) have been abandoned. It seems plausible that the pressure of a shift of demand may temporarily overrule the surplus regime, but some trace of the institutional features of the latter has to pass to the rent regime if they should be fully recovered at the end of the adjustment process. In the simple model described in sections 3 and 4 such institutional feature is represented by what we called a given *conventional* wage rate and by equation $w/p = \overline{\mathrm{w}}$, where "conventional" is used as a shorthand for the more articulated expression "depending on institutional factors". However, also such expression remains vague for an extension of the theory of effective demand from the short period to a theory of growth and fluctuations. It is not clear whether the *conventional* wage and the implicit conventions are those stressed by Keynes in his short period analysis or, instead, it reflects the conventional nature of the subsistence wage of the old classical long-period theory of population. According to the first connotation, the model should be extended to the monetary side of the economy. Instead of the equation $w/p = \overline{\mathrm{w}}$, we should assume a given *money* wage rate and a given money interest rate in order to close the system of price equations. By contrast, if we follow the classical approach, we should clarify which institutional factors can determine the *real* wage: What is at issue for a feasible two-way avenue mentioned above is whether the same conventional wage applies to the short and to the long period and whether in each case the wage is fixed at a certain level or it is only subjected to a minimum floor set by $w/p \geq \overline{\mathrm{w}}$.

Third question

We wonder how long-term expectations and investment plans are affected through the short term changes in prices and distributional variables. The method of comparative dynamics, applied to the model with obsolete machines, allows us to determine the change in quantities between steady states; in particular the change in K from the initial to the new amount induced by the change in demand D. Yet, the *process* of growth should be a main issue, considering also that the dynamics of the *absolute* quantities, in general, is subjected to path-dependency.[12] On this question, the composition of fixed capital in terms of obsolete and non obsolete machines seems to be an important factor, which perhaps has not received the due attention in the recent

[12] In (Parrinello 2014) the author has called the attention to a property of a disaggregated model of balanced growth at a geometric rate, noticed by Solow and Samuelson (1953). Such type of model is inherently unstable on the side of the absolute quantities of the capital goods and the path dependency of its equilibrium is the rule; only *relative* stability (i.e. the convergence of the proportions among quantities and among prices) is left to a demonstration .

literature dealing with effective demand and growth[13]. As a matter of fact, a normal state of an economy presents a stratification into layers of machines which embody different technologies and have been grouped in two categories only for our analytical purpose: obsolete and non obsolete machines. On the basis of this distinction, it is not necessary that fixed capital as a whole adjusts to a steady state in order to make possible for the distributive variables to converge toward their normal values. The adjustment of the proportions among capital goods which belong to only one layer would suffice: the layer of the non obsolete machines entering the sub-system of equations – first equation of system [2] in our simple steady state model – which determine the rate of profit, given the conventional wage rate. This property suggests that the speed of adjustment towards those normal values is decoupled, within certain limits, from the dynamics of obsolete fixed capital. The capital composition is subjected to the interference of a multiplicity of factors, in particular to the vagaries of technical progress, and the proportions among obsolete capital goods may not possess an equilibrium configuration which can act as an attractor. This is consistent with a gravitational pull attributed to normal prices and to the quantities of non obsolete capital associated with them.

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[13] Cfr. Parrinello (2014).