Sraffa's 'Given' Quantities of Output and Keynes's Principle of Effective Demand

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> [I]t is in the 'present' that the 'normal' rate of profits has always been firmly located. … [B]ecause this is the rate of profits which is being realised *in the present* …, it is also the rate of profits which that present experience will lead entrepreneurs in general to expect *in the future* from their current investment. (Garegnani, 1979, original emphases)

1. Introduction

Sraffa (1960) takes the quantities of output as 'given' when considering the determination of prices. He gives little hint at how the quantities of output have come to be what they are. A commonly accepted interpretation of the given quantities of output in the Sraffa system is that they reflect the state (level and composition) of effective demand (see e.g. Garegnani, 1984, 1990; Kurz, 1990, 1992, 1994; Cesaratto, 1995). It is in this vein when the 'Sraffian Keynesian' approach attempts to synthesise Sraffa and Keynes; that is, to provide an integrated framework in which the Sraffian system serves the determination of prices and the Keynesian principle of effective demand the determination of the quantities of output.

While the 'Sraffian Keynesian' approach proposes in a general agreement that it is the state of effective demand that determines output not only in the short but also in the long period, the approach varies widely in specific arguments and formulations. The initiative for the approach was taken by Garegnani's 1962 work (Garegnani, 1962), which has been followed by a series of his own elaborations (Garegnani, 1982, 1983, 1992). Some important work along these lines, exhibiting high hopes for some kind of formalisations to deal with the topic, was done in the 1980s (e.g. Eatwell, 1983; papers in Eatwell and Milgate, 1983 and in Bharadwaj and Schefold, 1990). The efforts of positive construction, however, seem to have been increasingly distracted by the theoretical brawls with the 'Kaleckian' steady-state approach. Meanwhile Serrano's (1995) idea of the 'Sraffian supermultiplier' was an important constructive contribution, if it subsequently came in for criticisms within the Sraffian camp (Trezzini, 1995, 1998, Park, 2000). In contrast to various attempts to 'formalise' the determination of output in the long period, Palumbo (1996), Garegnani and Palumbo (1998), and Palumbo and Trezzini (2003) take a critical stance to such attempts, arguing that the actual process of capital accumulation and output growth is too complex to be dealt with in a formalised way. Recently Trezzini (2005) and Garegnani and Trezzini (2010) pursue some particular Sraffian-Keynesian lines of research. That there is continuing interest in the approach is witnessed by a collection of essays published recently (which were originally discussed in a 1998 conference) (Ciccone *et al*, 2011, Parts III and IV).

The following pages are intended as another strand of attempt at synthesising Sraffa and Keynes, presumably more in the spirit of Garegnani (1962, 1982, 1983, 1992) and in an explicitly multi-sector framework. A key concept is a 'fully-adjusted position' of the economy (Vianello, 1985). The state of the economy is a fully-adjusted position when, in every industry, productive equipment (the size and composition of the means of production) and the state of demand are fully adjusted to each other so that (*i*) the productive equipment is utilised at the 'normal' level and (*ii*) the output produced in each industry is at the level exactly matching its total use (that is, use for the replacement of used-up capital equipment, new investment and consumption in the economy as a whole). It will be suggested that the Sraffa system of production prices *refers* to a fully-adjusted position, under the condition of free competition (just *referring*; that is, even if prices determined in the Sraffa system are established and the economy is in free competition, there is no need for the economy to be actually in a fully-adjusted position).

The framework we propose consists of three systems of equations. The first system is for a state of the economy which is not constrained by effective demand; the second and the third are for an effective-demand-constrained state of the economy. These systems of equations are paired in two ways. The first and the second deal with fully-adjusted positions, with the first serving as the baseline in comparison with which the second is specified. Whilst the second sets out the relations reflecting full adjustment under the constraint of effective demand, the third describes the 'realised' state of the economy under the same constraint of effective demand, which is usually not a fully-adjusted position. Consideration of all these three systems of equations, in the two ways of pairing, is, it shall be suggested, necessary for a comprehensive analysis of the *long-period* configuration of the economy.

The state of the economy that the first system of equations describes is the 'Warranted Growth (WG)' state of the economy, so dubbed in the spirit of Harrod (1939). The WG state is a fully-adjusted position holding for the existing configuration of capital equipment. At the beginning of a period, the economy is equipped with a given configuration of capital equipment. One can then, by aid of the system of equations to be presented, find what will be called the 'warranted investment' for each industry corresponding to that existing capital equipment. The warranted investment for a given capital equipment in a given period is the volume of investment which would bring about a fully-adjusted position corresponding to that capital equipment; thus, the resulting output in each industry is the (normal-) capacity output corresponding to the given capital equipment and precisely satisfies its total required use. The WG state being a fully-adjusted position, one can construct a price system *à la* Sraffa corresponding to the quantities of output in that state and obtain the prices of production for this state of the economy.

The warranted investment *must* be equal to the capacity saving corresponding to the given capital equipment and hence not autonomous. The reason to consider this state of the economy is that it will be used as the baseline in comparison with which the 'Effective-Demand-Constrained (EDC)' state of the economy is to be specified. An economy in the EDC state is one where the quantities of output are determined in accordance with the state of effective demand. To represent such an economy in the lines of *long-period analysis* is the objective of the second system of equations to be presented in the paper. Another key idea in this endeavour, in addition to the concept of a fully-adjusted position, is the autonomy of actual investment, that is, the idea that actual investment is usually at a different level from the warranted one (thus, actual output, determined in reference to actual investment, is also usually at an other-than-normal level). We shall conceive that, in a multi-sector model such as ours where the unit of analysis is an industry, the autonomy of investment reveals itself, and thus should be represented, *at the level of individual industries* and that the autonomy of investment at the aggregate level is simply the combined result of such industry-level autonomy. We shall also take, as the magnitude representing the autonomy of investment in an industry, the volume of (gross) investment in that industry *relative to its WG counterpart* (the very reason why we consider the WG state before the EDC state). As we are concerned with the long-period configuration of the economy, the EDC state we consider is as much a fully-adjusted position as the WG state is (we shall give an EDC fully-adjusted position the shorter name of a 'long-period position (LPP)' in distinction with the WG fully-adjusted position). Given the volumes of autonomously determined investment in the respective industries, the system of equations to be presented will yield such configurations of the means of production and the quantities of output that satisfy the two conditions of a fully-adjusted position. Included in the system are the Sraffian price equations corresponding to these quantities of output, from which one obtains the prices of production for the state in question. (If one assumes, as we do for the expediency of argument, constant returns to scale, these prices shall not be different from those to be established in the WG state; if no such assumption is made, of course, they may be different.) Though both a WG state and an LPP are fully-adjusted positions, there are differences between them. The critical, qualitative, difference is obviously that, for an LPP, full adjustment is envisaged under the condition of autonomous investment. Quantitative differences are to be observed in the size of the stock of means of production in each industry and, hence, in the level of the capacity output and of the capacity saving (Garegnani, 1962, 1982, 1983, 1992), with their relative sizes across the industries in an LPP too being generally different from those in a WG state.

Our standpoint is, in accordance with the Keynesian one (extended to the long period), that the long-period state in which the economy finds itself is an LPP, not a WG state. But, even when the quantities of output are those ensuing in an LPP (and thus, prices are production prices *à la* Sraffa), the 'realised' state of the economy which will appear to the eye of the statistical observer is not a fully-adjusted position. This is because the configuration of capital equipment existing in the economy at the beginning of the period under consideration—the starting point of our analysis—is usually different from that to be established in the LPP. Thus the rates of profits realised on the existing capital equipment are not uniform across the industries, despite the assumed prevalence of free competition, being affected by the degrees of utilisation of the capital equipment. Our third system of equations is to describe this 'realised' state of the economy where the quantities of output are those which are to be established in the LPP and commodities are valued in terms of the prices of production corresponding to those quantities of output, against the backdrop of the existing capital equipment, this now being seen to be utilised at an other-than-normal level. Here an additional aspect of the working of the economy, one regarding the depreciation of the means of production with respect to the degree of utilisation, should be taken into account.

There are two major problems facing a multi-sector framework such as ours which intends to provide a long-period analysis reflecting the condition of effective demand. The first is how to represent the autonomy of investment decision made in the respective industries over the long period, focus being on two aspects of the problem: *autonomy* and *long period*. To address both aspects is the reason why the consideration of the WG state is brought in prior to the system for the LPP. Given the existing stock of capital equipment and the 'fundamental parameters' (that is, the technique in use and the real wage rate (or the normal rate of profits)), the volume of warranted investment for each industry is dictated. We can then represent the autonomy of investment in terms of the autonomous decision on the *ratio*, in each industry, of the volume of actual investment relative to that of the WG state. The autonomy of investment, then, reveals itself in the form of the ratio being not necessarily unity. This way of representation is also an answer to the long period aspect of autonomous investment. The configuration of capital equipment changes over time as a result of investment in the previous period(s) and the volumes of the warranted investments follow the suit in the respective industries. Accordingly shall the volumes of autonomous investments change over time (due either to changes in the warranted investments or to changes in the ratios between the warranted and actual investments, or to both; the first kind of changes reflect the capacity-generating effects of investment in the long period and the second kind the effective-demand-constituting effects in the short period).

The second problem is related to the relationship between investment and saving. In accordance with the principle of effective demand, it is investment that generates the same volume of saving, not the other way around. It is noted that this principle in strict terms applies to the economy as a whole, regarding the aggregate investment and the aggregate saving. Equality between saving and investment is to be observed also at the level of industry, but the equality is achieved through a different mechanism from the one for the economy as a whole. The volume of saving that is brought into line with investment in each industry is not the same as that which is generated from the industry concerned (for example, from the pockets of the wage earners and the profit earners of that industry). Saving in the mind of savers is not industry-specific. Saving is *generated* of the same size as investment through the multiplier mechanism at the aggregate level, and this aggregate saving is *allocated* to the respective industries in line with the volume of actual investment that has been made in each industry. This necessitates the consideration of the *financial market*, the space where the allocation of the aggregate saving takes place.

The paper proceeds as follows. Section 2 presents some main ideas and concepts in the context of an aggregate economy. Section 3 is for the presentation of a multi-sector framework, where the three systems of equations are set out, representing respectively the WG state, and the LPP and the 'realised' state of the economy. There shall also be a discussion on how the prices of production *à la* Sraffa are understood to refer to a fully-adjusted position and how the quantities of output determined in the LPP can be interpreted as the 'given' quantities of output in Sraffa's original system. The arguments in Sections 2 and 3 proceed on the formulation of saving behaviour in terms of the social classes such that workers do not save whilst capitalists save a constant fraction of their income. This allows us not to deal explicitly with the financial market, keeping its working in the background. Section 4 brings the working of the financial market out into the open, for which we adopt the Kaldorian saving behaviour assumption in which the saving units are households and firms. Section 5 discusses some constraints which should be taken into account for the EDC state of the economy. The whole argument is illustrated in Section 6 by way of the 'Hicks-Spaventa' two-sector economy and here we find our construction serves well to illustrate Joan Robinson's famous classification of various Growth Ages. The appendix extends the basic model by introducing the government activity.

2. Main ideas in aggregate terms[1]

The dominant technique in operation in the economy is represented by the capital-output ratio (v) and the labour-output ratio (l), both engineering-specified in reference to the normal utilisation of the capital stock; while the realised capital-output ratio changes in response to the degree of utilisation of the capital stock, the realised labour-output ratio is assumed to remain the same as the engineering-specified ratio regardless of the degree of utilisation. The capital stock depreciates at the rate of δ per period.[2] The triplet $\{v, l, \delta\}$ represents the given technical parameters. There are two social classes in the economy, workers and capitalists: workers do not save and capitalists save a fraction s_c of their income (profits). In accordance with the Classical-Sraffian perspective, the real wage rate (w) is assumed to be exogenously given (or, essentially the same argument can be made assuming the normal rate of profits (r) is exogenously given).

The existing capital stock at period t is $\overline{K}_t$. The 'Warranted Growth (WG)' state of the economy corresponding to this capital stock is one where the capital stock is utilised at the normal level and the resulting quantity of output Y_t^*, which is the capacity output, exactly matches its use in the economy, that is, it is exhausted for the replacement of used part of the capital stock, net investment and consumption. Thus, the WG state is characterised by the following relations:

$$Y_t^* = \overline{K}_t / v$$
$$Y_t^* = (\delta + r)\overline{K}_t + wl\, Y_t^*$$
$$Y_t^* = (\delta + g^*)\overline{K}_t + wl\, Y_t^* + (1 - s_c) r \overline{K}_t$$

where g^* is the rate of net accumulation which will be observed under the stated conditions and hence called the 'warranted rate of (net) accumulation'. The first equation specifies the condition of the normal utilisation of the existing capital stock. The second is the 'price

[1] The second half of Park (2011) advances the same ideas if in a less refined form.

[2] Sraffians will not agree with this setting of the 'radioactive' depreciation rate; however, excuse can be granted for the expositional purpose.

equation': the value of output is divided into depreciation, profits and wages. The third is the 'quantity equation', which describes the use of produced output. With Y_t^* determined in reference to $\overline{K}_t$ and v in the first equation, the price equation determines the normal rate of profits r without reference to the quantity equation; then, the quantity equation, together with the price equation, determines g^*.

The WG state is a fully-adjusted position: first, the quantity of output is that which ensues from the normal utilisation of the capital equipment (the first equation); second, the quantity of output exactly matches its total use (the quantity equation). As a result, the rate of profits appearing in the price equation is the normal rate. The WG state is a fully-adjusted position holding for *the existing capital stock*.

The resulting Cambridge equation, $g^* = s_c r$, represents the equality between (net) investment and (net) saving. With $\overline{K}_t$ exogenously given and *r*, the normal rate of profits, determined solely in reference to the exogenously given variables ($\overline{K}_t$ and the technical variables), the resulting saving ($s_c r \overline{K}_t$) is the capacity saving corresponding to the given capital stock and is determined independently of the volume of investment. It should be the case, thus, that investment is brought into line with the capacity saving. This investment, $I_t^* \equiv g^* \overline{K}_t$, is the volume of investment which is *required* to ensure the normal utilisation of the capital stock; hence dubbed, in the spirit of Harrod (1939), the 'warranted (net) investment'. The warranted investment is not autonomous.

The autonomy of investment means that actual investment is autonomous from the capacity saving (thus, equal to the warranted investment except by chance). Corresponding to the actual investment will be the actual output Y_t which is accordingly different from Y_t^*. The degree of utilisation of the given capital equipment is defined as the ratio of the actual to the capacity output corresponding to that capital equipment: $u_t \equiv Y_t / Y_t^*$. The degree of utilisation being less than the unity means that the existing capital stock has been utilised at a level lower than the normal level.

We now pose the following question. *Given the actual net investment (or, rather, the actual gross investment J_t),*[3] *what level of the capital stock would have had that investment as the warranted investment?* We denote such a hypothetical capital stock by K_t. Had the existing capital stock been K_t, the actual gross investment J_t would have been the warranted (gross) investment and the capital stock would have been utilised at the normal level; thus, K_t would have been the configuration of the capital stock that was fully adjusted to output, the configuration of output in turn resulting from the autonomously determined gross investment J_t.[4] That is, the state of the economy we intend to contemplate is a *fully-*

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[3] The preference to consider in terms of *gross* investment is based on two reasons, one analytical and another economic; see footnote 5 and 10.

[4] The concept of K_t was first proposed in Park (1994), with inspiration from Harrod (1939), in the context of interpreting Keynes's theory of employment as a 'short-period analysis in the long-period framework', and was utilised by Park (2000) in the discussion of 'Sraffian supermultiplier'.

adjusted position where gross investment is autonomously given at the level of J_t. For that purpose, we express the volume of actual gross investment in the current period as a fraction (or a multiple) z_t of its warranted counterpart:

$$J_t = z_t J_t^* \equiv z_t(\delta + g^*)\overline{K}_t$$

The autonomy of investment may then be expressed by the proposition that $z_t = 1$ is not necessarily the case. This way of representing actual investment reflects an aspect of the *long-period analysis*. The long-period analysis pays attention to both of the two effects of *actual* investment: the effect of constituting effective demand in the current period and the effect of generating the productive capacity of the future periods. The increase in the productive capacity is always accompanied, for the given parameters, with the increase in the warranted investment. Thus, in the above conception of the determination of actual investment, the part J_t^* reflects the capacity-generating effect of (past) actual investment and the part z_t stands for its (present) effective-demand-constituting effect (and, with $z_t = 1$ not necessarily the case, the autonomy of investment).

The state of the economy in question is then described by the following set of relations:

$$\begin{gathered} Y_t = K_t/v \\ Y_t = (\delta + \tilde{r}_t)K_t + wl\,Y_t \\ Y_t = (\delta + \tilde{g}_t)K_t + wl\,Y_t + (1 - s_c)\tilde{r}_t K_t \\ (\delta + \tilde{g}_t)K_t \equiv J_t = z_t(\delta + g^*)\overline{K}_t \end{gathered}$$

The first equation expresses the first condition for a fully-adjusted position: output Y_t is produced by the normal utilisation of the hypothetical capital stock K_t. The price equation now has the rate of profits at $\tilde{r}_t$, which should reflect the difference from the WG state in the level of output and the size of the capital stock. The quantity equation incorporates the second condition for a fully-adjusted position ('output = its total use'). The fourth equation expresses the idea that actual gross investment is given at the level of J_t. The system of equations determines Y_t, K_t and $\tilde{r}_t$ for the given level of z_t. The state of the economy described here is a fully-adjusted position corresponding to a given state of effective demand: it is an *effective-demand-constrained* (EDC) state.[5]

It turns out that $\tilde{r}_t = r$; this is, with the 'fundamental parameters' unchanged, a natural result for a fully-adjusted position. Also, the ratio of net investment to the capital stock in this

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[5] This system of equations makes clear one of the reasons, an analytical one, why we take gross rather than net investment when representing the autonomy of investment. If we took net investment, the actual net investment would be bound to be nil when the warranted net investment was nil; then, the last equation in the system would lose its operational significance (though, with the fourth equation gone, one should and could make z_t, the key variable in our framework, appear in some of the other equations).

state, $\tilde{g}_t \equiv (J_t/K_t) - \delta,$ is equal to its WG counterpart, g^* (thus, the Cambridge equation derived for the EDC state, $\tilde{g}_t = s_c\tilde{r}_t,$ is nothing but a copycat of its counterpart for the WG state, $g^* = s_c r$).[6]

Differences exist, however, between the EDC state and its WG counterpart. An obvious, but the most important, qualitative, difference is that in the former an autonomously given volume of investment determines output and its corresponding fully-adjusted capital stock, whilst in the latter the existing capital stock dictates the volume of investment that shall bring about its utilisation at the normal level. There are also quantitative differences. The comparison of the two states of the economy reveals:

$$\frac{K_t}{\bar{K}_t} = \frac{Y_t}{Y_t^*}(\equiv u_t) = z_t$$

The EDC state is a uniformly scaled-down (or scaled-up) version of the WG state: the factor that determines the scale is z_t.[7] The degree of utilisation being, for example, less than the unity means that K_t is smaller than the existing stock; that is, the capital stock should have been K_t, smaller than the existing one, if there was to be the normal utilisation of the capital equipment given the current state of effective demand, which is represented by the actual gross investment J_t, smaller than its warranted counterpart. Both the EDC and the WG states being fully-adjusted positions, we give the name of '*long-period position* (LPP)' to an EDC fully-adjusted position in order to distinguish it from its WG counterpart (thus, K_t may be called the 'long-period position capital stock').

The attribution of 'long-period' to such a state of the economy may be legitimated on the following reasoning. The long-period is characterised with the free movement of (financial) capital in pursuit of a higher rate of return and thus implies changes in the configuration of capital equipment. In this pursuit, entrepreneurs endeavour to adjust the configuration of their capital equipment to the demand for their products so that the capital equipment can be utilised at the normal level (and the return on their capital can also be at the normal level). In our present case where the current state of effective demand is the actual gross investment being J_t, that configuration of the capital equipment is K_t.

This reasoning leads to the next. It is obvious that, in a growing economy in particular, K_t (at its particular level) shall not be the final goal which entrepreneurs would wish eventually to achieve: as far as net investment is positive, the capital stock keeps growing and (unless z_t keeps shrinking in inverse proportion with, or faster than, the resulting growth of

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[6] It should be noted, however, that $\tilde{g}_t$ is not a measure of how the EDC capital stock accumulates over time (the 'rate of accumulation'): it is not that J_t is actually grossly added to K_t to change the size of the EDC capital stock but it is that K_t is *determined* in reference to J_t; as K_{t+1} is determined in reference to J_{t+1}, the evolution of the EDC capital stock (K_{t+1}/K_t) shadows the evolution of the actual investment (J_{t+1}/J_t); see below.

[7] This is not necessarily the case in a multi-sector framework.

the warranted investment) so does the actual gross investment. This means that the LPP capital stock, K_t in its general meaning, will also keep growing. In this context, K_t is better conceived simply as a configuration that the entrepreneurs regard as the one which is 'normal' with regard to the current state of effective demand. Then, precisely for that reason, it will act as a 'reference point' that entrepreneurs take into account when they make decisions regarding investment next period.[8] Though K_t is a non-observed configuration of capital equipment, it is expected to exert concrete influence on the investment behaviour of entrepreneurs. Such is, thus, our understanding of the long period position that the LPP capital stock is determined in reference to the state of effective demand in the current period and, at the same time, determines the state of effective demand in the coming periods. It turns out that this understanding fits nicely into a paraphrasing of Garegnani's (1979) expression, quoted as the epigraph of the present paper: it is in the 'present' that the long period position is firmly located; because this is the state of the economy which is being regarded as 'normal' with reference to the state of effective demand *in the present*, it is also the state of the economy that present experience will lead entrepreneurs in general to take into account when they make decisions on their investment *in the future*.

Existing, so to speak, beneath the surface of the economy, the long-period position is usually not what is to appear to the eye of the statistical observer. The 'realised' state of the economy will look like the following:[9]

$$Y_t = \overline{K}_t/(v/z_t)$$
$$Y_t = (z_t\delta + z_t r)\overline{K}_t + wl\,Y_t$$
$$Y_t = (z_t\delta + z_t g^*)\overline{K}_t + wl\,Y_t + (1 - s_c)(z_t r)\overline{K}_t$$

With the existing capital stock $\overline{K}_t$ and the current gross investment $J_t = z_t(\delta + g^*)\overline{K}_t$, the current output is Y_t and the realised (observed) capital-output ratio is (v/z_t). It is important to treat the rate of gross profits $z_t(\delta + r)$ and the rate of gross accumulation $z_t(\delta + g^*)$ as one variable respectively, until one specifies how the capital stock depreciates as the degree of utilisation of the capital stock changes. This is because the efficiency of a machine will be affected by how intensively it is used. The above equations are based on the assumption of 'linear' depreciation such that the depreciation rate is proportional to the degree of utilisation. Then, the realised depreciation rate is $z_t\delta$, the rate of net profits realised on the existing capital stock $z_t r$, and the realised rate of net accumulation $z_t g^*$.[10] The actual employment

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[8] If the much-used expression 'a centre of gravitation' does not connote the eventual convergence to a situation of an *unchanging* configuration, one may also use that expression instead of our term which has a much weaker connotation.

[9] Both the EDC and the WG states are 'equilibrium' states; therefore, the 'realised' state of the economy here is in the sense that the output of the EDC state is *assumed* to prevail.

[10] This is another reason, an economic one, why we take gross, not net, investment when considering autonomous investment. The degree of utilisation is endogenously determined

of labour is $l\,Y_t$.

The role of the long period position as a ‘reference point’ comes to the fore when we consider the evolution of the (‘realised’) economy over time. The autonomy of investment does not necessarily mean that the entire part of z_t is given exogenously. The ratio of the actual to the warranted investment can be conceived to evolve over time, for example, in the following way:

$$z_{t+1} = \bar{z} + \alpha[(K_t/\bar{K}_t) - 1]\,z_t, \text{ with } 0 < \alpha < 1$$

That is, if the long-period position capital stock is smaller than the actual stock, which means that the capital stock should have been smaller than the actual one if there was to have been the normal utilisation, entrepreneurs are so discouraged in their investment behaviour that they set the ratio of the actual to the warranted investment in the next period below the level $\bar{z}$ (whose meaning will be discussed shortly). The formulation clearly incorporates the ‘reference point’ role of the long-period position.

The actual gross rate of accumulation $G_t(\equiv J_t/\bar{K}_t)$ in period t is related to the warranted gross rate $G^*(\equiv J_t^*/\bar{K}_t = \delta + g^*)$ through z_t:

$$G_t = \frac{z_t J_t^*}{\bar{K}_t} = z_t G^*$$

Using the recurrence relation for z_t above and noting that $K_t/\bar{K}_t = (J_t/G^*)/\bar{K}_t = G_t/G^*$, one gets

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and so shall depreciation be if depreciation is a function of the degree of utilisation. In this situation, it seems economically more realistic to suppose that entrepreneurs make plans on the sum of depreciation and net increase of the capital equipment, the two elements not distinguished, and leave the partition between them endogenously determined, rather than fix the net increase first whilst bearing later the burden of depreciation endogenously determined as a result of that net increase. If the depreciation rate is a function of time only, so that depreciation is in force regardless of how the capital equipment is utilised, then the realised depreciation rate is δ, the realised rate of net profits $z_t(\delta + r) - \delta$ and the realised rate of net accumulation $z_t(\delta + g^*) - \delta$ (from this, incidentally, one can notice that our framework allows for the shrinking of the economy, even if the WG state depicts a growing economy: it may be the case that z_t is so small that $z_t(\delta + g^*) - \delta < 0$ even with $g^* > 0$). In general one can specify, as part of the technical variables, the depreciation rate as a function of the degree of utilisation: $\delta_t = \delta(u_t)$ where, over the domain $0 < u_t < u_t^{max}$ (with u_t^{max} usually larger than unity), $d\delta_t/du_t$ can be positive or negative or in alternate signs (each, linearly or at varying rates of change). The case in the main text is where $\delta_t = \delta u_t$, and the case mentioned just above is where $\delta_t = \delta$.

$$G_{t+1} = z_{t+1}G^* = \bar{z}G^* + \alpha[(G_t/G^*) - 1]\,G_t$$

With this expression it may become possible to interpret the part $\bar{z}G^*$ as standing for the *expected rate of output growth*: entrepreneurs have in mind a certain expectation regarding the future growth of output and, taking it as the baseline of their investment decisions, adjust the rate of accumulation period by period around that expected rate of output growth in the light of the realised degree of utilisation (which reflect the degree of adjustment between the productive capacity and the actual output). One may also say that $\bar{z}G^*$ (or $\bar{z}$ alone, as G^* is determined by the given fundamental parameters) represents the state of 'animal spirits': the value of $\bar{z}$ which lies between zero and the unity ($0 < \bar{z} < 1$) stands for a state of animal spirits 'below the par' ('low' animal spirits) and that which is larger than unity ($\bar{z} > 1$) for a state of animal spirits 'above the par' ('high' animal spirits).

The long-period position capital stock K_t, being determined in accordance with the actual investment, evolves over time in step with the latter. It turns out that the evolution of the rate of net accumulation along the path of long-period positions, K_{t+1}/K_t, is precisely the same, if one period behind, as that of the actual rate of net accumulation: $K_{t+1}/K_t = G_{t+1}$.[11]

It goes without saying that the above formulation of the evolution of the rate of actual accumulation is not the only possible one; some other formulations have been in use in the literature.[12] However, the above formulation proves to facilitate discussion regarding some

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[11] See footnote 6 above.

[12] They are:

(1) $z_t = 1$ for all *t*. This is the usual case of steady-state growth (the path of warranted growth): the capital stock is utilised continuously at the normal level. The case is better interpreted as the *required* condition for continuous normal utilisation and thus is in contradiction to the autonomy of investment.

(2) $z_t = \bar{z}$ for all *t*. The 'Kaleckian' approach (Rowthorn, 1981 and Dutt, 1990, to name just two from among the vast literature) focuses on a steady-state growth (at the gross rate of $\bar{z}G^*$) with the possibility of the degree of utilisation being permanently different from the normal level.

(3) $z_{t+1} = z_t + \alpha[(K_t/\overline{K}_t) - 1]$. The baseline is the current ratio (the current rate of accumulation) and an adjustment around the baseline is made in *no* reference to the current ratio (the current rate of accumulation). One stationary point exists, which is unstable: once K_t is ever different from $\overline{K}_t$, the actual rate of accumulation explodes in either direction over time. This obviously approximates Harrod's (1939) position.

(4) $z_{t+1} = 1 + [1 - (1/z_t)]\bar{z}$, with $\overline{K}_t = K_t$ for all *t*. This is an interpolation from Serrano's (1995) 'Sraffian supermultiplier'. Serrano considers the 'long-period' path of output where the capital stock is utilised always at the normal level (hence, $\overline{K}_t = K_t$ for all *t*); the driving force of growth is non-induced investment ('autonomous investment'), which grows at an exogenously given rate ($\bar{G} \equiv \bar{z}G^*$). The recurrence relation shows the evolution

important points related to our conceptualisation of the effective-demand-constrained long-period analysis. The economy described by the above recurrence relation converges to the state of steady growth where the gross rate of accumulation is, if the economy is a growing one, different from the warranted rate.[13] This means that the economy settles down at a state where the capital equipment is *not* utilised at the normal level. Would this not contradict the rationality of entrepreneurs who, realising the current pattern of their investment (represented here by $\bar{z}$ and α) is the culprit of the permanent mismatching between the capital equipment and output, will adjust their behaviour pattern in the attempt to remove the mismatch, whether that attempt is eventually successful or not (most probably, unsuccessful in reality)?[14] This is indeed the case: the utilisation of the productive equipment at a level *constant and permanently different* from the normal one cannot be conceptually maintained if one is to accommodate the rationality of entrepreneurs mentioned above. As much as the steady state with continuous normal utilisation (the continuous warranted growth) is denied by the autonomy of investment (Garegnani, 1982, 1992; Ciccone, 1984; Palumbo and Trezzini, 2003), so the steady state with constant under- or over-utilisation is refuted by economic rationality (Committeri, 1986; Ciccone, 1986; Palumbo and Trezzini, 2003). The steady state is to be distanced away in the analysis of economic growth (and, furthermore, our study will show that in a multi-sector framework the steady state, requiring uniform growth across the industries, is a remote possibility.)

Then, the above formulation of investment behaviour over time must be taken as nothing but a heuristic tool. The important point, which the recurrence relation under consideration

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of the rate of net accumulation which is *required* to ensure the normal utilisation of the capital stock period by period; the rate evolves over time as if the baseline was the warranted rate ($1 \times G^*$), pre-determined in accordance with the given fundamental parameters, and the variation from it was a fraction of the exogenously given rate of growth of non-induced investment, the fraction reflecting the difference between the warranted rate and the rate of accumulation of the previous period. As Trezzini (1995, 1998) and Palumbo and Trezzini (2003) aptly point out, this attempt contradicts the Keynesian principle of effective demand by stipulating that the capital stock be utilised always at the normal level (even if it does not follow the path of steady-state growth).

(5) $z_{t+1} = 1 + \alpha[(K_t/\bar{K}_t) - 1]$. This can be considered a variation over the Harrodian case, surprisingly with the opposite result. It is different from Harrod's case by taking the warranted rate itself as the baseline, with the adjustment continuing to be made with no reference to the current rate. There is a unique stationary point, which is globally stable, where the pre-determined warranted rate prevails.

[13] The recurrence relation converges to a value between δ and $\bar{z}G^*$ (exclusively) if $G^* > \delta$, and equals G^* if $G^* = \delta$.

[14] As is witnessed by the debates between the 'Kaleckians' and the 'Sraffians' which started in the 1980s and continue to date (e.g. the papers in the recent issue of *Metroeconomica*, 2012), this is the main point of criticism targeted to the Kaleckians by the other side of the debates.

intends to convey as a heuristic tool, is that during most of the time the economy is being operated at a degree of utilisation that is different from the normal level, and this is the direct implication of the principle of effective demand. Over a multiple number of periods, this makes significant difference in the configuration of productive capacity of the economy and thus the capacity to employ labour. At the same time, as the relation also serves to convey, it cannot be denied that entrepreneurs continually make attempts to achieve full adjustment between the capital stock and output. There is a tendency in the real economy for the adjustment, if only approximate, between the configuration of the capital equipment and that of output so that the degree of utilisation of the productive equipment actually installed tends to approach the normal level. But this adjustment, even if fully achieved, should be taken to be the result of '[t]he elasticity [of] the capitalist economy … in reacting to incentives for a more rapid growth by bringing about additional productive capacity, or, symmetrically, by … erasing the visible traces of the losses in output due to a low such incentive' (Garegnani, 1992, p. 53). The tendency of the adjustment, whether full or only approximate, between the capital equipment and output should not be taken as an evidence of the latter following the former (supply-led growth). The truth is the opposite: the adjustment is a decisive evidence of the productive capacity being determined in relation to output (demand-led growth).

The recurrence relation above thus incorporates two points: the non-necessity of the normal utilisation of productive equipment in the short period and the tendency of productive equipment to adjust (if not fully) to output in the long period. The relation demonstrates the latter point, in particular, on the basis of the idea that there is a configuration of the capital equipment in reference to which the current actual configuration of the capital equipment is judged and that the investment behaviour in the following periods is adjusted in the light of that judgement. This 'reference point' configuration of the capital equipment is an essential part of the LPP. We take the 'long-period position' not as some state of the economy that will be reached in a distant future, usually as a stable, steady-growth state, but as a state of the economy that 'exists in the present' and, being considered 'normal' for the currently given state of effective demand, directs the movement of the economy over time. The long-period analysis the present paper suggests is an endeavour that analyses primarily how such 'present-existing' LPPs are configured corresponding to the current state of effective demand *period by period*. The evolution of LPPs over time may be conceived by way of some formalised relation that intends to describe that evolution, but this is, we suggest, not absolutely necessary for the long-period analysis; there may be many possible, even plausible, alternative formalisations; or, as some insist, any attempt at such formalisations may be misleading. Whatever the stance, however, the crucial point should be that that evolution shadows the evolution of effective demand whilst at the same time guiding it.

Equipped with these main ideas and concepts, we may now move on to a presentation of a multi-sector framework, where it turns out that some simplistic (quantitative) conclusions of the aggregate economy are replaced with far more complex and interesting ones.

3. A multi-sector framework

Consider an economy consisting of n single-product industries (hence, all produced means of production are circulating capital).[15] The dominant technique in use in the economy is represented by a semi-positive $n\times n$ material input coefficients matrix ($\mathbf{A}$) and a positive $n\times 1$ labour coefficients vector ($\mathbf{l}$). A material input coefficient a_{ij} and a labour input coefficient l_i stand, respectively, for the quantity of the jth means of production and the quantity of labour required for the production of one unit of the ith output *at the normal utilisation*. Following Sraffa (1960, p. v), we resort to the 'temporary working hypothesis' of constant returns to scale.[16]

A. Prices of production and a fully-adjusted position

Prices of production, which Sraffa considers in his price system, are the exchange ratios of produced commodities that 'if adopted in the market restores the original distribution of products and makes it possible for the process to be repeated' (Sraffa, 1960, p. 3). Consider a stationary economy. Prices of production are the exchange ratios that will reproduce the *same* configurations (both the scales and the compositions) of output and the means of production that existed when the produced commodities started to be exchanged after the 'harvest'. If the same physical configurations in question are reproduced, in turn, the same prices of

[15] Differently from our discussion in terms of the aggregate economy, the present paper chooses, for the multi-sector framework, the case of single-product industries only. This is because the existence of fixed capital requires some additional analysis and this will bring about more-than-usual complexity in the joint-production framework. How intensively a machine is utilised over a 'year' may affect its *physical* efficiency, and the effect, if any, will reveal themselves in the length of *physical* lifetime (measured in the number of 'years') over which the machine can be in operation. In the joint-production framework, this means that the number of productive processes physically associated with the machine will vary with how intensively it is utilised. A fully-adjusted position is free of this problem, for in this position the machine shall be utilised, by definition, at the normal level. However, the problem will be in full force in the 'realised' economy (see Subsection E of the present section and footnote 26 there). This problem is separate from the well-known problem of 'truncating' fixed capital, that is, of determining its *economic* lifetime (the conventional discussion of truncation assumes the normal utilisation of fixed capital). (Section 2 touched upon the effect in question in the case of a fixed capital with 'radioactive' depreciation; see footnote 10 and its related argument in the main text.) With the means of production consisting solely of circulating capital, by contrast, one shall be faced with the problem of interpreting the 'degree of utilisation' of the stock of circulating capital; see below.

[16] The assumption is solely for the expediency of argument, the immediate objective of our argument being to present an analytical framework where Sraffa's way of determining prices and Keynes's way of determining the quantities of output meet together. Variable returns to scale, understood in the way of Sraffa (1925,1926), can be accommodated in our framework without much problem if in a more complicated way; see Subsection D below. (By contrast, variable returns understood in the Marshallian way would cause problems; see footnote 25.)

production will be established in the market in the next 'year', which will in turn reproduce the same physical configurations, and so on each 'year'. In this process, the two conditions for a 'fully-adjusted position' seem absolutely necessary. The means of production should be utilised at the level that producers consider 'normal'; if considered not 'normal', the existing relationship between the configuration of output and that of the means of production shall not be 'repeated'. It should also be the case that the produced quantity of each type of output is neither more nor less than the quantity that is required for use either as the means of production or for consumption in the economy as a whole; otherwise, again, either the configuration of output or that of the means of production or both shall not be considered appropriate for being 'repeated'. For a growing economy, let the 'configurations' of output and of the means of production which are to be repeated mean the relations between them which are defined by the dominant technique; then, we can repeat the same argument as above.

Let us denote by x_i^* the quantity of the *i*th output that satisfies the two conditions of a fully-adjusted position. With free competition prevailing, then, the following relations hold:

$$(1+r)[x_i^*]\mathbf{Ap} + w[x_i^*]\mathbf{l} = [x_i^*]\mathbf{p}$$

where $[x_i^*]$ stands for the diagonal matrix generated from x_i^*'s, **p** for the $n\times 1$ vector of prices, r for the uniform rate of profits obtainable at the normal degree of utilisation, and w for the real wage rate. This set of equations represents the value relationships which will be established when the first condition of a fully-adjusted position is satisfied (in free competition); that is, when, in the respective industries, the ratios between the quantities of the means of production ($[x_i^*]\mathbf{A}$) and the quantities of labour input ($[x_i^*]\mathbf{l}$) on the one hand and the quantities of output ($[x_i^*]$) on the other bear those specified by the dominant technique in use.

Obviously the above equations are reduced to

$$(1+r)\mathbf{Ap} + w\mathbf{l} = \mathbf{p} \tag{1a}$$

Insofar as the quantities of output are those ensuing from the normal utilisation of the capital equipment, prices of production are determined in reference to the input coefficients defining the dominant technique in use. Normalisation of prices is expressed by the following condition:

$$\mathbf{dp} = 1 \tag{1b}$$

where **d** is a given semi-positive $1\times n$ vector of the unit basket of consumption goods. Equations (1a) and (1b) together determine **p** and w (or r) for a given value of r (or w).

For the relations (1a) to be able to repeat itself over the cycles of production, however, the quantities of output should in addition satisfy the second condition of a fully-adjusted position. This condition is represented by the 'quantity equations', and it is here that we can distinguish between the state of the economy which is not constrained by effective demand and the state of the opposite character, even if both are equally fully-adjusted positions.

B. The Warranted Growth state

At the beginning of period t, the ith industry is equipped with the means of production of such a configuration as $\bar{\mathbf{k}}_i \equiv (\bar{k}_{i1}, \bar{k}_{i2}, \cdots, \bar{k}_{in})$.[17] We assume that the proportions among these means of production are in accordance with the dominant method of production used in the industry. Then, if the level of output that the industry can produce by utilising $\bar{\mathbf{k}}_i$ *at the normal level* is denoted by $\bar{x}_i$, the following relation holds:

$$\bar{\mathbf{k}}_i = \bar{x}_i \mathbf{a}_i \tag{2a}$$

where $\mathbf{a}_i$ is the ith row of $\mathbf{A}$ (denoting the method of production adopted in the ith industry for one unit of output). The given method of production and the exiting configuration of capital equipment together determine $\bar{x}_i$.[18]

$\bar{x}_i$ is the quantity of output which will result from the normal utilisation of capital equipment. Thus the following price equations are established in the WG state:

$$(1+r)[\bar{x}_i]\mathbf{A}\bar{\mathbf{p}} + w[\bar{x}_i]\mathbf{l} = [\bar{x}_i]\bar{\mathbf{p}}$$

where $[\bar{x}_i]$ is the diagonal matrix generated from $\bar{x}_i$'s. With the assumption of constant returns to scale, these price equations are reduced to equations (1a); hence, $\bar{\mathbf{p}} = \mathbf{p}$. Together with the normalisation condition (1b), they determine the prices of commodities in the WG state.

The WG state of the economy requires not only that every industry utilise its existing capital equipment at the normal level but also that the level of output produced in each industry be exactly matching its total use. Let g_i^* denote the 'warranted rate of net accumulation' of the ith industry. With $[g_i^*]$ the diagonal matrix generated from the g_i^*'s, the two conditions for the WG state of the economy is expressed by the following 'quantity equations':

$$\bar{\mathbf{x}} = \bar{\mathbf{x}}\mathbf{A} + \bar{\mathbf{x}}[g_i^*]\mathbf{A} + h^*\mathbf{d} \tag{2b}$$

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[17] To avoid the cluttering of subscripts, we omit the subscript for time t in the following pages unless necessary.

[18] If the proportions among the existing means of production do not match the method of production, the normal level of output may be defined as $\bar{x}_i = \min[k_{i1}/a_{i1}, k_{i2}/a_{i2}, \cdots, k_{in}/a_{in}]$ instead of the way described in (2a). This more realistic case, however, complicates the definition of the degree of utilization of a given capital equipment.

where h^* is the 'level' of consumption (that is, the number of the unit consumption baskets) to be obtained in the WG state.[19] The system of equations (2b) has g_i^*'s and h^* as the unknowns. To determine them we need one more relation. The required relation is the macroeconomic equilibrium condition: the equality between aggregate investment and aggregate saving.[20] For the convenience of argument, we assume that workers do not save whilst capitalists save a fraction s_c of their income (profits).[21] Then one has the following Cambridge equation:

$$\bar{\mathbf{x}}[g_i^*]\mathbf{Ap} = s_c r \bar{\mathbf{x}}\mathbf{Ap} \tag{2c}$$

The means of production are valuated at $\bar{\mathbf{p}} = \mathbf{p}$. The relation (2c) implies that the economy-wide warranted rate of net accumulation ($\bar{\mathbf{x}}[g_i^*]\mathbf{Ap}/\mathbf{xAp}$) is $g^* = s_c r$; however, the industry warranted rates of accumulation may differ across the industries. (The level of aggregate employment is $\bar{\mathbf{x}}\mathbf{l}$; there is no guarantee that this level matches the total labour available in the economy.)

The system of equations consisting of (2a) to (2c), together with (1a) and (1b), describes the WG state of the economy. Given the technique in use, one of the distributive variables *and* the existing configuration of the means of production in each industry in a particular period *t*, one can obtain the warranted rates of accumulation of the respective industries and the level of consumption in that period. The volume of warranted gross investment of the *i*th industry is then

$$J_i^* \equiv (1+g_i^*)\bar{x}_i\mathbf{a}_i\mathbf{p} = G_i^*\overline{\mathbf{k}}_i\mathbf{p} \tag{3}$$

This volume of investment is *not* autonomous in the sense that it is dictated by the given parameters, not autonomously determined by entrepreneurs.

C. The long-period position

The long-period position (LPP) is a fully adjusted position generated under the constraint of effective demand. This means, as is the case with the WG state, that there is full adjustment between the quantities of output of the respective industries and the configuration of the

[19] Thus, it is being assumed that different income groups—wage earners and profit earners—consume the same unit bundle of commodities.

[20] For the aggregate economy discussed in Section 2, the quantity equation has already incorporated the saving behaviour; thus, in conjunction with the price equation, the equality between saving and investment is derived.

[21] In the next section, we use a different understanding of saving behaviour, in terms of households and corporations, which allows us to consider the working of the financial market more clearly.

means of production of the corresponding industries so that the capital equipment is utilised at the normal level in each of the industries; and also that the quantity of output of each commodity precisely matches its total use in the economy as a whole. The crucial difference is that the LPP is determined by a given state of effective demand.

If we denote by x_i the quantities of output that will ensue from an LPP and by $[x_i]$ the diagonal matrix generated from x_i's, then the value relations to be established in that position are expressed by the following price equations:

$$(1+r)[x_i]\mathbf{A}\tilde{\mathbf{p}} + w[x_i]\mathbf{l} = [x_i]\tilde{\mathbf{p}} \tag{4}$$

which is, again with the assumption of constant returns to scale, reduced to (1a); hence, with (1b), one gets $\tilde{\mathbf{p}} = \mathbf{p}$.

The quantity relations in the LPP are described by the following:

$$\mathbf{x} = \mathbf{x}\mathbf{A} + \mathbf{x}[\tilde{g}_i]\mathbf{A} + h\mathbf{d} \tag{5a}$$

where $\mathbf{x}$ is a $1 \times n$ vector of outputs, $[\tilde{g}_i]$ the diagonal matrix generated from the ratios $\tilde{g}_i$'s that actual net investments bear to the LPP capital stocks (both in value terms) in the respective industries, and h the level of consumption, all defined in reference to the LPP. The quantities of produced output $\mathbf{x}$ bear, to the means of production in the respective industries, the ratios (in physical terms) specified by the dominant technique (which means the normal utilisation of the capital equipment) and also precisely match the demand for them in the economy as a whole. The two conditions of a fully-adjusted position are satisfied.

The factor that determines the demand for commodities is, in our EDC economy, autonomously determined investment in each industry.[22] Investment, when made autonomously by entrepreneurs, does not necessarily equal the volume of the warranted investment. Such autonomy of investment must be in operation in each of the industries of the economy, because there is generally no reason why entrepreneurs operating in different industries should coordinate their investment behaviour or why entrepreneurs in some particular industries should take the lead whilst those in others should follow the suit. Thus, the autonomy of investment in a multi-sector framework is expressed by the actual gross investment J_i undertaken in the ith industry being a fraction (or a multiple) z_i of the warranted gross investment of the same industry:

$$J_i = z_i J_i^*$$

or, in a more explicit form,

[22] Of course, if we expand our analytical scope, the government expenditure and net exports also will come to compose the effective demand of the economy.

$$J_i \equiv (1 + \tilde{g}_i)x_i\mathbf{a}_i\mathbf{p} = z_i(1 + g_i^*)\bar{x}_i\mathbf{a}_i\mathbf{p} \qquad (5b)$$

The means of production in the LPP are valued at the prices of production $\tilde{\mathbf{p}} = \mathbf{p}$.

As is with the WG state again, the macroeconomic equilibrium requires that the volume of aggregate investment be equal to the volume of aggregate saving:

$$\mathbf{x}[\tilde{g}_i]\mathbf{Ap} = s_c r\mathbf{xAp} \qquad (5c)$$

What is described by the system of equations consisting of (5a) to (5c), together with (1a) and (1b), is the state of the economy where autonomous investment rules the roost; once investment is given autonomously (equations (5b)), such quantities of output are produced and such configurations of the capital equipment are installed in the respective industries that there is full adjustment between the two (so that the productive equipment in each industry is utilised at the normal level) and the quantities of output precisely satisfy the demand for them in the economy as a whole (equations (5a)); moreover, in the economy as a whole, investment generates the saving of the same volume (equation (5c)). The economy is in an EDC fully-adjusted position, that is, in an LPP. In this state of the economy, the ratios $\tilde{g}_i$'s need not be uniform across the industries and are generally different from the industry warranted rates of accumulation, whilst the economy-wide ratio ($\mathbf{x}[\tilde{g}_i]\mathbf{Ap}/\mathbf{xAp}$) is the same as that of the WG state ($s_c r$) as has been observed in Section 2. The aggregate level of labour employment, **xl**, need not equal the total amount of labour available in the economy.

The evolution of the economy over time is due primarily to changes in the actual gross investment. Here appears more of what has been hidden in the analysis conducted in Section 2 in terms of the aggregate economy. Gross investment in each industry in period *t* determines the configuration of the capital stock in that industry in period *t*+1. The rate of gross accumulation ($G_i \equiv J_i/\bar{x}_i\mathbf{a}_i\mathbf{p}$) being different across the industries, the relative sizes of the existing capital stock and hence the relative sizes of the capacity output, across the industries in period *t*+1, shall generally be different from those in period *t*. In period *t*+1, we begin with a different configuration of the existing capital equipment in the respective industries. This means the WG state in period *t*+1 will yield a different set of the warranted rates of accumulation from those in period *t*. The *industry* warranted rates of accumulation varies over time. (But, as far as the saving behaviour of the social classes and the real wage rate remain the same, the economy-wide warranted rate of accumulation does not change, the net rate continuing to be $g^* = s_c r$.)

Suppose that, in each industry, the ratio of the actual gross investment to the warranted gross investment evolves in the same way as we have considered in Section 2, but generally with different values of the 'state of animal spirits' ($\bar{z}_i$) and of adjustment coefficient (α_i) across the industries:

$$z_{i,t+1} = \bar{z}_i + \alpha_i[(K_{i,t}/\bar{K}_{i,t}) - 1]\, z_{i,t}, \text{ with } 0 < \alpha_i < 1$$

where $K_{i,t}(\equiv x_i\mathbf{a}_i\mathbf{p}$ in period *t*) is the size of the long-period position capital stock and $\bar{K}_{i,t}$

($\equiv \bar{x}_i \mathbf{a}_i \mathbf{p}$ in period *t*) is the size of the existing capital stock; thus, $K_{i,t}/\bar{K}_{i,t} = x_{i,t}/\bar{x}_{i,t} = u_{i,t}$.[23] The specification implies that the process of accumulation in an individual industry affects, and also is affected by, the process of accumulation in the remaining industries of the economy; for $x_{i,t}$ and therefore $K_{i,t}$ are determined, in principle, in reference to all the industries of the economy. Translating the recurrence relation in terms of the growth rates, one has

$$G_{i,t+1} = z_{i,t} G^*_{i,t} + \alpha_i [(G_{i,t}/\tilde{G}_{i,t}) - 1]\, G_{i,t}$$

The difference from the Section 2 case is the appearance of not only $G^*_{i,t}$ (the time-varying warranted rate of accumulation) but also $\tilde{G}_{i,t} (\equiv J_{i,t}/K_{i,t})$ in the place of $G^*_{i,t}$. Both $G^*_{i,t}$ and $\tilde{G}_{i,t}$, as much as $G_{i,t}$, are affected by actual investments in all the industries in the previous period (whilst, in Section 2, $G^* = \delta + s_c r^*$ is not responsive to actual investment). Thus, the evolution of the capital equipment (hence, output) in the individual industries is much more complex than that in the economy as a whole.[24] An implication of this is that, unless the relative sizes of the industries at the beginning of our analysis are already compatible with uniform growth across the industries, it would be extremely improbable that the economy should reach such a state subsequently (and this is another reason why the long-period analysis should distance itself away from steady-state growth, which requires uniform growth in the multi-sector economy).

D. Sraffa's 'given' quantities of output

It must have become clear by now that the LPP is, we suggest, where Sraffa's Classical approach to the determination of production prices and Keynes's principle of effective demand reside harmoniously. On the one hand, prices *à la* Sraffa are determined in reference to the quantities of output resulting from the normal utilisation of the capital equipment. On the other, foremost in reference to autonomously given investments in the respective industries of the economy, one can find the quantities of output and the configuration of capital equipment which satisfy the conditions of a fully-adjusted position; in this process, prices determined *à la* Sraffa plays a role (in valuing the volume of industry investments). The quantities of output determined in the LPP—x_i's determined in the system of equations

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[23] In the case of the aggregate economy, $u_t = z_t$; however, in the multi-sector framework, $u_i = z_i(1 + g^*_i)/(1 + \tilde{g}_i)$ as is easily derived from equation (4b). (In the aggregate economy, $g^* = \tilde{g}$.)

[24] As the rates of output growth vary across the individual industries, some of which may be producing pure consumption goods, the composition of the consumption basket (**d**) may well change over time. But the long period analysis we suggest will accommodate this case easily: if the consumption basket changes its composition, that change is revealed in a particular period and the systems of equations—the price, the WG quantity and the EDC quantity equations—for that period change accordingly.

consisting of (5a) to (5c), together with (1a) and (1b)—stand, we suggest, as the 'given' quantities of output in Sraffa's original system. In other words, the system of equations (4), reproduced below,

$$(1+r)[x_i]\mathbf{Ap} + w[x_i]\mathbf{l} = [x_i]\mathbf{p} \tag{4}$$

is the production system—a Keynesian instance, so to speak—that Sraffa presents in his book (Sraffa, 1960), where the quantities of output appear as 'given'.

Our argument up to now has proceeded on the assumption of constant returns to scale. Essentially the same way of analysis can apply to the cases of variable returns to scale, if these are understood in Sraffa's (1925, 1926) way.[25] Increasing returns to scale, understood as a general characteristic of technical progress ('division of labour'), will define the set of technical coefficients for each stage of technical progress (after the choice of technique has taken place in each stage), reflecting not only changes in the values of technical coefficients of the existing industries but also, if any, the emergence of new industries and the disappearance of some of old industries.[26] *Once* given the set of technical coefficients, which by definition refer to the normal utilisation of the capital equipment in the respective (old and new) industries, the analysis of the WG state and the LPP (and the 'realised' state of the economy, to be discussed in the next subsection) can proceed in the same way as we do for the case of constant returns. Diminishing returns are understood to come from the use of 'land' (standing for a factor of production whose quantity, either in its entirety or in the lots of particular qualities, is constant) and are dealt with by Sraffa in the joint production framework. There will be several 'productive processes' which use land (along with material inputs and labour) to produce a given type of output. Whether these productive processes represents the use of land of different qualities (the case of extensive rent) or the use of different methods of production with the land of the same quality (the case of intensive rent), we shall have the knowledge of the technical coefficients of the respective productive processes, which refer to the normal utilisation of the capital equipment in those processes.

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[25] The Marshallian understanding of returns to scale is that they (whether increasing, diminishing or constant) are a matter of production technique where the values of technical coefficients are mechanically related to the levels of output in the respective industries. Thus, according to this understanding, some or all elements of $\mathbf{A}$ and $\mathbf{l}$ are (decreasing, increasing or constant, respectively) functions of the quantities of output of the respective industries. If we followed this understanding, our framework should probably be faced with the problem of multiple solutions in the case of variable returns to scale (by contrast, Sraffa's way is free of such problem).

[26] As was mentioned for the aggregate economy (see footnote 10), our framework allows for an industry to shrink its size over the cycles of production: gross investment, that is, effective demand for its product, is not sufficient even to cover the depreciation of the existing capital equipment. Its size falling below a certain threshold, the industry shall not survive in the economy.

Then, analysis can go ahead, again, as with the case of constant returns: investment (warranted and autonomous, respectively) is considered for the respective productive processes; the WG state and the LPP are located as fully-adjusted positions corresponding to these investments.

E. The 'realised' state of the economy

The determination of prices in reference to a fully-adjusted position does not necessarily mean that the configurations of output and of the means of production and the accompanying industry rates of profits and depreciation rates of the capital equipment will appear to the eye of the statistical observer as the same as those of that position. For one thing, the configuration of the LPP capital equipment is not necessarily the same as that of the existing capital equipment: $x_i\mathbf{a}_i \neq \bar{\mathbf{k}}_i$. The situation where this equality necessarily holds is the WG state of the economy, whilst the non-necessity of this equality is the evidence of the autonomy of investment. The state of the economy to appear as the 'realised' one when the gross investment is $J_i = z_i J_i^*$ in the *i*th industry will be described by the following system of equations:[27]

$$J_i = (z_i + g_i)\bar{\mathbf{k}}_i\mathbf{p} \tag{6a}$$

$$x_i = \sum_{j=1}^{n}(z_j + g_j)k_{ji} + hd_i \tag{6b}$$

$$(z_i + \bar{r}_i)\bar{\mathbf{k}}_i\mathbf{p} + wx_il_i = x_ip_i \tag{6c}$$

$$\sum g_i\,\bar{\mathbf{k}}_i\mathbf{p} = \sum s_c\bar{r}_i\,\bar{\mathbf{k}}_i\mathbf{p} \tag{6d}$$

At the beginning of a period the existing capital equipment is observed to be $\bar{\mathbf{k}}_i$ and it is known that the actual gross investment is made of the volume of J_i which bears the ratio z_i to the warranted investment. If the LLP corresponding to these givens is established, it is 'observed' that the existing capital equipment depreciates at the rate of z_i and the rate of net accumulation over that capital equipment is $g_i(= z_ig_i^*)$. Output is produced at the level of x_i, which is different from the capacity level of output expected from the existing capital equipment (however, it is exhausted as gross investment and consumption in the economy as a whole: k_{ji} is the existing *i*th means of production used in the *j*th industry and d_i the

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[27] Here we are assuming the equivalent of 'linear' depreciation of fixed capital with respect to the degree of utilisation adopted in Section 2 (see footnote 10 and its related argument in the main text). Only that part of the existing stock of circulating capital which has been used up is counted as part of the 'costs'; the un-utilised part, if any, is not affected at all in its efficiency and can be used as if newly produced, in the next cycle of production; that is, there are no 'carrying costs'. Positive 'carrying costs' would obviously affect the realised depreciation rate and thus the realised rate of net profits and the realised rate of net accumulation. It is also assumed that the quantity of labour input changes in exact proportion with the quantity of output produced, thus always bearing the technical labour input coefficient to the latter in each industry.

quantity of the ith output in the unit consumption basket). The rate of net profits realised on the existing capital equipment is observed to be $\bar{r}_i(= z_i r)$ and labour employment is $x_i l_i$, the wage bill being $w x_i l_i$. In the economy as a whole, the volume of net investment is equal to the volume of net saving.[28]

The 'realised' state of the economy is *not* a fully-adjusted position. Still, in this state, the prices to be 'observed' are the prices of production, which are determined by the price system consisting of equations (1a) and (1b). This is because, given the current state of effective demand, producers take the 'normal' output and the 'normal' configuration of the capital equipment corresponding to that state of effective demand as the 'reference point' when they set the prices.[29] The long period position 'exists in the present'.

4. The financial market

Up to now the saving behaviour has been contemplated in terms of the social classes, with workers saving nothing. The macroeconomic equilibrium condition, (2c) or (4c), relates the aggregate (net) investment to the aggregate (net) saving. Equation (2c) represents 'Say's Law': it is (aggregate) saving that generates (aggregate) investment of the same amount so that the capital equipment is utilised at the normal level. In contrast, equation (4c) incorporates the principle of effective demand: it is (aggregate) investment that generates (aggregate) saving of the same amount.

Now, in both cases, the equality of saving and investment must be observed also in each industry. But, in both cases, the volume of saving *generated in an individual industry* can fall short of or exceed the volume of investment undertaken in that industry. In the case of the WG economy, the gross capacity saving generated in the ith industry is $(1 + s_c r)\bar{\mathbf{k}}_i\mathbf{p}$ whilst the gross investment is $(1 + g_i^*)\bar{\mathbf{k}}_i\mathbf{p}$. In the EDC economy, the gross investment is undertaken of the volume of $(z_i + z_i g_i^*)\bar{\mathbf{k}}_i\mathbf{p}$ whilst the gross saving is generated of the volume of $(z_i + s_c z_i r)\bar{\mathbf{k}}_i\mathbf{p}$. Thus, in both cases, investment exceeds or falls short of saving according as g_i^* is larger or smaller than $s_c r$ (which is the economy-wide, thus average, warranted rate of accumulation). In an industry where saving exceeds investment, the excess

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[28] The appearance of the configuration of the existing capital stock ($\bar{\mathbf{k}}_i$), the rates of depreciation different from unity (z_i) and differential rates of profits ($\bar{r}_i$) should not lead to the wrong idea that the system of equations in question describes a *short-period* state of the economy. The relations, (6b) and (6c) in particular, hold only when the quantities of output and prices are those which are established in the LLP corresponding to gross investments J_i. Nor is it the system for *determining* an LPP. What it does is nothing but transcribing what would appear on surface, against the backdrop of the existing configuration of the capital equipment, when the economy is in the LLP corresponding to the given gross investments.

[29] Thus, our interpretation of the determination of prices in the way of the Sraffa system in the 'observed' economy seems concordant with the empirical studies on 'full-cost pricing' where the unit costs on which a mark-up is to be added are 'historical costs', that is actually incurred costs (for example, the work by Hall and Hitch).

saving flows out to some of the 'saving-deficient' industries; in an industry where saving falls short, the additionally required saving of the volume flows in from some of the 'saving-excessive' industries. In the economy as a whole these outflows and inflows cancel each other. To consider this inter-industry movement of saving, one need consider the space where they take place, that is, the financial market.

The formulation of saving behaviour in terms of the social classes, however, does not provide an interesting analysis of the working of the financial market.[30] The formulation that does so is Kaldor's (1966) one where the saving units are households and firms. The following pages of the present section modify the arguments of Section 3 in the light of this different formulation of saving behaviour.

Corporations retain, on average (regardless of the industry where they operate), a fraction s_f $(0 \le s_f \le 1)$ of their profits for net investment. In the WG economy, then, aggregate retained profits in the *i*th industry are $s_f r \bar{\mathbf{k}}_i \mathbf{p}$. For the portion of net investment that requires more than retained profits, corporations resort to the issue of new shares. The value of aggregate net investment that is financed through the new issue of shares is, following Kaldor, represented by $\phi_i^* g_i^* \bar{\mathbf{k}}_i \mathbf{p}$ with $|\phi_i^*| < 1$. Corporations' plans of net investment and their financing in the *i*th industry are then represented by the following equation:

$$g_i^* \bar{\mathbf{k}}_i \mathbf{p} = s_f r \bar{\mathbf{k}}_i \mathbf{p} + \phi_i^* g_i^* \bar{\mathbf{k}}_i \mathbf{p}$$

which is simplified to

$$g_i^* = s_f r + \phi_i^* g_i^* \tag{7}$$

In the economy as a whole, the supply of shares by corporations is $\sum_{i=1}^{n} \phi_i^* g_i^* \bar{\mathbf{k}}_i \mathbf{p}$.

Households purchase corporate shares out of their saving. Whilst corporations in an industry save with the specific purpose of investing in themselves, households save without having in mind any specific industries to purchase the shares of (see Keynes, 1936, ch. 12). Household saving is not industry-specific, and it means that household saving is to be

[30] We have adopted the formulation of saving behaviour in question because it, more familiar and simpler, facilitates the argument to a great extent. The working of the financial market in this case would be described as follows (for the EDC economy). The volume of gross saving having eventually found the place in the *i*th industry can be expressed as $(1 + s_c r)\, x_i \mathbf{a}_i \mathbf{p} + \theta_i x_i \mathbf{a}_i \mathbf{p}$, which is equal to J_i. Here $\theta_i x_i \mathbf{a}_i \mathbf{p}$ represents the inter-industry flows of saving: $\theta_i > 0$ for a net *inflow* of saving into the *i*th industry beyond the volume of saving that is generated thanks to the output in that industry; $\theta_i < 0$ for a net *outflow* of saving out of the *i*th industry so that the volume of saving that will be used in supporting the investment in the industry is smaller than the volume of saving that has been generated in association with the output in the industry. The aggregate result of these inter-industry flows of saving is, of course, *nil*: hence, we must have $\sum \theta_i x_i \mathbf{a}_i \mathbf{p} = 0$.

considered, at least initially, at the level of the economy as a whole. It is through the working of the share market that aggregate household saving is allocated to the respective industries: household saving *flows into* each of the industries in the form of the demand for shares supplied by the respective industries; $\phi_i^* g_i^* \bar{\mathbf{k}}_i \mathbf{p}$ is precisely the description of how household saving is *allocated* to the respective industries, the working of the financial market.

Households earn income from two sources: labour (from which they earn wages) and the holding of shares (which returns dividends and capital gain). We follow Kaldor's formulation of the behaviour of household saving: net saving out of wages is represented by a fraction s_w $(0 \le s_w \le 1)$ of their wages and net consumption out of the share holding by a fraction of c of capital gain (Γ). Thus, the total net saving of households is $S_h = s_w w \bar{\mathbf{x}} \mathbf{l} - c\Gamma$. Capital gain is calculated as $\Gamma = \sum_{i=1}^{n}(1-\phi_i^*) g_i^* \bar{\mathbf{k}}_i \mathbf{p}$, where the valuation ratio (the ratio between the share market's valuation of the corporations in the economy and the replacement costs of their means of production) is set to unity.[31]

Equilibrium in the share market requires equality between the total supply of and the total demand for new shares:

$$S_h \equiv s_w w \bar{\mathbf{x}} \mathbf{l} - c \sum_{i=1}^{n}(1-\phi_i^*) g_i^* \bar{\mathbf{k}}_i \mathbf{p} = \sum_{i=1}^{n} \phi_i^* g_i^* \bar{\mathbf{k}}_i \mathbf{p} \tag{8}$$

Now, equations (7) and (8) together replace equation (2c) for the WG state of the economy. The macroeconomic equality between the aggregate investment and the aggregate saving (which consists of the corporate saving and the household saving) is implied by the two new equations. One can now additionally determine ϕ_i^*'s.

As far as the working of the financial market is concerned, the EDC state is not dissimilar from the WG state. Equations (7) and (8) are replaced by the following two, with ϕ_i as the EDC counterpart of ϕ_i^*:

$$\tilde{g}_i x_i \mathbf{a}_i \mathbf{p} = s_f r x_i \mathbf{a}_i \mathbf{p} + \phi_i \tilde{g}_i x_i \mathbf{a}_i \mathbf{p}$$

which boils down to

$$\tilde{g}_i = s_f r + \phi_i \tilde{g}_i \tag{9}$$

and

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[31] In the aftermath of the Kaldor paper, there have been debates on whether the valuation ratio in the long-period equilibrium can deviate from unity. Kaldor's stance was that it can. But we take up the position that the share market's valuation of corporations at a value other than their replacement costs prompts (or forces) corporations to adjust their strategy of share issuance (probably because of the impacts that the share market's valuation exerts on the demand for shares); thus, in equilibrium, the valuation ratio must be unity.

$$s_w w\mathbf{xl} - c\sum_{i=1}^{n}(1-\phi_i)\tilde{g}_i x_i \mathbf{a}_i \mathbf{p} = \sum_{i=1}^{n}\phi_i \tilde{g}_i\, x_i \mathbf{a}_i \mathbf{p} \qquad (10)$$

Equations (9) and (10) are the replacement of equation (4c) as the condition for the aggregate equality of investment and saving in the EDC state of the economy.[32] These equations, in conjunction with equations (4a) and (4b), additionally determine ϕ_i's, which represent the working of the financial market.

There is a noticeable feature arising from this formulation of saving behaviour, in comparison with that assumed in Section 3. Previously the ratio between the aggregate investment and the aggregate capital stock was independent of investment decisions in the respective industries, as is seen in equation (4c). This is no longer the case: the ratio in question in the current case is obtained from equations (9) and (10) as

$$\frac{\sum \tilde{g}_i x_i \mathbf{a}_i \mathbf{p}}{\sum x_i \mathbf{a}_i \mathbf{p}} = (1-c)s_f r + s_w w\left(\frac{\sum x_i\, l_i}{\sum x_i \mathbf{a}_i \mathbf{p}}\right)$$

The aggregate labour-capital ratio on the right-hand side depends on the industry quantities of output, which are in turn determined in reference to investment decisions across the industries and *also* to the working of the financial market.

The arguments in the coming two sections (and in the appendix) are based on this mechanism of the financial market.

5. Constraints

The long-period analysis of the EDC state, as has been suggested in the previous two sections, requires consideration of some more constraints beside the constraint of effective demand. These constraints can eventually be expressed as those to be imposed on the 'behavioural variable' of our main concern: z_i's.

First of all, recall that the volume of economy-wide gross investment for the WG state is that which will maintain the normal utilisation of productive capacity in the economy as a whole (and also in each industry of the economy). We take this volume of gross investment as the baseline with which to compare the economy-wide volume of gross investment made in the EDC state. Thus, it will be the case in general that unless there is a spur from outside (that is, outside of the private sector, such as the government or the foreign sector), the total volume of investment undertaken in the EDC state does not exceed that in the WG state:

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[32] Alternatively, one can consider the case in terms of g_i (the rate of accumulation on the existing capital equipment): under the assumption of 'linear' depreciation with respect to the degree of utilisation, $g_i\mathbf{k}_i\mathbf{p} = s_f(z_i r)\mathbf{k}_i\mathbf{p} + \phi_i g_i \mathbf{k}_i \mathbf{p}$ which is reduced to $g_i = s_f(z_i r) + \phi_i g_i$; and $s_w w\mathbf{xl} - c\sum_{i=1}^{n}(1-\phi_i)g_i\mathbf{k}_i\mathbf{p} = \sum_{i=1}^{n}\phi_i g_i\, \mathbf{k}_i\mathbf{p}$.

$$\sum_{i=1}^{n} J_i \le \sum_{i=1}^{n} J_i{}^*$$

or, using (3),

$$\sum_{i=1}^{n} z_i J_i{}^* \le \sum_{i=1}^{n} J_i{}^* \tag{11}$$

This constraint will be called the *constraint of normal utilisation* (NU). If constraint (11) holds in strict inequality, this means that the state of economy-wide effective demand is insufficient for the normal utilisation of productive capacity in the respective industries.

Whether constraint (11) is in operation either as equality or as strict inequality, it may be the case that the volume of investment in some individual industries can be larger than that in the WG state. This is because a greater volume of investment shall make its way into those industries in which 'animal spirits' are higher. Industries in the WG state utilise their productive capacity at the normal level and this is represented by $z_i = 1$. If autonomous investment in an individual industry is such that $z_i > 1$, this means that productive capacity in that industry will be stretched to be utilised *above* the normal level. This 'over'-utilisation is possible, if only up to a certain limit; as is well documented in the literature (the representative of which is Steindl, 1952), for various—technical or economic—reasons, the decision on the normal degree of utilisation of productive capacity is made in allowance of some margin for 'over'-utilisation. If we denote that limit of 'over'-utilisation by z_i^{max}, then z_i must observe the following condition:

$$0 < z_i \le z_i^{max}, \text{ with } z_i^{max} > 1 \tag{12}$$

We may call this the *constraint of full utilisation* (FU), using the expression 'full' as meaning the physical limit of utilisation of productive capacity. This constraint is specified for each industry; however, combined with the constraint of NU above, it must be the case that $1 < z_i \le z_i^{max}$ for some industries implies $0 < z_j < 1$ for some other industries.

We may interpret the full employment of labour in its strict sense as the state in which there is no further labour force available. This yields the *constraint of full employment of labour* (FE): with $\bar{L}$ denoting the total available labour force

$$\sum_{i=1}^{n} x_i l_i \le \bar{L}$$

From equations (4b), (7) and (9), one obtains

$$\frac{x_i}{\bar{x}_i} = \frac{(1+s_f r-\phi_i^*)(1-\phi_i)}{(1+s_f r-\phi_i)(1-\phi_i^*)} z_i \equiv \Phi_i z_i$$

This result lets us express the constraint of the full employment of labour as follows:

$$\sum_{i=1}^{n} z_i \Phi_i \bar{x}_i l_i \leq \bar{L} \tag{13}$$

Of course, one can adopt a less strict interpretation of full employment, such that an increase in the participation rate, accompanying an economy-wide increase in investment, allows for the total amount of labour employed in the economy to be larger than $\bar{L}$ (up to a certain limit). But our purpose is sufficiently served by the strict interpretation.

Sraffa (1960, p. 5, fn. 1) explicitly states that his 'formulation [for the determination of prices] presupposes the system's being in a self-replacing state': for each commodity, the aggregate quantity produced in the economy should not be less than the aggregate quantity used up as the means of production across the industries:

$$x_i - \sum_{j=1}^{n} x_i a_{ji} \geq 0$$

or using the definition of Φ_i above

$$\Phi_i z_i \bar{x}_i - \sum_{j=1}^{n} \Phi_j z_j \bar{x}_j a_{ji} \geq 0 \tag{14}$$

where a_{ji} is the *ji*-th element of **A**. This is the *constraint of self-replacing state* (SR). If constraint (14) holds as equality for all *i*, then the economy is one without any surplus; if strict inequality holds for at least one industry, then the economy is a surplus economy.

In equilibrium, ϕ_i's are determined in reference to z_i's and the 'fundamental parameters' (the technology, the real wage rate and the saving or consumption propensities): $\phi_i = \phi_i(z_1, z_2, \cdots, z_n; \mathbf{A}, \mathbf{l}, w, s_c, c)$. Now, it must be the case that $|\phi_i| < 1$ for all *i*. This implies that some constraint is to be imposed on the combinations of z_i's that will keep the equilibrium values of ϕ_i within the permitted range:

$$(z_1, z_2, \cdots, z_n) \in \{(z_1, z_2, \cdots, z_n): -1 < \phi_i < 1, \forall i\} \tag{15}$$

This constraint shall be christened the *constraint of financial market* (FM).

Those combinations of z_i's which satisfy all the constraints of NU, FU, FE, SR and FM will define the set of possible long-period positions in the EDC state. A particular combination of z_i's among them will be called a '*point of effective demand*' (PED). A long-period position is a PED that satisfies the share market equilibrium (10). Expressed in terms of z_i, condition (10) is transformed into

$$s_w w \sum z_i \Phi_i \bar{x}_i l_i - c s_f r \sum \Phi_i z_i \bar{x}_i \mathbf{a}_i \mathbf{p} = s_f r \sum z_i \left(\frac{\phi_i}{1-\phi_i}\right) \Phi_i \bar{x}_i \mathbf{a}_i \mathbf{p} \qquad (16)$$

Thus we shall call equation (16), with all the endogenous variables having been determined, the *equation of long-period positions* (LP).

6. An illustration

Consider an economy which consists of two industries, one producing the means of production ('machine', commodity 1) and the other producing the consumption good ('corn', commodity 2); the machine is a circulating capital so that it is used up in a unit production period. Thus, the technique in use is represented by the following:

$$\mathbf{A} = \begin{pmatrix} a_{11} & 0 \\ a_{21} & 0 \end{pmatrix}, \quad 0 < a_{i1} < 1;$$

$$\mathbf{l} = \begin{pmatrix} l_1 \\ l_2 \end{pmatrix}, \quad l_i > 0$$

The unit basket of consumption is $\mathbf{d} = (0 \quad 1)$; thus, $p_2 = 1$. For a given level of w, the system of (1a) and (1b) determines r and p_1. In accordance with the saving behaviours of corporations and households and with the condition that the valuation ratio is unity in all the industries, the system consisting of equations (2a), (2b), (7) and (8) for i =1, 2, with $\mathbf{A}$ and $\mathbf{l}$ as above determines the WG levels of output and the normal rates of accumulation: $(\bar{x}_1, \bar{x}_2)$ and (g_1^*, g_2^*). The volume of warranted investment in an individual industry is $J_i^* = (1 + g_i^*)\bar{x}_i \mathbf{a}_i \mathbf{p}$.

The EDC state will observe relationships (4a), (4b), (9) and (10). The autonomous volume of gross investment in an individual industry bears a ratio z_i to J_i^*. Then, the five constraints that z_i's must satisfy are as follows:

(i) normal utilisation (NU):

$$z_2 \leq \left(1 + \frac{\bar{x}_1 \mathbf{a}_1 \mathbf{p}}{\bar{x}_2 \mathbf{a}_2 \mathbf{p}}\right) - z_1 \left(\frac{\bar{x}_1}{\bar{x}_2}\right)\left(\frac{\mathbf{a}_1 \mathbf{p}}{\mathbf{a}_2 \mathbf{p}}\right) \qquad (11')$$

(ii) full utilisation (FU):

$$0 < z_i \leq z_i^{max}, \quad i = 1, 2 \qquad (12')$$

(iii) full employment (FE):

$$z_2 \leq \left(\frac{\bar{L}}{\bar{x}_2 l_2 \Phi_2}\right) - z_1 \left(\frac{\bar{x}_1}{\bar{x}_2}\right)\left(\frac{\Phi_1}{\Phi_2}\right)\left(\frac{l_1}{l_2}\right) \qquad (13')$$

(iv) self-replacing state (SR):

$$z_2 \le z_1 \left(\frac{\overline{x}_1}{\overline{x}_2}\right)\left(\frac{\Phi_1}{\Phi_2}\right)\left(\frac{1-a_{11}}{a_{21}}\right) \tag{14'}$$

(v) financial market (FM)

$$z_2 = \left(\frac{\overline{x}_1}{\overline{x}_2}\right)\left(\frac{\Phi_1}{\Phi_2}\right)\left(-\frac{s_w w l_1 - s_f r(c+\frac{\phi_1}{1-\phi_1})\mathbf{a}_1\mathbf{p}}{s_w w l_2 - s_f r(c+\frac{\phi_2}{1-\phi_2})\mathbf{a}_2\mathbf{p}}\right) z_1 \equiv \zeta z_1 \tag{16'}$$

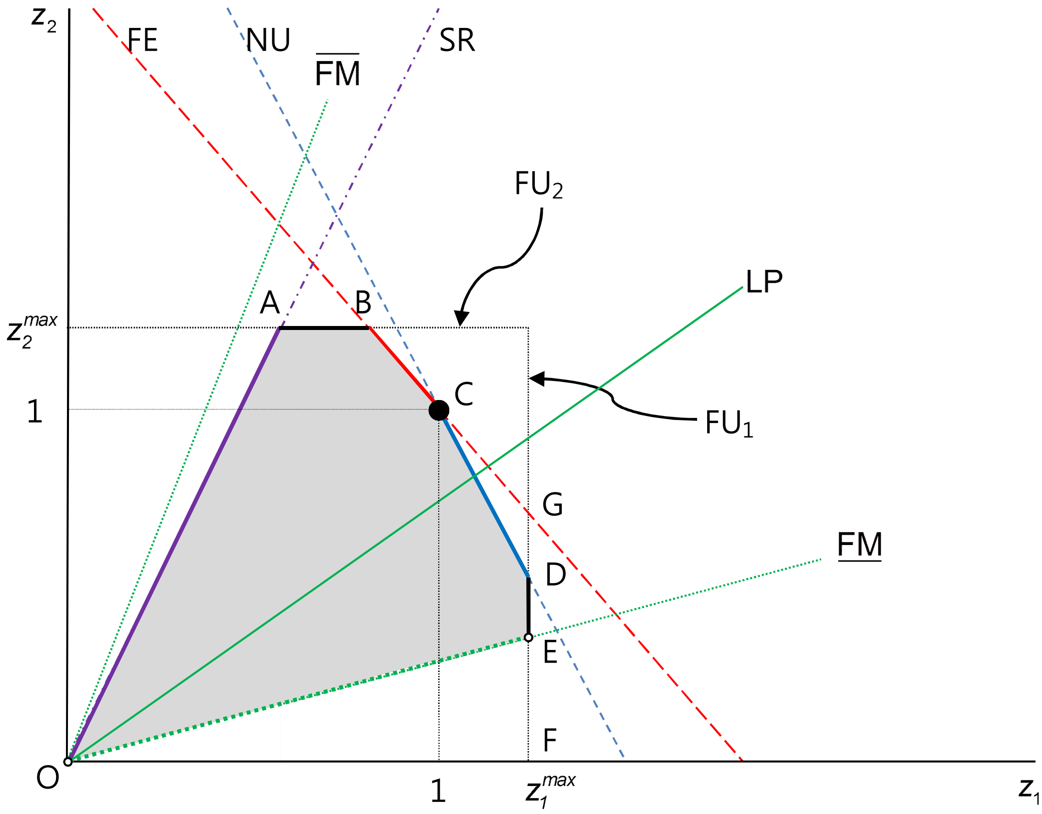
with

$$\max(0, \underline{\zeta}) \le \zeta \le \min(\overline{\zeta}, \infty) \tag{15'}$$

where $\underline{\zeta}$ is the infimum of ζ and $\overline{\zeta}$ the supremum of ζ for $|\phi_i| < 1$ for all i (strict inequality applies if $\underline{\zeta}$ and $\overline{\zeta}$ are respectively the case in (15')).

The following figure, which incorporates the five constraints, can serve as an aid in discussing some issues regarding effective demand in the long-period (for an expositional purpose, we have drawn the FE line to go through point C where $z_1 = z_2 = 1$, so that the WG state also employs the entire available labour).[33]

[33] The figure is drawn with z_i's as the variables for the two axes. For expositing certain aspects of an EDC state, it may be more convenient to draw the figure in reference to the ratio of the EDC level of output to its WG counterpart for each industry ($u_i \equiv x_i / \overline{x}_i = z_i(1+g_i^*)/(1+\tilde{g}_i)$). But we leave the exercise to the reader.



Long period positions that are possible in an EDC state are represented by the combinations of z_1 and z_2 constituting the inside and the boundary (except segment *OE*) of polygon *OABCDE* (a 'Diamond'). We have drawn the lines such that line $\overline{\text{FM}}$, which corresponds to $\zeta = \bar{\zeta} < \infty$, lies above line SR and line $\underline{\text{FM}}$, which corresponds to $\zeta = \underline{\zeta} > 0$, cuts the internal part of segment *DF*. The positions of lines FE, SR, $\overline{\text{FM}}$ and $\underline{\text{FM}}$ will change with z_1 and z_2. A point on line LP inside or on the Diamond represents an exogenously given combination of z_1 and z_2 that satisfy all the constraints and also equation (16') with all the endogenous variables determined in appropriate ranges. Such a point is, thus, a PED. It immediately turns out that line LP is the set of PEDs which will result when z_1 and z_2 maintain a constant ratio (for this, the endogenous variables must adjust, with the result that FE, SR, $\overline{\text{FM}}$ and $\underline{\text{FM}}$ change their slopes).

Point *C* represents the 'Golden Age Growth (GAG)' state: productive capacity is utilised at the normal level both in the economy as a whole and in each industry and, at the same time, the available labour force is fully employed (hence, line LP should go through *C*). The figure depicts an economy where the GAG state is also a self-replacing state (below line SR). It is possible for point *C* to lie above the SR line; if so, the GAG state is a non-self-replacing state even though the technique in use allows the economy to be viable.[34] The name of the 'Golden Age Growth' state for point *C* follows the same christening of such a state of the economy by Joan Robinson (1962): 'starting with near full employment and a composition of the stock of plant appropriate to the desired rate of accumulation, near full employment is maintained' (p. 52).

The figure helps identify several causes for labour unemployment over the long period. The first refers to the case related to the area above line SR. The levels of gross investment in the respective industries should not go beyond segment *OA*; if they did, the economy would not be in a self-replacing state. But the full employment of labour requires investment to go beyond the *OA* segment. Unemployment of this type can exist even if the constraint of NU is not binding; that is, even if the economy is capable of investing and willing to invest at the level that is more than enough to attain the full employment of labour, this latter objective cannot be realised.

The second and the third are due to the constraint of the full utilisation of productive capacity in the respective industries, represented by segments *AB* and *DE*. No investment can be made, even if it is desired, if this investment requires the utilisation of productive capacity beyond the full level, for the full degree of utilisation is the maximum of physically possible levels of utilisation. In the figure, the full employment of labour requires investment in the corn industry above z_2^{max} (segment *AB*); this is physically impossible (and this is so even if both the willingness to invest and the capability of investment are more than enough to achieve full employment). This case is reminiscent of Joan Robinson's 'Creeping Platinum Age', in which 'the ratio of basic plant is too high' (Robinson, p. 57), that is, the machine-producing industry is too large relative to the corn-producing industry so that, to maintain the long-period configuration of the economy (in particular, a self-replacing state), productive capacity in the latter industry has to be stretched to be utilised up to its physical limit, with the result, however, that even this over-utilisation is not sufficient to guarantee full employment. Another case is one in which the full employment of labour could be attained if the utilisation of productive capacity of the machine industry could go beyond z_1^{max} (segment *DE*); however, again, this is physically impossible (and in the particular case of the figure, effective demand at the normal level, too, is not sufficient to guarantee full employment). This case corresponds to Robinson's 'Galloping Platinum Age', in which '"animal spirits" are high, and a large mass of non-employed labour is available, but the desired rate of growth cannot be attained because of lack of basic plant to produce plant' (Robinson, p. 56): the utilisation of productive capacity in the machine-producing industry is stretched up to the limit but with no success in attaining full employment. Unemployment of this type can exist even though 'animal spirits' are higher than the level enough to guarantee full employment, as is the case if the relative slope of lines FE and NU is reversed.

The fourth case is, perhaps, what can be appropriately called 'long-period structural unemployment' due to the general lack of effective demand. Along segment *CD*, effective

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[34] For the distinction between a viable economy and a self-replacing economy, see Sraffa (1960, p. 5) and Chiodi (2002).

demand is just enough to guarantee continuous normal utilisation of productive capacity for the economy as a whole; however, even this level of overall effective demand, except at point *C*, is not sufficient to attain the full employment of labour. The reason is that the total volume of investment has been 'mis-allocated' among the industries, so that the composition of the aggregate output is not appropriate to bring about the full employment of labour, given the technique in use. This is what Robinson calls the 'Limping Golden Age', in which 'a steady rate of accumulation of capital may take place below full employment' (1960, p. 53), which would eventually settle at the 'Leaden Age' in which 'Malthusian misery checks the rate of growth of population … [so that] the rate of accumulation and the rate of growth of the labour force [are] equal, the ratio of non-employment being great enough to keep the latter down to equality with the former' (p. 54).

The last case is one that can be explained in reference to segment *OE* (with the relative slope of FE and NU reversed). Given the fundamentals parameters (the technique in use, the rate of profits and the saving propensities), it is impossible to achieve full employment even if 'animal spirits' are high enough. For whilst full employment requires an appropriate adjustment in the behaviour of corporations as regards their plans for external financing, the share (more generally, financial) market is not capable of accommodating the required adjustment. Full employment is possible only with changes in the 'fundamentals', which will enhance the capability of the financial market in this regard by removing the constraint expressed by line $\underline{\mathrm{FM}}$ (similar remarks can be made if $\overline{\mathrm{FM}}$ is below SR).

Only if line LP cuts through segment *BC* can full employment be attained when the state of effective demand is high enough to guarantee it; this may happen even if the economy-wide level of effective demand is *lower* than that corresponding to the WG state (shift the NU line to the left until it passes through point *B*; then, the general level of effective demand is lower but full employment is attained). However, the economy is restrained in another sense: restrained by the lack of further available labour even though 'animal spirits' can be higher than the level that corresponds to full employment. Robinson would describe this situation as the 'Restrained Golden Age', where '[w]ith a stock of plant appropriate to the desired rate of accumulation (which exceeds the rate of growth of population) and full employment already attained, the desired rate of accumulation cannot be realised, because the rate of growth of output per head … is not sufficient to make it possible' (p. 54).

With effective demand constraining the performance of the economy, however, the most usual state in which the economy finds itself will be well *inside* the Diamond (and on line LP): effective demand is such that *neither* the full employment of labour *nor* the normal utilisation of productive capacity *in any industry* is attained. Over time, however, the economy may show the tendency to move towards a fully-adjusted position with the stock of means of production fully adjusted to the state of demand so that it is utilised at the normal level. The state of effective demand exerts its effect *in the course of such adjustment* (whether full or not), that is, in the form of the below-normal utilisation of productive capacity, let alone the unemployment of labour, over the periods in which the economy goes through the process of adjustment. A result of this process, after the elapse of some periods, is a smaller size of productive capacity and thus a lower level of output at the normal utilisation of that productive capacity.

Suppose that the government—which we have not taken into consideration until now—tries, by increasing (or initiating) government expenditure, to help the economy to attain the full employment of labour.[35] An increase in government expenditure will be represented by a movement of a PED (and a change in the slope of LP); the precise direction of its movement dependent, among others, on the allocation of the expenditure among the products of the respective industries (one may call the government decisions of this kind *industrial policy*).[36] Increase in the government expenditure can achieve full employment only in the case of Limping or Restrained Golden Age, that is, only if the economy finds itself on a point of segment *BCG* (with government expenditure, effective demand in the economy as a whole can be larger than the limit that the private sector is subject to; hence, the part *CG* is now capable of being attained). This implies that mere increase in government expenditure, however large, may not always secure the full employment of labour unless the response of the private sector is such as to guarantee that line LP cuts segment *BCG*. But the adjustment of ϕ_i's in the private sector in response to the increase in government expenditure *and* its allocation among the commodities affects the slopes of the lines representing constraints FE, SR, $\overline{\mathrm{FM}}$ and $\underline{\mathrm{FM}}$ in addition to the slope of line LP. The success of fiscal policy (in the sense of increase in government expenditure) in its aim to attain the full employment of labour requires the company of appropriate *industrial policy* (in the sense of allocating government expenditure among the industries).

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[35] See Appendix.

[36] But the natural direction of movement will exclude South-West (output does not decrease in all the industries).

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Appendix: Introducing government expenditure

With the introduction of government expenditure, we need additionally consider taxation and the financing of budget deficit if any. Equation (1a) is to be modified, income tax being considered, to

$$[1+(1+t_r)r]\mathbf{Ap}+(1+t_w)w\mathbf{l}=\mathbf{p} \tag{A1}$$

where t_r = tax rate on profits and t_w = tax rate on wages.

Gross investment in the private sector is made in the same way as before (we are still taking the WG case without the government as the baseline):

$$(1+\tilde{g}_i)x_i\mathbf{a}_i\mathbf{p} = z_i(1+g_i^*)\bar{x}_i\mathbf{a}_i\mathbf{p} \tag{A2}$$

and the counterpart of equation (9) is

$$\tilde{g}_i = s_f(1-t_r)r + \phi_i\tilde{g}_i \tag{A3}$$

Government expenditure takes the form of purchasing the various commodities produced in the economy. There are several possible ways of expressing it in the model. One such way is as follows. We assume that budget deficit is financed through T-bonds (public debt) only and T-bonds are perpetuals (consols). The government makes two kinds of decisions in relation to government expenditure: the total volume of expenditure and its allocation among the products of the respective industries. One may call the former decision (proper) *fiscal policy* and the latter *industrial policy*. As regards fiscal policy, the government tries to maintain a certain ratio between public debt and GNP (γ). Public debt is calculated as the total volume of government expenditure (G) plus the interest payment by the government on T-bonds (ρB_{-1}, where B_{-1} is the value of the previously issued T-bonds and ρ is the rate of interest on T-bonds) minus the total amount of tax revenue: thus,

$$G = \gamma\mathbf{xp} + T - \rho B_{-1} \tag{A4}$$

The government allocates G for purchasing the product of the ith industry by the amount of

$$G_i = \beta_i G = \beta_i(\gamma xp + T - \rho B_{-1}), \text{ with } \sum \beta_i = 1 \tag{A5}$$

Then, the physical quantity of the ith commodity purchased by the government is

$$q_i = \beta_i G / p_i \tag{A6}$$

The market-clearing condition for each commodity requires the following relations, which is an extension of equations (4b):

$$\mathbf{xA} + \mathbf{x}[\tilde{g}_i]\mathbf{A} + h\mathbf{d} + \mathbf{q} = \mathbf{x} \tag{A7}$$

where $\mathbf{q} \equiv (q_1, q_2, \cdots, q_n)$.

Interest payment on T-bonds becomes part of the income of the private sector which purchases the bonds, and tax is levied on the interest income; if the tax rate on the interest income is t_B, total tax revenue is

$$T = t_r r\mathbf{xAp} + t_w w\mathbf{xl} + t_B \rho B_{-1} \tag{A8}$$

It is assumed for simplicity that T-bonds are purchased by households only.

Households now have three sources of income: labour, the holding of corporate shares and the holding of T-bonds. For simplicity and in analogy with the formulation of household saving in the main text, we assume that saving out of wages is represented by the saving propensity s_w and consumption out of the total of property income by the fraction c of the sum of capital gain and the after-tax interest income; thus, the after-tax household saving is

$$S_h = s_w w(1-t_w)xl - c\left[\sum (1-\phi_i)\tilde{g}_i \mathbf{x}_i \mathbf{a}_i \mathbf{p} + (1-t_B)\rho B_{-1}\right] \tag{A9}$$

Households use this saving for two purposes: purchasing corporate shares and purchasing T-bonds. If the household portfolio decision is represented by η that is a fraction of the household saving which is used for purchasing corporate bonds, we have the following two equilibrium conditions, one for the share market and the other for the T-bond market:

$$\eta S_h = \sum \phi_i \tilde{g}_i \mathbf{x}_i \mathbf{a}_i \mathbf{p} \tag{A10}$$

$$(1-\eta)S_h = \gamma \mathbf{xp} \tag{A11}$$

The part of the household saving which is used in purchasing corporate shares is sorted out into the respective industries in accordance with the long period condition of the uniformity in the valuation ratios. If (A.10) holds, (A.11) too holds in equilibrium. One can easily check that for the economy as a whole the following familiar national accounting relation holds:

$$\sum J_i + G = S_f + S_h + (T - \rho B_{-1}) \tag{A.12}$$

where $S_f = [1 + s_f(1-t_r)r]\mathbf{xAp}$ = after-tax aggregate gross corporate saving. That is, for the economy as a whole, the sum of gross investment and government expenditure is equal to the sum of after-tax private sector saving (which in turn consists of after-tax gross corporate saving and after-tax household saving) and 'net' tax revenue (which is obtained by subtracting the payment of interest on T-bonds from the tax revenue collected by the government).

The equation system for an EDC economy with government expenditure consists of equations (A.1) to (A10) plus an equation for the normalisation of prices: there are $6n$+5 unknowns (p_i, r, x_i, $\tilde{g}_i$, q_i, ϕ_i, h, G, G_i, T and ρ) in the same number of independent equations.

In analogy with the result in the main text, the ratio that the EDC level of output bears to its WG counterpart (without government expenditure) is

$$\frac{\hat{x}_i}{\bar{x}_i} = \left(\frac{1-\hat{\phi}_i}{1-\phi_i^*}\right)\left(\frac{1+s_f r-\phi_i^*}{1+s_f(1-t_r)r-\hat{\phi}_i}\right)\left(\frac{\mathbf{a}_i\mathbf{p}}{\mathbf{a}_i\hat{\mathbf{p}}}\right)z_i \equiv \hat{\Phi}_i z_i \qquad \text{(A13)}$$

where $\hat{x}_i$, $\hat{\phi}_i$ and $\hat{\mathbf{p}}$ are the values determined in relation to government expenditure. (Note that, differently from the case without government expenditure, the ratio depends also on the prices of commodities; this is due to the introduction of tax rates in the price system.)

The impact of the government expenditure on the quantity of output in the *i*th industry can be expressed as an equivalent change in the amount of private investment, because, comparing $\hat{\Phi}_i$ with its counterpart in the case of the absence of government expenditure (Φ_i), one gets from equation (A13)

$$\frac{\hat{x}_i}{\bar{x}_i} = \Phi_i\left(\frac{\hat{\Phi}_i}{\Phi_i}\right)z_i \equiv \Phi_i\hat{z}_i \qquad \text{(A14)}$$

Thus, the introduction of government expenditure can be represented by the movement of PED from the position corresponding to the case where there was no government expenditure. Whether $\hat{z}_i > z_i$ and also whether $\hat{z}_i$ moves in step with G depend on how ϕ_i's are adjusted, which is in turn affected by how the government allocates G among the commodities. But it is natural to expect that PED should move with increases in G in the direction except South-West.