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# THE MARKET FOR SAVINGS IN THE THEORY OF GENERAL INTERTEMPORAL EQUILIBRIUM

Sergio Parrinello

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*Abstract of the paper*

*A recent debate on the theory of general intertemporal equilibrium with production is focused on whether this theory is immune from the criticism to the aggregate version of the neoclassical theory of value and distribution. This article resumes two controversial and related issues of that debate: 1) whether a market of aggregate values (saving) for each period is implicit in that theory and is as much relevant for the determination of an equilibrium as the markets for dated physical commodities which appear in the generally accepted form of the corresponding model; 2) whether the possibility of reverse capital deepening and reswitching of techniques can intrude into the model through that hidden market and become a source of non meaningful equilibria. The arguments presented will lead to an affirmative response to question 1). Furthermore they will provide, also in the light of a recent (2009) contribution by Garegnani to the same question, a revised version of the quasi-equilibrium model which he used to describe the possibility 2).*

# THE MARKET FOR SAVINGS IN THE THEORY OF GENERAL INTERTEMPORAL EQUILIBRIUM

Sergio Parrinello

## Introduction*

The recent debate on the theory of general intertemporal equilibrium among Garegnani (2000; 2003), Mandler (2005), Parrinello (2005; 2008), Foley (2008), Petri (2004) and Schefold (2008) seems to be centred on two main issues.

1) It has been questioned whether a value aggregation of physical quantities is necessary in the theory of general intertemporal equilibrium; in particular whether an equilibrium condition between aggregate saving and aggregate investment for each period is as much determinant as an equilibrium condition in the market of a physical good or service.
2) It has been debated whether the possibility of *reverse capital deepening* and *reswitching* of techniques, which was proved for the aggregate version of the neoclassical theory of capital and distribution, can be a *specific* source of non meaningful intertemporal equilibria or, instead, can be neutralized by the same sufficient conditions (e.g. the weak axiom of revealed preferences or the representative consumer) that since long time ago have been adopted to prove the existence, uniqueness and stability of general equilibrium.

The debate on issues 1), 2) will be resumed in the light of Garegnani (2009). We shall use the abbreviation "*I*ntertemporal" instead of "model of general intertemporal economic equilibrium"; and the abbreviation "*A*–temporal" instead of "model of general a–temporal economic equilibrium". The adjective "a–temporal" encompasses the terms "static" or "one–period", although we are aware of possible objections to the use of such expressions interchangeably. Clearly question 2) above arises only if the answer to the former 1) is that aggregation is necessary. Ultimately the controversy seems to resolve itself into the acceptance or refusal of a syllogism of the following type. 1) Each *In*-

* This article has benefited from live discussion with Garegnani and Schefold and from mail exchanges with Foley and Petri on its earlier drafts. Furthermore the author acknowledges the suggestions received from two anonymous referees. The responsibility for errors and omissions remains only mine.

tertemporal can be formally converted into an *A*–temporal and shares the same properties, in terms of existence, uniqueness and stability of equilibrium which have been already proved for the static general equilibrium model. 2) Hahn–Garegnani's model, which is taken as a prototype for the discussion at issue, is an *I*ntertemporal. 3) Hence Hahn–Garegnani's model is formally equivalent to an *A*–temporal and cannot exhibit properties which require the demonstration of special theorems.

The premise 1) of the syllogism above is ambiguous because it presupposes that all *A*–temporals have a unique analytical structure and the same equilibrium properties. Schefold (2008) has convincingly demonstrated how an *I*ntertemporal can be reduced to an *A*–temporal and that any solution to the former must be also a solution to the latter. However, the second part of the premise 1) does not follow from the first one. The one–period or static model to which an *I*ntertemporal is converted possesses a special structure which reflects that of its ancestor. The existing theorems of existence, uniqueness and stability, which have been proved for a standard *A*–temporal without capital used for production, may not be extendable to the *A*–temporal corresponding to the *I*ntertemporal. Garegnani (2000; 2003) has argued that non meaningful equilibria can exist in the *I*ntertemporal as a result of *reverse capital deepening* and *reswitching* of techniques. Elsewhere, I (Parrinello 2005; 2008) stressed that the theorems of *tâtonnement* stability cannot be extended from the *A*–temporal to the *I*ntertemporal, because the adjustment mechanism is different in the two models. We should specify more carefully what is the structure of the *I*ntertemporal, or its equivalent *A*–temporal, which opens the door to different equilibrium and disequilibrium properties related to the theory of capital.

Sections 1, 2, 3 summarize the elementary notions of one period versus multi–period budget constraints, of dependence among equilibrium conditions and of perfect substitutes — as an introduction to the main argument. We shall reiterate in sections 4, 5 that a market for saving may and in a sense *must* exist in each period of the *I*ntertemporal and intervenes in a special way in the determination of equilibrium. Sections 6a, 6b specify the individual behaviour which underlies the markets for saving in the *I*ntertemporal. We shall deal in section 7 with the distinction among dated Walras laws (see Garegnani, 2009). Sections 1, 2, 3, 4, 5, 6, 7 should convince the reader that a market for saving must exist in each period of the *I*ntertemporal if its structural form has to be reduced into an a–temporal form in which each agent is subjected to a unique budget constraint. More importantly, the properties of the market demand and supply functions of the corresponding *A*–temporal cannot be assumed as if the saving markets would not exist, but they must be derived from the properties of the demand and supply functions of the *I*ntertemporal taken in its structural form. Section 8 clarifies the role of multiple numeraires as a specific feature of the *I*ntertemporal. Section 9 presents a reformulation of the method of quasi–equilibrium used by Garegnani to describe some properties of the *I*ntertemporal by focusing on the market for saving only. The final section suggests some lines for the development of the debate on question 2) presented above.

## 1. The budget equations

Dorfman, Samuelson and Solow (DOSSO, 1958) have warned their readers against a supposed equivalence between the intertemporal versus the a–temporal theory (in their words: dynamic versus static models) of general equilibrium:

> But time does make a difference in economics — witness years of controversy over the theory of capital. To treat dynamic problems as nothing but special cases of static ones may simply rob us of the insights that a more direct theory might yield. After all, $n$ commodities at each of $T$ dates are not *simply* $nT$ separate commodities. There is a structure: sometimes it is useful to view them as $T$ groups with date in common, sometimes as n groups with physical characteristics in common (DOSSO, 1958: 265).

It seems that this warning has been occasionally neglected. I will recall here different structures of the budget constraints related to the quotation above.

### 1.1. Alternative structures behind the wealth constraint

Let us denote by $T$ row–vectors the quantities of $nT$ commodities which enter into the budget constraint(s) of a typical agent:

$$\mathbf{x}_1 = \left(x_{1,1},\ \ldots,\ x_{1,n}\right)$$
$$\mathbf{x}_2 = \left(x_{2,1},\ \ldots,\ x_{2,n}\right)$$
$$\cdots\cdots\cdots\cdots\cdots\cdots$$
$$\mathbf{x}_T = \left(x_{T,1},\ \ldots,\ x_{T,n}\right)$$

where each element $x_{t,j}$ of the $\mathbf{x}_t$ vectors is defined $x_{t,j} \equiv \omega_{t,j} - c_{t,j}$, the (positive or negative) difference between a given endowment $\omega_{t,j}$ and a quantity $c_{t,j}$ of commodity $t, j$ to be chosen, $t = 1, \ldots, T; j = 1, \ldots, n$. Let us write the price column–vectors which are taken as given by the agent:

$$\mathbf{p}_1 = (p_{1,1}, \ldots, p_{1,n})$$
$$\mathbf{p}_2 = (p_{2,1}, \ldots, p_{2,n})$$
$$\ldots\ldots\ldots\ldots\ldots\ldots$$
$$\mathbf{p}_T = (p_{T,1}, \ldots, p_{T,n})$$

In the standard *A*–temporal the agent chooses the quantities $\overline{\mathbf{x}}_1, \overline{\mathbf{x}}_2, \ldots, \overline{\mathbf{x}}_T$ under the unique budget equation

$$\mathbf{x}_1\mathbf{p}_1 + \mathbf{x}_2\mathbf{p}_2 +, \ldots, \mathbf{x}_T\mathbf{p}_T = 0 \qquad \text{I.1}$$

In the *I*ntertemporal the agent is supposed to make his/her choices (promises of exchange) at one point in time, say at time 0, the start of period t = 1, according to the usual interpretation of the Arrow–Debreu approach. In the *I*ntertemporal *without markets for saving*, the agent must choose $\mathbf{x}_1, \mathbf{x}_2, \ldots, \mathbf{x}_T$ under the budget constraints

$$\left.\begin{array}{l} \mathbf{x}_1\mathbf{p}_1 = s_1 \\ \mathbf{x}_2\mathbf{p}_2 = s_2 \\ \ldots\ldots\ldots \\ \mathbf{x}_T\mathbf{p}_T = s_T \end{array}\right] \qquad \text{I.2}$$

where $s_t, t = 1, \ldots, T$, is a slack variable (a scalar) subject to $s_t \geq 0, t = 1, 2, \ldots, T$. This condition means that the agent cannot obtain through promises of exchange, made at time 0, a quantity of commodities for consumption in period $t$ whose value exceeds that of his income in $t$. In this case an optimal choice $\overline{\mathbf{x}}_1, \overline{\mathbf{x}}_2, \ldots, \overline{\mathbf{x}}_T$ cannot be derived from a solution to a maximum utility problem under a unique budget constraint like I.1.

Instead in the *I*ntertemporal *with markets for saving*, the agent chooses not only $\overline{\mathbf{x}}_1, \overline{\mathbf{x}}_2, \ldots, \overline{\mathbf{x}}_T$ but also an optimal plan of savings or dissavings under the budget constraints

$$\left.\begin{array}{l} \mathbf{x}_1\mathbf{p}_1 = s_1 \\ s_1 + \mathbf{x}_2\mathbf{p}_2 = s_2 \\ \ldots\ldots\ldots \\ s_{T-1} + \mathbf{x}_T\mathbf{p}_T = 0 \end{array}\right] \qquad \text{I.3}$$

where $s_1, s_2, \ldots, s_T$ are dated savings which will be shortly interpreted (subsection 1.2)[1].

[1] We are assuming, here, like in Hahn's model, that saving is null in the last period.

In this model an optimal plan $\overline{\mathbf{x}}_1, \overline{\mathbf{x}}_2, ..., \overline{\mathbf{x}}_T$ coincides with that under the unique constraint I.1 of the *A*–temporal, because I.1 can be obtained from 1.3 through an ordered substitution of one equation for group t into the equation for $t+1$, $t=1, \ldots, T$. Following this route to find $\overline{\mathbf{x}}_1, \overline{\mathbf{x}}_2, ..., \overline{\mathbf{x}}_T$, the equations I.3 will serve to determine an optimal plan of saving $\overline{s}_1, \overline{s}_2, ..., \overline{s}_T$ by substitution of $\overline{\mathbf{x}}_1, \overline{\mathbf{x}}_2, ..., \overline{\mathbf{x}}_T$ into the same equations. Hence, equation I.1, which appears in the a–temporal reduction of the *I*ntertemporal, does not mean that the markets for aggregate saving do not exist. On the contrary, such markets must exist if we want to use the unique budget I.1 instead of I.3 without affecting an optimal choice $\overline{\mathbf{x}}_1, \overline{\mathbf{x}}_2, ..., \overline{\mathbf{x}}_T$. Therefore an *I*ntertemporal with complete markets possesses two structural features which are not shared by an *A*–temporal which is not the a–temporal equivalent of the *I*ntertemporal:

1) a market for saving exists for each period;
2) the demand and supply of commodities in each period, although they are the result of a one–shot choice of the individual at a point in time and can be calculated under a unique budget constraint, must satisfy distinct budget constraints for the different periods where a demand or supply of dated savings intervenes.

### 1.2. Different interpretations of the variable $s_t$

It might be claimed[2] that the budget equation I.1 can be *directly* assumed in the *I*ntertemporal instead of being derived from the budget constraints I.3, because the assumption of complete markets — in particular forward markets — and of promises of exchange made only at a point of time, seems to make the separate markets for saving redundant. We reply that the assumption of a market for saving, associated with each period t, does not presuppose that contracts for the direct exchange of quantities of numeraire available in different periods are actually stipulated at time 0. It is also not appropriate to view $s_t$ in the budget equations I.3 as a carrier of purchasing power over time, because there is no need for additional stipulations of purchase and sale after time 0. No actual direct transaction may even occur on this market, only promises of exchange between dated commodities might be settled at time 0. Still a market for saving remains a necessary theoretical construction which belongs to the structure of the *I*ntertemporal in the following sense. Let us re–write the budget equations I.3 in the form:

[2] This section owes much to an argument suggested by Garegnani in a recent exchange with Petri and the present author.

$$\left.\begin{array}{l} \mathbf{x}_1\mathbf{p}_1 = \sigma_1 \\ \mathbf{x}_2\mathbf{p}_2 = \sigma_2 \\ \cdots\cdots\cdots\cdots \\ \mathbf{x}_T\mathbf{p}_T = \sigma_T \end{array}\right] \qquad \text{I.3'}$$

with

$$\sum_{t=1}^{T} \sigma_t = 0$$

A positive $\sigma_\tau$ in $\mathbf{x}_\tau\mathbf{p}_\tau = \sigma_\tau$ means that some quantities of goods available in period $\tau$, embedded in $\mathbf{x}_\tau$ and with a total value equal to $\sigma_\tau$, are left at disposal of other agents. To the value $\sigma_\tau$ must correspond a negative value

$$\sum_{t \neq \tau}^{T} \sigma_t = -\sigma_\tau$$

which means that the agent will receive for the former saving a disposal of commodities in periods $t = 1, 2, ..., T, \ t \neq \tau$ whose values exceeds his incomes in $t = 1, 2, ..., T, \ t \neq \tau$. A similar argument holds for a negative $\sigma_\tau$, i.e. dissaving.

From a different perspective, the theoretical existence of a market for saving can be explained by looking at a promise, *made at time 0*, for the delivery of a commodity available in period $\tau$ against a quantity of numeraire available at time 0, as equivalent to a promise, *made at time 0*, for the exchange of that quantity of numeraire between time 0 and period $\tau$ combined with a promise of delivery of the commodity in period t against a quantity of contemporary numeraire. This type of equivalence — via arbitrage — between a single and a double transaction would hold also in the case of three contemporary commodities, but it would not require the introduction of additional markets. Instead this kind of decomposition of one transaction in two transactions becomes necessary within a theory of general equilibrium which includes forward markets, because one of the two markets — the market on which a good available in two different periods is exchanged for itself — is absorbed by an aggregate market for this type of intertemporal deliveries as a result of the condition of equal effective rates of return $t$ (see section 5). In the next sections a market for saving will be supposed to exist in this sense, without assuming that it serves to move *purchasing* power across time; which would be indeed its typical role if the contracting activity should be (more realistically) spread over different periods of time.

## 2. Different types of dependence among equilibrium conditions

We need to clarify what does one mean by saying that an equilibrium condition depends on the others. A distinction already arises in the *A*–temporal with: $n$ commodities, the corresponding market demand $D_i(.)$ and supply $O_i(.)$ functions and prices $P_i$. For example, let us consider the following statements.

1) The equality between total intended expenditure and revenue

$$\sum_{i}^{n}[D_i(.)-O_i(.)]P_i=0 \qquad \text{II.1}$$

depends on the equilibrium conditions $D_i(.)=O_i(.)$ $i=1, \ldots, n$.

2) Any equilibrium condition, say $D_n(.)=O_n(.)$, depends on the others, $D_i(.)=O_i(.)$ $i=1, \ldots, n-1$ since the functions $D_i(.)$, $O_i(.)$ satisfy the Walras law:

$$\sum_{i}^{n}[D_i(.)-O_i(.)]P_i\equiv 0 \qquad \text{II.2}$$

The identity sign in II.2 means that the equality is valid also at disequilibrium prices. Instead equation II.1 is satisfied only at equilibrium prices, *if the identity II.2 is not assumed.*

Therefore it is useful to distinguish the following types of dependence:

- a *subordinate* dependence of type II.1 in which a condition is a linear combination of the structural equations of the model and can be omitted.
- an *equivalent* dependence of type II.2 in which *any* equilibrium condition depends on the others because the equilibrium conditions are assumed to satisfy the Walras law. In this case no specific condition *has to* be omitted instead of others[3].

[3] It is also true that an equilibrium condition of the structural model can be derived from the other equilibrium conditions jointly with a linear combination of all equilibrium conditions, but the latter cannot be interpreted as a structural equation, because it cannot be known if we do not already know *all* structural equations of the model.

## 3. A pseudo market for saving in the *I*ntertemporal

Let us re–consider Hahn–Garegnani's model. A simple version of this model is presented in the Appendix A by the equations $A_1 \div A_8$. The symbols used are:

- $A_0, B_0, L$ endowments of goods available at the beginning of period t = 0 and of labour in period t = 0;
- $A_1, B_1$ quantities of goods produced in period t = 0 and available in period t =1;
- $(P_{a0}, P_{b0}, P_{a1}, P_{b1}, W)$ discounted nominal prices of goods and nominal wage rate;
- $l_a, l_b, a_a, b_a, a_b, b_b$ technical coefficients;
- $C_{a0}, C_{b0}, C_{a1}, C_{b1}$ quantities of goods consumed in period t = 0,1;
- (.) denotes function.

The Walras law (see identity A.8) includes only the values of excess supplies on markets for physical goods. Let us provisionally define the functions for saving and investment by

$$S_0(\cdot) \equiv [A_0 - C_{a0}(\cdot)]P_{a0} + [B_0 - C_{b0}(\cdot)]P_{b0} \qquad \text{III.1}$$

$$I_0(\cdot) \equiv I_{a0}(.)P_{a0} + I_{b0}(.)P_{b0} \qquad \text{III.2}$$

with

$$I_{a0}(.) \equiv a_a A_1 + a_b B_1; \; I_{b0}(.) \equiv b_a A_1 + b_b B_1$$

The equality $S_0(.) = I_0(.)$ depends in subordinate sense. It is implicitly satisfied by the equilibrium solution and is as much superfluous as the equation $\sum_i^n [D_i(.) - O_i(.)]P_i = 0$ in the *A*–temporal as mentioned in section 2 (or its analogue equation in the *I*ntertemporal). Such interpretation dismisses the existence of a market for aggregate saving. We shall argue that the relevant saving market emerges through a different route. We need first some elementary explanation of the role of perfect substitutes in the theory of general equilibrium.

### 4. When the aggregation of physical quantities becomes necessary

Let us convert Hahn's model into an $A$–temporal in which $A_0$, $B_0$, $A_1$, $B_1$, $L$ are distinct quantities of goods $a_0$, $b_0$, $a_1$, $b_1$ and labour available in the same period. Suppose that $a_0$, $b_0$ are imperfect substitutes for consumption, whereas are perfect substitutes as factors of production. This means that the marginal rate of technical substitution (MRTS) between the two inputs of $a_0$, $b_0$ is given and constant. Let us assume MRTS = 1 by an appropriate choice of the units of measure. On the basis of this convention we may find in equilibrium two cases:

- either $P_{ao} = P_{bo}$, the price ratio is equal to MRTS and the distribution of a given expenditure between the two inputs is indifferent for the producer;
- or $P_{ao} \neq P_{bo}$, the price ratio is different from MRTS and the producer will use only one input (the cheapest).

If $P_{ao} = P_{bo}$, we replace the price equations A.1, A.2 of the original model with

$$P_{a1} = l_a W + k_a P_0 \qquad \text{IV.1}$$

$$P_{b1} = l_b W + k_b P_0 \qquad \text{IV.2}$$

where $k_a$, $k_b$ denote the input–coefficients of a good made of the goods $a_0$, $b_0$ used in indefinite proportions and $P_0$ is the price of this composite good. The functions of aggregate saving and investment are:

$$S(.) = \left(A_0 - C_{ao}(.)\right) + \left(B_0 - C_{bo}(.)\right) \qquad \text{IV.3}$$

$$I(.) = k_a A_1 + k_b B_1 \qquad \text{IV.4}$$

Notice that the aggregation of saving and investment in III.1, III.2 is in value, whereas that in IV.3, IV.4 is in pure physical units. In this case, a market for an aggregate replaces the markets of its components and

$$S(.) = I(.) \qquad \text{IV.5}$$

is an equation which describes an equilibrium condition. The Walras law A.8 is replaced by

$$P_0\left[\left(A_0 + B_0\right) - \left(C_{ao}(.) + C_{bo}(.) + I(.)\right)\right] +$$
$$+W\left(L - L_D(.)\right) + \left(A_1 - C_{a1}(.)\right)P_{a1} + \left(B_1 - C_{b1}(.)\right)P_{b1} \equiv 0 \qquad \text{IV.6}$$

We observe that the equations $A_0 = C_{ao}(.) + I_a(.)$; $B_0 = C_{bo}(.) + I_b(.)$ are not equilibrium conditions like in the original Hahn's model, where the supply of each initial endowment meets a demand for consumption and investment of the same commodity. Now the same equations serve only to determine the physical composition of the investment after the determination of the equilibrium prices.

Suppose now that in equilibrium $P_{ao} \neq P_{bo}$ instead of $P_{ao} = P_{bo}$. For example, let $P_{ao} < P_{bo}$. In this case the producers will use only good $a_0$ as a means of production and two distinct conditions of market equilibrium become determinant:

$$A_0 = C_{ao}(\cdot) + I(.)$$
$$B_0 = C_{bo}(\cdot)$$

combined with the price equations:

$$P_{a1} = l_a W_1 + k_a P_{ao}$$
$$P_{b1} = l_b W_1 + k_b P_{ao}.$$

We conclude that, if goods $a_0$, $b_0$ are perfect substitutes for production and their relative price is equal to the respective marginal rate of technical substitution, the existence of an aggregate market for the two goods becomes a necessity also in the *A*–temporal. We could have assumed that goods $a_0$, $b_0$ are perfect substitutes in consumption and imperfect substitutes for production and an analogous conclusion would have followed.

## 5. When value aggregation becomes necessary

Let us go back to the original Hahn's model where all goods are imperfect substitutes in consumption and in production. In the *A*–temporal which is not a reduction of this model we cannot find a condition of a uniform rate of return on capital invested or saved, despite we can play with certain identities which resemble such a condition. We may start from the identity

$$\left(\frac{P_{a1}}{P_{b1}}\right)\cdot\left(\frac{P_{bo}}{P_{ao}}\right)\cdot\left(\frac{P_{ao}}{P_{a1}}\right) \equiv \frac{P_{bo}}{P_{b1}}; \qquad \text{V.1}$$

and define the relative prices:

$$p_{ao} \equiv \frac{P_{ao}}{P_{bo}}, \quad p_{a1} \equiv \frac{P_{a1}}{P_{b1}}, \quad p_{bo} \equiv \frac{P_{bo}}{P_{bo}} \equiv 1,$$

$$p_{b1} \equiv \frac{P_{b1}}{P_{b1}} \equiv 1,\ 1+i_a \equiv \frac{P_{a0}}{P_{a1}},\ 1+i_b \equiv \frac{P_{b0}}{P_{b1}}.$$

and then write V.1 in the form

$$\frac{p_{a1}}{p_{a0}}\left(1+i_a\right) \equiv \frac{p_{b1}}{p_{b0}}\left(1+i_b\right) \qquad \text{V.2}$$

We may even arbitrarily call $i_a$, $i_b$ the own rates of interest on goods $a_0$, $b_0$. The form V.2 remains an identity which is an expression of the law of a unique price (Jevons' law) and it applies also if the markets are out of equilibrium. It belongs to the sphere of exchange, not to that of production processes.

Now let us return to the *I*ntertemporal and use the current prices $p_{i,t}$ $i = a, b$; $t = 0, 1$ to define the factors of return on capital invested in the two industries:

$$1+r^a \equiv \frac{p_{a1}}{a_a p_{a0} + b_a p_{b0} + l_a w},\ 1+r^b \equiv \frac{p_{b1}}{a_b p_{a0} + b_b p_{b0} + l_b w}$$

We define

$$1+\tilde{r}_a \equiv \frac{p_{a0}}{a_a p_{a0} + b_a p_{b0} + l_a w},\ 1+\tilde{r}_b \equiv \frac{p_{b0}}{a_b p_{a0} + b_b p_{b0} + l_b w}$$

the corresponding factors of profit calculated at the prices of the current period. The price equations A.1, A.2 (see Appendix A) imply $r^a = r^b$. In terms of the definitions above we can write:

$$\frac{p_{a1}}{p_{a0}}\left(1+\tilde{r}_a\right) = \frac{p_{b1}}{p_{b0}}\left(1+\tilde{r}_b\right) \qquad \text{V.3}$$

By contrast with identity V.2, equation V.3 reformulates the equilibrium condition $r^a = r^b$ as an equality between the factors of return on each type of investment, multiplied by a factor of appreciation. In the corresponding *A*–temporal the same equation V.3 can be obtained from the price equations A.1, A.2, but must be interpreted as a relation between the relative prices of contemporary commodities. It still reflects the absence of profits under perfect competition, but it cannot mean equality between the rates of return on investment or saving. Instead in the *I*ntertemporal the equation V.3 extends itself to all rates of return — *pure numbers per period of time* — on capital invested or lent. In the presence of capital goods a market for aggregate saving must exist. This market can be called by different, but equivalent, names: market of saving, of credit or of future income. It is distinct from the pseudo market which in section III was defined by the saving function III.1 and the investment function III.2. We notice that, in the case of perfect substitutes in production or in consumption, some markets for separate goods merge themselves into an aggregate market. Instead, in each period of the *I*ntertemporal, the market for saving adds to the markets of the individual components of the aggregate itself.

## 6. The wealth equation and the dated budget constraints in the Hahn–Garegnani's model

Let us reconsider the theory of individual choices underlying the Hahn–Garegnani's model. For the sake of the argument we replace the distinction between households and firms with that between:

- $m_s$ owners of the initial endowments which are only consumers and savers;
- $m_p$ agents which possess only their labor force and act as consumers and producers.

Let us briefly call them "savers" and "producers" respectively. Furthermore we assume from now on that wages are paid *post factum*, i.e. at the end of the production period, in order to make easier the comparison of our formalization with that often used in the recent debate.

### 6.1. The wealth equation of the saver and his dated budget constraints

Let lower case letters denote the quantities of saver i corresponding to the market quantities already defined. He is assumed to maximize his utility function

$$u^i\left(c^i_{a0},\ c^i_{b0},\ c^i_{a1},\ c^i_{b1}\right)$$

subject to

$$a^i_0 P_{a0} + b^i_0 P_{b0} = c^i_{a0} P_{a0} + c^i_{b0} P_{b0} + s^i_0 \text{ for } t = 0$$

$$s^i_1 = c^i_{a1} P_{a1} + c^i_{b1} P_{b1} \text{ for } t = 1 \qquad \text{VI.1}$$

Could $s_0^i$ be different from $s_1^i$? Of course; as indicated in section I. Income or wealth which are not consumed are not a supply of saving by definition. Nothing prevents one to call "saving" or "dissaving"

$$s_0^i \equiv (a_0^i P_{ao} + b_0^i P_{bo}) - (c_{ao}^i P_{ao} + c_{bo}^i P_{bo})$$

on the basis of the plus or minus sign of this expression, but this value is simply a slack variable. Instead, if we assume the existence of a market for saving, the equality $s_0^i = s_1^i$ must hold and the optimal consumption plan of the saver determined under the unique wealth constraint

$$a_0^i P_{ao} + b_0^i P_{bo} = c_{ao}^i P_{ao} + c_{bo}^i P_{bo} + c_{a1}^i P_{a1} + c_{b1}^i P_{b1} \qquad \text{VI.2}$$

coincides with the choice under the two dated constraints VI.1. A solution to the maximum problem determines the demand for two consumption goods in the two periods, $c_{ao}$, $c_{bo}$, $c_{a1}$, $c_{b1}$, and, by substitution into VI.1 an optimal amount of saving $\bar{s}^i$ which is the optimal common value of $s_0^i$, $s_1^i$.

## 6.2. The wealth equation of the producer and his dated budget constraints

The producer $k$ maximizes his utility function $u^k\left(c_{ao}^k, c_{bo}^k, c_{a1}^k, c_{b1}^k\right)$ and chooses the optimal outputs $a_1^k$, $b_1^k$ under the budget constraints:

$$\imath_0^k = c_{ao}^k P_{ao} + c_{bo}^k P_{bo} + \left(a_a a_1^k + a_b b_1^k\right) P_{ao} + \left(b_a a_1^k + b_b b_1^k\right) P_{bo} \text{ for } t = 0$$

$$l^k W + a_1^k P_{a1} + b_1^k P_{b1} = c_{a1}^k P_{a1} + c_{b1}^k P_{b1} + \left(l_a a_1^k + l_b b_1^k\right) W + \imath_1^k \text{ for } t = 1 \qquad \text{VI.3}$$

The market for saving brings about $\imath_0^k = \imath_1^k$. Let $\imath^k$ denote the common value of $\imath_0^k$, $\imath_1^k$. A positive optimal value of $\imath^k$ means a demand for saving. The choice of $c_{ao}^k$, $c_{bo}^k$, $c_{a1}^k$, $c_{b1}^k$, that maximizes $u^k(.)$, maximizes also the profits of the producer

$$\Pi^k = (a_1^k P_{a1} + b_1^k P_{b1}) - \left[\left(l_a a_1^k + l_b b_1^k\right) W + \left(a_a a_1^k + a_b b_1^k\right) P_{ao} + \left(b_a a_1^k + b_b b_1^k\right) P_{bo}\right].$$

Here we meet a case of badly defined demand and supply functions due to the assumptions of constant returns and perfect competition. Let us assume that the market prices are not arbitrarily given to the producers, but are equal to the respective unit costs of production. This implies that his/her maximum profit is equal to zero and the corresponding demand for inputs and supply of outputs, combined with a demand for saving $i^k$ remain undetermined. For the sake of argument, it is sufficient to assume that an optimal choice $\bar{c}^k_{ao}$, $\bar{c}^k_{bo}$, $\bar{c}^k_{a1}$, $\bar{c}^k_{b1}$ is associated with other market signals: $\bar{a}^k_1$, $\bar{b}^k_1$, $\bar{i}^k$[4]. Since by assumption maximum $\Pi^k = 0$, the demand and supplies of the physical goods and labour service which are chosen under the unique budget constraint

$$l^k W = c^k_{ao} P_{ao} + c^k_{bo} P_{bo} + c^k_{a1} P_{a1} + c^k_{b1} P_{b1} \qquad \text{VI.4}$$

coincides with those under the separate constraints VI.3.

## 7. Walras laws and saving functions for each period

Let us aggregate the quantities produced in period 0 and consumed in period 1 over the two types of agents:

$$A_{a1} = \sum_k a^k_1,\ B_{b1} = \sum_k b^k_1,\ C_{a1} = \sum_i c^i_{a1} + \sum_k c^k_{b1},\ C_{b1} = \sum_i c^i_{b1} + \sum_k c^k_{b1}.$$

$S(.) \equiv \sum_i^{m_s} \bar{s}^i$ defines the market supply function of saving with $\bar{s}^i$ denoting an optimum value $s^i$ for saver $i$. $I(.) \equiv \sum_k^{m_p} \bar{i}^k$ defines the market demand for saving with analogous meaning of $i^k$ for producer $k$. Notice the difference between the functions $S(.)$, $I(.)$ and the functions of saving and investment defined in section III by $S_0(.) \equiv [A_0 - C_{ao}(\cdot)]P_{ao} + [B_0 - C_{bo}(\cdot)]P_{bo}$; $I_0(\cdot) \equiv I_{ao}P_{ao} + I_{bo}P_{bo}$. We have seen that the equation $S_0(.) = I_0(.)$ is not to be reckoned among the determinants of equilibrium, whereas $S(.) = I(.)$ play the same role of the others equilibrium conditions on the markets of physical goods.

Let us add the budget equations of period $t = 0$:

for all savers:

$$a^i_0 P_{ao} + b^i_0 P_{bo} = c^i_{ao} P_{ao} + c^i_{bo} P_{bo} + s^i \qquad i = 1, \ldots, m_s$$

[4] The amounts $\bar{a}^k_1$, $\bar{b}^k_1$, $\bar{i}^k$ can be determined after the determination of a general market equilibrium by an assumption of symmetry among $m_p$ producers, that is each of them is supposed to satisfy a fraction $1/m_p$ of the market demand for each output.

for all producers:

$$i^k = c_{a0}^k P_{a0} + c_{b0}^k P_{b0} + \left(a_a a_1^k + a_b b_1^k\right) P_{a0} + \left(b_a a_1^k + b_b b_1^k\right) P_{b0} \qquad k = 1, \dots, m_p$$

and obtain the Walras law for period $t = 0$

$$\left[A_0 - D_{a0}(.)\right] P_{a0} + \left[B_0 - D_{b0}(.)\right] P_{b0} \equiv S(.) - I(.) \qquad \text{VII.1}$$

Similarly let us add the budget equations of period $t = 1$:

for all savers:

$$s^i = c_{a1}^i P_{a1} + c_{b1}^i P_{b1} \qquad i = 1, \dots, m_s$$

for all producers:

$$l^k W + a_1^k P_{a1} + b_1^k P_{b1} = c_{a1}^k P_{a1} + c_{b1}^k P_{b1} + \left(l_a a_1^k + l_b b_1^k\right) W + i^k \qquad k = 1, \dots, m_p$$

and obtain the Walras law for period $t = 1$

$$\left[L - L_D(.)\right] W + \left[S(.) - I(.)\right] + \left[A_1 - C_{a1}(.)\right] P_{a1} + \left[B_1 - C_{b1}(.)\right] P_{b1} \equiv 0 \qquad \text{VII.2}$$

This model exhibits six equilibrium conditions associated with the markets of goods a, b in period $t = 0, 1$; with the labour market and with the market for saving, but only four relative prices — the relative price of goods a, b available at $t = 0, 1$ the real wage and the rate of interest on the numeraire. The existence of two Walras laws, which share the value of the same excess supply $S(.)$ — $I(.)$, allows one to eliminate a seeming overdetermination. Garegnani (2009) has pointed out that the existence of distinct budget constraints for each period and the corresponding dated Walras laws should be taken into account: two laws in our case. The dependence of one equilibrium condition for each period is of the equivalent type according to the distinction made in section II. There is no reason to take the equation $S(.) = I(.)$ as dependent, instead of an equilibrium condition on the markets of physical goods or on the labour market, except for the mathematical convenience to reduce the number of the budget constraints to only one in order to find a solution to the system of equations by the method of substitution. Hence the properties of equilibrium (existence, uniqueness and stability) depend on the properties of the functions S(.), I (.) on the same foot as on those of the demand and supply functions of physical commodities.

We observe that we cannot assert — as in the traditional *A*–temporal — that any condition of market equilibrium depends on all the other equilibrium conditions of the model as a consequence of a unique Walras law. Instead in the

*I*ntertemporal, a condition associated with one period depends on the conditions of the same period only. Furthermore, since the same excess supply of saving appears in the budget equations of two contiguous periods — and it sets the link between them — a disequilibrium on the market for saving of the model implies a disequilibrium on two other markets for physical commodities, one for each period. We shall exploit this property in section 9 for a reformulation of the method of quasi equilibrium used by Garegnani (2000; 2003).

## 8. A digression about the choice of the numeraire

In the *A*–temporal the relative price of each commodity is defined in terms of a unique numeraire and it is usual to assume that the market excess demand for the commodity directly affects such a relative price. Similarly, the prices, that appear in the price equations and in the Walras law(s) of the *I*ntertemporal and are called *discounted* prices, are defined relatively to a unique dated commodity chosen as a numeraire. However, we have explained elsewhere (Parrinello 2005; 2008) that the analysis of disequilibrium and stability in the *I*ntertemporal requires the choice of a different numeraire for each period. In particular the same commodity available in period $t$ should be used to define only the relative prices of the other commodities available in the same period $t$, $t = 1, 2, \ldots, T$. Then the price of each non numeraire commodity in $t$ is assumed to change relatively to the numeraire in $t$, in response to an excess demand for the same commodity and according to the sign preserving rule. Instead an excess demand for saving in period $t$, combined with a disequilibrium on the markets of the numeraire available in period $t$ and $t+1$, will affect — according to the same rule — the price of the numeraire in period $t$ relatively to the numeraire in period $t+1$; that is the rate of interest on this good, without changing the price of the dated numeraire, equal to one by definition.

## 9. A revised method of quasi–equilibrium

In Appendix B the equations $B_1 \div B_{11}$ formalize a revised version of the quasi-equilbrium model used in Garegnani (2000; 2003) to describe a disequilibrium in terms of aggregate savings and investment, relatively to the model of Appendix A. After the replacement of the Walras law A.8 with two dated Walras laws B.10–B.11, a disequilibrium can be represented explicitly on the market for aggregate savings — only one such market in the simple model with two dates — and implicitly on the markets of the numeraire, good $b$ dated $t = 0, 1$ and assuming the other markets in equilibrium. The explicit market is defined by the demand and supply for aggregate savings as functions of the rate of interest on the numeraire $\tilde{S}(r_b)$, $\tilde{I}(r_b)$, with $r_b \equiv P_{bo}/P_{b1}-1$, $P_{b1}=1$. Figure 1 represents a possible shape of the curve of excess supply of saving $E(r_b)\equiv \tilde{S}(r_b)-\tilde{I}(r_b)$, in particular if the model is generalized by allowing for the choice of techniques. Each point on the curve $E(r_b)$, which is not an intersection with the $r_b$ axis, describes a disequilibrium on three markets: the market for aggregate saving and the markets of commodity $b$ available at $t = 0, 1$. The points of intersection, where $E(r_b)=0$, are points of full general intertemporal equilibrium, provided that all prices implicit in $E(r_b)$ are non negative (therefore $r_b \geq -1$ must hold).

Outside a full equilibrium, the rate of interest $r_b$ is supposed to increase (decrease) in response to an excess demand (supply) of saving, whereas the other prices adjust correspondingly and keep all markets in equilibrium, except the market for saving and the markets for good $b$ at $t = 0, 1$.

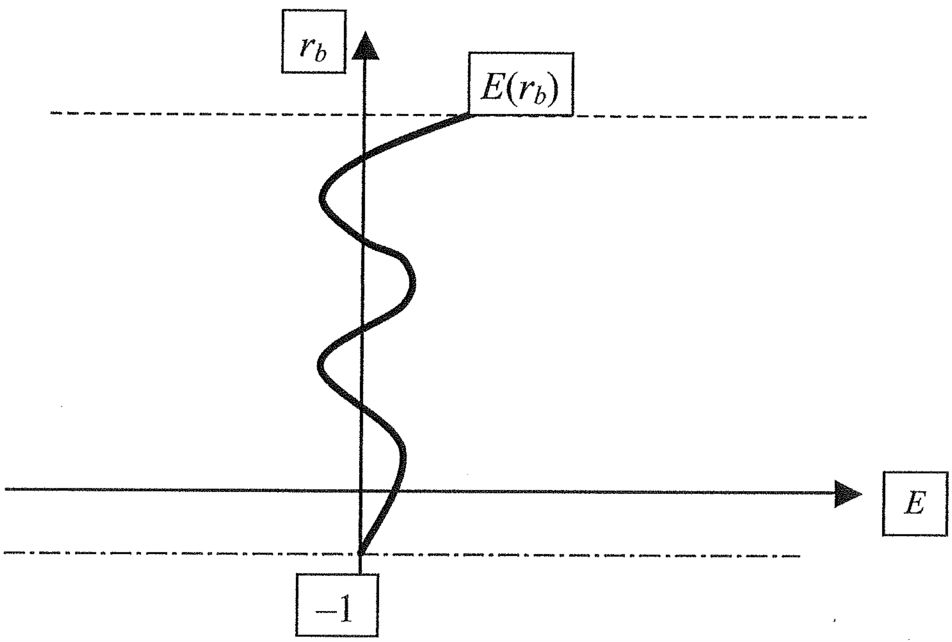


**Figure 1**

## 10. What progress in the debate next?

We do not pretend that our argument has settled the controversy around question 2) of the introduction: "whether the possibility of *reverse capital deepening* and *reswitching* of techniques, which was proved for the aggregate version of the neoclassical theory of capital and distribution, can be a *specific* source of non meaningful intertemporal equilibria". However it should be undisputed about question 2) of the same introduction that the function $I(r_b)$, and therefore $E(r_b)$, is the door through which those "perverse" properties of an equilibrium can enter both into the *I*ntertemporal and into its corresponding *A*–temporal. Some participants in the debate will be inclined to close that door; others to keep it open as much as possible. I believe that more progress in such a theoretical controversy can be achieved if some difficulties are acknowledged on each side and eventually overcome. The "door" should not be closed by applying the standard sufficient conditions (e.g. the absence of income effects) for meaningful equilibria — existence, uniqueness and stability — directly to the *A*–temporal associated with the *I*ntertemporal. In my opinion by–passing the structural form of the *I*ntertemporal and attributing behavioural assumptions directly to its reduced *A*–temporal form is a methodological infringement. On the other side, who has good reasons to leave the "door open" in the *general* *I*ntertemporal should deal with the choice of techniques subject to initial capital stocks arbitrarily given; therefore outside a long period analysis, which was at the centre of the debate during the Sixties[5]. Then, within a short period context, the composition of the demand for consumption plays a specific role and, in general, the number of methods of production, which belong to a profit maximizing technique, is not equal to the number of products, also in the absence of scarce natural resources. However, it is not necessary to deal with a general *I*ntertemporal in which the initial endowments are assumed to be arbitrarily given, if the problem at issue is the proof of possible lack of meaningful equilibria related to *reverse capital deepening* and *reswitching* of techniques. The theorems of existence, uniqueness and stability for the *general* *I*ntertemporal cannot exclude the case in which the initial capital stocks are taken just in the proportions consistent with constant relative prices and with a uniform rate of profit associated with equal own rates of interest. Therefore a counterexample under such a *special* *I*ntertemporal is sufficient for the effectiveness of the criticism[6].

[5] This point was raised by Schefold in an informal exchange and can be easily accepted.

[6] A final suggestion: since each equilibrium, under the assumption of complete competitive markets, is Pareto–efficient, a model of a multi–period planned economy with associated shadow–prices can throw some light on the existence and uniqueness of an intertemporal equilibrium.

## Appendix A: A simple version of Hahn–Garegnani's model

Let us assume that the techniques are given and all markets are cleared in equilibrium, with the possible exception of an excess supply on the labour market with a null salary. The economy lasts only two periods $t = 0, 1$, with two non storable goods $a$, $b$ dated $t = 0, 1$. The production is characterized by constant returns to scale represented by fixed coefficients. The endowments of goods $a$, $b$ are given for $t = 0$ and all production is consumed at the end of period $t = 1$. We write the equations of the model by using the symbols defined in section 3.

Price equations under perfect competition:

$$P_{a1} = l_a W + a_a P_{a0} + b_a P_{b0} \qquad \text{A.1}$$
$$P_{b1} = l_b W + a_b P_{a0} + b_b P_{b0} \qquad \text{A.2}$$

Equilibrium on the goods markets in period t = 0:

$$A_0 = C_{a0}(\cdot) + (a_a A_1 + a_b B_1) \qquad \text{A.3}$$
$$B_0 = C_{b0}(\cdot) + (b_a A_1 + b_b B_1) \qquad \text{A.4}$$

Equilibrium on the labour market:

$$L \geq l_a A_1 + l_b B_1 \text{ with } (L - l_a A_1 - l_b B_1)W = 0 \qquad \text{A.5}$$

Equilibrium on the goods markets in period t = 1:

$$A_1 = C_{a1}(\cdot) \qquad \text{A.6}$$
$$B_1 = C_{b1}(\cdot) \qquad \text{A.7}$$

all variables being subjected to non negativity conditions.

The unknowns of the model are the prices $(P_{a0}, P_{b0}, P_{a1}, P_{b1}, W)$ and the quantities $A_1$, $B_1$. The written conditions are seven, but one of them is dependent in equivalent sense. Given a certain numeraire, e.g. assuming $P_{b1} = 1$, the system is determinate in principle and the solution to equations A.1–A.7 determines also the equilibrium rate of interest on the numeraire through the identity $r_b \equiv P_{bo} / P_{b1} - 1$. The Walras law can be written in the form:

$$[A_0 - D_{a0}(.)]P_{a0} + [B_0 - D_{b0}(.)]P_{b0} + (L - L_D(.))W + \\ + (A_1 - C_{a1}(.))P_{a1} + (B_1 - C_{b1}(.))P_{b1} \equiv 0 \qquad \text{A.8}$$

with

$$D_{ao}(.) \equiv C_{ao}(.) + I_{ao}(.);\ D_{bo}(.) \equiv C_{bo}(.) + I_{bo}(.);$$
$$I_{ao}(.) \equiv a_a A_1 + a_b B_1;\ I_{bo}(.) \equiv b_a A_1 + b_b B_1;$$
$$L_D \equiv l_a A_1 + l_b B_1.$$

## Appendix B: A quasi–equilibrium model

Let us reformulate the model formalized in Appendix A by replacing the identity A.8 with two dated Walras laws and by adding the equilibrium condition between the aggregate saving function and the aggregate investment function. For simplicity the equilibrium condition on the labour market is assumed to be satisfied by a strict equality. The notation (.) distinguishes the functions from the variables, for example $C_{a1}\left(P_{ao}, P_{bo}, P_{a1}, P_{b1}, W\right)$ as distinct from $C_{a1}$. In the following only the variables $A_1, B_1$ are written and the functions $A_1\left(P_{ao}, P_{bo}, P_{a1}, P_{b1}, W\right)$, $B_1\left(P_{ao}, P_{bo}, P_{a1}, P_{b1}, W\right)$ are not well defined in the presence of constant returns to scale.

$$P_{a1} = l_a W + a_a P_{ao} + b_a P_{bo} \qquad \text{B.1}$$
$$P_{b1} = l_b W + a_b P_{ao} + b_b P_{bo} \qquad \text{B.2}$$
$$A_0 = C_{ao}(\cdot) + (a_a A_1 + a_b B_1) \qquad \text{B.3}$$
$$B_0 = C_{bo}(\cdot) + (b_a A_1 + b_b B_1) \qquad \text{B.4}$$
$$L = l_a A_1 + l_b B_1 \qquad \text{B.5}$$
$$A_1 = C_{a1}(\cdot) \qquad \text{B.6}$$
$$B_1 = C_{b1}(\cdot) \qquad \text{B.7}$$
$$P_{b1} = 1 \qquad \text{B.8}$$
$$S(.) = I(.) \qquad \text{B.9}$$

The Walras law for period t = 0:

$$\left[A_0 - C_{ao}(.) - a_a A_1 - a_b B_1\right] P_{ao} + \left[B_0 - C_{bo}(.) - b_a A_1 - b_b B_1\right] P_{bo} \equiv S(.) - I(.). \qquad \text{B.10}$$

The Walras law for period $t = 1$:

$$\left[S(.) - I(.)\right] + \left[L - l_a A_1 - l_b B_1\right] W + \left[A_1 - C_{a1}(.)\right] P_{a1} + \left[B_1 - C_{b1}(.)\right] P_{b1} \equiv 0. \qquad \text{B.11}$$

Only 7 equilibrium equations are independent and in principle can determine the 7 unknowns: the prices $P_{ao}, P_{bo}, P_{a1}, P_{b1}, W$ and the quantities $A_1, B_1$. Then the other equilibrium quantities are also determined, in particular the common equilibrium value of $S$, $I$.

Now release the equation B.9, solve the equations B.1, B.2, B.3, B.4, B.5, B.6, B.7, B.8 by taking $P_{bo}$ as a parameter. We obtain the functions (hopefully single valued) $P_{ao}(P_{bo})$, $P_{a1}(P_{bo})$, $W(P_{bo})$. Substitute $P_{ao}$, $P_{a1}$, $W$, $P_{b1}$ in the functions $S(.), I(.)$ with $P_{ao}(P_{bo})$, $P_{a1}(P_{bo})$, $W(P_{bo}), P_{b1}=1$. Then the functions $\tilde{S}(P_{bo})$, $\tilde{I}(P_{bo})$ follow and can be used like in Garegnani (2000). Let us check whether this construction is possible without resorting to additional assumptions, like the condition $A_0^D / B_0^D = A_0^S / B_0^S$ used in Garegnani (2000; 2003).

Solve equations B.1, B.2 assuming $W$, $P_{bo}$ as independent variables. We obtain the functions:

$$\begin{cases} P_{ao} = P_{ao}(W,\ P_{bo}) \\ P_{a1} = P_{a1}(W,\ P_{bo}) \end{cases} \qquad \textbf{(a)}$$

Choose B.3, B.5, B.6 as the independent equations of the model and substitute $P_{ao}, P_{a1}$ in the consumption functions $C_{ao}(\cdot)$, $C_{a1}(\cdot)$, with the functions **(a)**. Through substitution of $A_1$ with equation B.6 in equations B.3, B.5:

$$\begin{cases} A_0 = \tilde{C}_{ao}(W,\ P_{bo}) + a_a \tilde{C}_{a1}(W,\ P_{bo}) + a_b B_1 \\ L = l_a \tilde{C}_{a1}(W,\ P_{bo}) + l_b B_1 \end{cases} \qquad \textbf{(b)}$$

Equations **(b)** can determine the functions:

$$\begin{cases} W = W(P_{bo}) \\ B_1 = B_1(P_{bo}) \end{cases} \qquad \textbf{(c)}$$

Through substitution of $W$ with $W = W(P_{bo})$ in the equations **(a)**:

$$\begin{cases} P_{ao} = \tilde{P}_{ao}(P_{bo}) \\ P_{a1} = \tilde{P}_{a1}(P_{bo}) \end{cases} \qquad \textbf{(d)}$$

Through substitution of the $P_{ao}$, $P_{a1}$, $W$ functions into the consumption functions:

$$\begin{cases} C_{ao} = \tilde{C}_{ao}(P_{bo}) \\ C_{a1} = \tilde{C}_{a1}(P_{bo}) \end{cases} \qquad \textbf{(e)}$$

hence $A_1 = \tilde{C}_{a1}(P_{bo})$

Finally, through substitution of $W = W(P_{bo})$, $P_{ao} = \tilde{P}_{ao}(P_{bo})$, $P_{a1} = \tilde{P}_{a1}(P_{bo})$ into the saving and investment functions $S(.), I(.)$ we derive:

$$\begin{bmatrix} S = \tilde{S}(P_{bo}) \\ I = \tilde{I}(P_{bo}) \end{bmatrix} \qquad \textbf{(f)}$$

The consumption function $C_{b1} = \tilde{C}_{b1}(P_{bo})$ can be derived from the second Walras law B.10. Notice that a disequilibrium $\tilde{S}(P_{bo}) \neq \tilde{I}(P_{bo})$ must be accompanied with a disequilibrium on at least two other markets as a consequence of the Walras laws B.10, B.11. In our case such markets are the markets of commodity $b$ available in periods t = 0, 1. In the end we have derived the functions $\tilde{S}(r_b)$, $\tilde{I}(r_b)$ where $r_b \equiv P_{bo} / P_{b1} - 1$ ; $P_{b1} = 1$.

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