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## **Technical Change and Triple-Switching** in the Corn-Tractor Model

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#### **Abstract**

With triple-switching, each of two techniques are cost-minimizing in two disjoint intervals of the wage or rate of profits. Technology that supports multiple switch points between two techniques can only be a temporary phenomenon, as one technique supplants another with technical progress. A perturbation analysis of a triple-switching example in the corn-tractor model illustrates this claim. A parameter space, defined by two selected coefficients of production, is partitioned by loci corresponding to fluke switch points. The analysis of the choice of technique does not qualitatively vary within each of the resulting regions. Technical progress corresponds to specific trajectories through this parameter space.

#### Keywords

Cambridge Capital Controversy; Fixed Capital; Reswitching of techniques

**JEL Codes** 

B51; C67; D24

#### 1. Introduction

\* The author thanks an anonymous reviewer for helpful comments. All errors remain the responsibility of the author.

The reswitching of techniques is probably the most surprising result from the Cambridge capital controversy<sup>1</sup>. Kurz & Salvadori (1995) is a standard textbook presentation of the analysis of prices of production and of the choice of technique. Switch points, in which two techniques are both cost-minimizing at a given wage or rate of profits, are found as the zeros of certain polynomials of high degree. Reswitching occurs when two techniques have multiple switch points on the wage frontier at economically meaningful rates of profits<sup>2</sup>. These zeros can be complex and, if real, need not be positive and below the maximum rate of profits. Nevertheless, no obvious rationale exists for not expecting many economically feasible switch points to exist<sup>3</sup>. Then one technique will be cost-minimizing in at least two disjoint intervals of the rate of profits, if more than one switch point is on the wage frontier.

Empirical research<sup>4</sup> indicates, however, that the reswitching of techniques is rare. Kurz (2020) argues that these empirical investigations, although impressive, still suffer from limitations not overcome in data collection. Only circulating capital is assumed. Heterogeneous commodities are produced in each industry, and the input coefficients vary among processes operated in an industry. Accounting conventions may assign a firm to different industries in different years, depending on the mix of products produced by each firm. Still, it is not clear why reswitching should not be common, if these and other limitations in data are overcome in future work.

Schefold (2023) uses simulation to investigate the rarity of reswitching and other capital-theoretic phenomena. He randomly generates coefficients of production for alternate techniques. Wage curves are nearly affine functions. Only one, two, or maybe a few more techniques contribute their wage curves to the frontier, except near extremes for the rate of profits. The continuous variation in the cost-minimizing technique with distribution, as postulated in marginalist theory, is difficult to sustain. The reswitching of techniques does not seem likely on the wage frontier.

This article argues that reswitching can be empirically hard to observe for complementary reasons. A numerical example is created, for the corn-tractor model, that is just barely an instance of triple-switching. Fluke switch points are on the wage axis and the axis for the rate of profits. A switch point is a fluke if it is a knife edge case in which almost all perturbations of model parameters destroy its defining properties. A

<sup>&</sup>lt;sup>1</sup> Cohen & Harcourt (2003) and Lazzarini (2011) survey the controversy.

<sup>&</sup>lt;sup>2</sup> The rate of profits is also known as the interest rate in long period models without risk.

<sup>&</sup>lt;sup>3</sup> "Numerical examples and the realization that switching points are roots of *n*-th degree polynomials (and therefore numerous) have convinced us that reswitching may well occur in a general capital model" (Bruno, Burmeister, & Sheshinski; 1966: 527).

<sup>&</sup>lt;sup>4</sup> Han & Schefold (2006) and Zambelli (2018) compare techniques drawn from Leontief matrices constructed from actual national income and product accounts.

perturbation analysis partitions the parameter space with fluke switch points<sup>5, 6</sup>. The intersections of such partitions are double-fluke cases. For instance, the wage curves, with such parameters, are tangent at a switch point that is also on the wage axis. A picture of how triple-reswitching can arise emerges from an analysis of how the parameter space is divided into regions by these partitions. Technical innovation in the production of one type of tractor leads to certain trajectories through the parameter space. The emergence of triple-switching requires specific evolutions of coefficients of production. Further evolution of technology seems to remove the possibility of triple-switching<sup>7</sup>.

The corn-tractor model is an extension of the Samuelson-Garegnani model. Samuelson (1962) attempts to provide a rigorous defense of aggregate marginalist theory, as in the Solow-Swan model of economic growth. Samuelson's model consists of any number of techniques, each associated with a different type of capital good, called a 'tractor' here <sup>8</sup>. Labor and tractors can produce a new tractor, or they produce the consumption good, called 'corn'. Garegnani (1970), in his general treatment of an economy in which multiple commodities are produced, considers only the case of circulating capital. He shows that Samuelson's conclusions depend decisively on the critical assumption that, for each type of tractor, coefficients of production do not vary, other than by a scale factor, between the tractor and corn industries. Steedman (2019) extends the model to a special case of fixed capital. He treats depreciation as in Sraffa's model of joint production, instead of as radioactive decay, as in Samuelson's approach.

An original contribution of this article is to present perspicacious numerical examples illustrating that coefficients of production supporting multiple switch points between two techniques arise only temporarily, as one technique replaces another with technical progress. The perturbation of selected coefficients of production, to partition parameter spaces and thereby illustrate the effects of technical change, is also a contribution. Both numerical examples, of double and triple-switching, are partitioned by fluke switch points. This article also validates assertions in Steedman (2019) with the numerical examples. In contrast to Samuelson (1962), for example, double-switching can arise when each capital good is produced with the same physical capital intensity as when it is used to produce the consumption good. A numerical example also validates the assertion that triple-switching can arise when this assumption is relaxed.

The remainder of this article consists of two sections and an appendix. The next section analyzes an example in the corn-tractor model. The technology is specified for a numeric example. The system of equations for prices of production is specified and

<sup>&</sup>lt;sup>5</sup> Vienneau (2021) illustrates partitions of parameter spaces in other models of fixed capital.

<sup>&</sup>lt;sup>6</sup> The same structure of partitions of the parameter space can emerge in models of single production, although I do not have a numerical example.

<sup>&</sup>lt;sup>7</sup> Apodictic statements in this article are mostly based on numerical experimentation. I worry and hope that some specialist in the appropriate branch of algebraic geometry, if it exists, will be inspired to make my work redundant.

<sup>&</sup>lt;sup>8</sup> Salvadori & Steedman (1988) stress that Samuelson's conclusions do not follow if there is one type of tractor with coefficients varying as with the different types.

solved. A selected part of the parameter space is partitioned by fluke switch points. An analysis of structural economic dynamics shows how triple-switching can appear and disappear with technical progress. The final section concludes. The appendix modifies the example to partition the parameter space in a case in which double-switching, but not triple-switching, can occur.

#### 2. An Example in the Corn-Tractor Model

The corn-tractor model is a fixed capital model, an adaption of the Samuelson-Garegnani model. The consumption good, corn, can be produced by labor working with any one of a number of different types of tractors. Each type of tractor is produced by labor with an input of that type of tractor. Each type of tractor lasts for a specified number of years in the production of new tractors and of corn. In the general model, its lifetime can vary between industries, and these lifetimes can vary among types of tractors. This is an example of joint production. Every process for producing a new tractor, except the last, also produces tractors one year older than the tractors used as inputs. The production of corn also yields a joint product of tractors one year older. Each type of tractor works with constant efficiency, whether in producing new tractors or in producing corn. With these assumptions, no choice of the economic life of a machine arises. The tractor will be used for its full physical life in each industry.

Parameter	Type I Tractors	Type II Tractors
Tractor input per new tractor (a)	$a_I \approx 0.3062258$	$a_{II} = 2/5$
Labor input per new tractor (b)	$b_I \approx 233.6967$	$b_{II} = 20$
Years tractors last in tractor	$n_I = 1$	$n_{II}=2$
industry (n)		
Tractor input per bushel corn $(\alpha)$	$\alpha_I = 1$	$\alpha_{II} = 20$
Labor input per bushel corn $(\beta)$	$\beta_I = \alpha_I \cdot b_I / a_I$	$\beta_{II} = 850$
Years tractors last in corn industry	$\nu_I = 1$	$v_{II}=2$
$(\nu)$		

Table 1: Parameters for Technology for an Example in the Corn-Tractor Model

A technique is identified with a type of tractor. Six parameters (Table 1) specify a technique. The numerical example consists of a choice between two types of tractors. The first lasts only one year. That is, the production and operation of the first type of tractor is an example of circulating capital. The second type lasts two years in both the production of new tractors and of corn. The ratio of labor to tractors does not vary between industries for the first type of tractors. In other words, physical capital-intensity does not vary between industries. The production of corn is physically more capital-intensive than the production of new tractors ( $\alpha_{II} \cdot b_{II} - a_{II} \cdot \beta_{II} > 0$ ) for the initial parameters for the

second type of tractors<sup>9</sup>. As Steedman (2019) notes, this special case is sufficient to yield triple-switching.

I chose the parameters in Table 1 to illustrate a double-fluke case. The parameters for type II tractors are arbitrary, but such that the convexity of the corresponding wage curve changes once along its length. The convexity cannot vary more than once for tractors that last two years. The parameters for type I tractors are constrained to provide switch points on the wage axis and the axis for the rate of profits. These constraints result in a knife-edge case in which certain perturbations of parameters result in triple-switching.

#### 2.1 A System of Equations for Prices of Production and the Choice of Technique

Prices of production are such that the same rate of profits is obtained in each operated process <sup>10</sup>. Operated processes must be such that the technique comprised of those processes supports an economy in a self-replacing state.

For a technique in the corn-tractor model, prices of production, with labor advanced, satisfy the system of equations in Displays 1 and 2:

$$p_0 \cdot a \cdot c_n(r) + w \cdot b = p_0 \tag{1}$$

$$p_0 \cdot \alpha \cdot c_{\nu}(r) + w \cdot \beta = 1 \tag{2}$$

The variable  $p_o$  is the price of a new tractor of the given type, w is the wage, r is the rate of profits, and  $c_n(r)$  is given by Display 3:

$$c_n(r) = \frac{r \cdot (1+r)^n}{(1+r)^{n-1}} \tag{3}$$

Chapter X in Sraffa (1960) provides a derivation.

Suppose the distribution of income is exogenously specified, for instance, by taking the rate of profits as given. Then the solution of the above system is given by Displays 4 and 5:

$$w = \frac{1 - a \cdot c_n(r)}{\beta + \alpha \cdot b \cdot c_v(r) - a \cdot \beta \cdot c_n(r)} \tag{4}$$

$$p_0 = \frac{b}{\beta + \alpha \cdot b \cdot c_{\nu}(r) - \alpha \cdot \beta \cdot c_n(r)}$$
 (5)

Equation 4 is the wage curve for the given type of tractors. If the lifetime of tractors is the same in both industries and physical capital-intensity does not vary between industries, the denominator is a constant,  $\beta$ . The wage curve for a tractor that lasts multiple years is not a straight line, even under these assumptions. The price of a new tractor, however, is a constant, independent of distribution, under the assumption of a constant physical capital-intensity.

<sup>&</sup>lt;sup>9</sup> This assumption is useful, in two-sector models of growth for restricting the direction of income effects and for other reasons: "For uniqueness of momentary equilibrium [...], it is sufficient but not necessary to have the consumer good more capital intensive when the propensity to save out of wages is lower." (Dixit 1976: 129).

<sup>&</sup>lt;sup>10</sup> Vienneau (2024a) provides a perturbation analysis of the choice of technique when persistent and long-lasting differences in the rate of profits exist among industries.

In the numeric example, the physical capital-intensity is the same in both industries for type I tractors. The wage curve for type I tractors is an affine function. Since type II tractors last for more than one year, the wage curve for type II tractors is not a straight line. Since physical capital-intensity varies between industries for type II tractors, the price of new type II tractors varies with distribution. Steedman (2019) observes that Samuelson (1962) claims both properties hold under the assumption of constant physical capital-intensity only because he treats depreciation as radioactive decay, instead of following the more rigorous approach of Sraffa (1960).

Figure 1 plots the wage curves for the numeric example. The contrast between them is hardly visually stunning for this example. The cost-minimizing technique at a given rate of profits is the technique with the highest wage. The wage frontier is the outer frontier of all wage curves. Its construction, in the case of circulating capital and pure fixed capital, shows which technique is cost-minimizing, given the exogenous specification of the distribution of income. It is downward-sloping and cuts each axis at a finite value, given a finite number of techniques. In general, the wage frontier can be of any convexity. Like the convexity of the wage curve for type II tractors, the convexity of the wage frontier can vary along its length.

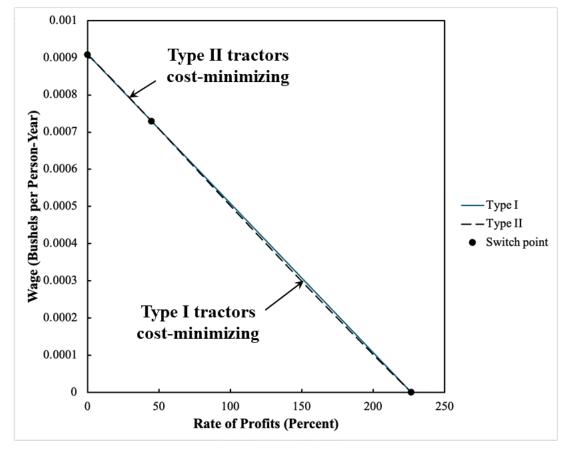


Figure 1: Wage Curves for a Double-Fluke Case

The cost-minimizing technique is not unique at switch points. A switch point is an intersection of wage curves on the outer frontier. Three switch points exist in this case,

one on the wage axis, one at the maximum rate of profits, and one at a rate of profits of approximately 45 percent. Switch points are found by solving a cubic equation. The coefficients of this cubic depend on the parameters in Table 1. The next section provides an analysis of how roots of this cubic vary with selected model parameters.

In a stationary state, tractors of each age are operated in parallel, both in the tractor industry and in the corn industry. At the end of each year, the oldest tractors are discarded and the appropriate number of new tractors are added to the stock. The sum of the prices of production of the stock of tractors is the value of capital. Following Steedman, I take a non-physical measure of capital-intensity to be the ratio of the value of capital to the value of net output. The capital-output ratio is a dimensionless number, while the units for the ratio of the value of capital to employment depends on the choice of the numeraire. In a stationary state, net output consists solely of corn, which is consumed. Net output per worker is an unambiguous physical quantity here.

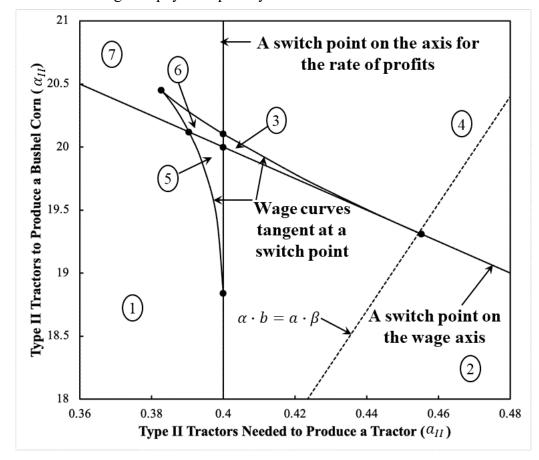


Figure 2: Partitioning a Part of the Parameter Space with Fluke Cases

In the numerical example, the capital-output ratio is a constant, independent of the rate of profits, for type I tractors. It increases and then decreases, with the rate of profits, for type II tractors. Since a switch point exists on the wage axis, the capital-output ratio does not vary, with the type of tractors, at the two switch points with a positive rate of profits. In the jargon, real Wicksell effects are zero at these switch points (Harris 1973). Around the switch point at approximately 45 percent, a lower rate of profits is associated with the

adoption of a technique with a longer-lasting tractor, even though this increases, across stationary states, neither the capital-output ratio nor consumption per worker.

#### 2.2 Perturbations of Selected Parameters

Almost any perturbation of the model parameters destroys fluke properties of the example in the previous section. Figure 2 illustrates perturbations in  $a_{II}$  and  $a_{II}$ . A switch point is on the axis for the rate of profits only for a specific value of  $a_{II}$ . Likewise, a switch point is on the wage axis only for the depicted partition of the parameter space, dividing, for instance, regions 1 and 7. The example in the previous section has parameters found at the intersection of these two partitions. The two other partitions occur for parameter values at which a switch point is repeated and the two wage curves are tangent at this switch point. The regions bounded by these partitions of the selected part of the parameter space are numbered.

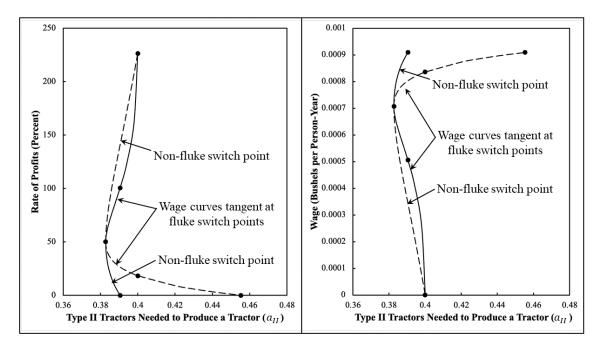


Figure 3: The Distribution of Income at Switch Points with Tangent Wage Curves

The dashed line depicts the combination of coefficients of production for which the ratio of labor to tractors does not vary between industries for tractors of type II. To the left and above this line the physical capital-intensity of production is less, for type II tractors, in producing new tractors than it is in producing corn. To the right and below, the tractor industry for type II tractors has a larger physical capital-intensity than corn production. The three points in which the dashed line intersects partitions of this parameter space are also points on partitions in Figure A-1 in the appendix.

I would have liked to have drawn the partitions as three-dimensional manifolds in a four-dimensional space, where  $(a_{II}, b_{II}, \alpha_{II}, \beta_{II})$  is a point in the space. But I can

visualize a tesseract only momentarily, if at all (Heinlein 1941). Figure 2 is constructed by selecting only two parameters to perturb.

Double-fluke cases arise at intersections of the partitions. The partition between regions 2 and 3 is tangent to the partition between regions 3 and 4 at their point of intersection. Similarly, the partition between regions 1 and 5 is tangent to the partition between regions 2 and 5. The two partitions between regions 6 and 7 are tangent at their point of intersection, as well.

This last double-fluke switch point can perhaps admit of some elaboration. Figure 3 shows the rate of profits and the wage at switch points for each of two fluke cases. The solid lines correspond to the partition between regions 1 and 5 and the lower partition in Figure 2 between regions 6 and 7. The dashed lines correspond to the partition between regions 3 and 4 and the upper partition. Three switch points exist for the parameters along these two partitions. Two of these switch points are repeated roots, which is the fluke case under consideration. All three switch points coincide on the wage frontier at the double-fluke switch point to the extreme left.

Region	Cost-	Notes on Switch Points
	Minimizing	
	Technique	
1	Type II	No switch point. Type II tractors are dominant with
		sufficiently low coefficients of production in
		producing Type II tractors.
2	Type II, Type I	Around the switch point, a lower rate of profits is
		associated with a technique with a longer-lasting
		tractor, a greater capital-output ratio, and more
		consumption per person-year.
3	Type I, Type II,	Around the first switch point, a lower rate of profits
	Type I	is associated with a technique with a tractor lasting for
		less time, a greater capital-output ratio, and more
		consumption per person-year. Around the second
		switch point, a lower rate of profits is associated with
		a technique with a longer-lasting tractor, a lower
		capital-output ratio, and less consumption per person-
		year.
4	Type I	No switch point. Type I tractors are dominant with
		sufficiently high coefficients of production in
		producing Type II tractors.
5	Type II, Type I,	Around the first switch point, a lower rate of profits
	Type II	is associated with a technique with a longer-lasting
		tractor, a greater capital-output ratio, and more
		consumption per person-year. Around the second
		switch point, a lower rate of profits is associated with

		a technique with a tractor lasting for less time, a lower	
		capital-output ratio, and less consumption per person-	
		year.	
6	Type I, Type II,	Around the first and third switch point, a lower rate	
	Type I, Type II	of profits is associated with a technique with a tractor	
		lasting for less time, a greater capital-output ratio, and	
		more consumption per person-year. Around the	
		second switch point, a lower rate of profits is	
		associated with a technique with a longer-lasting	
		tractor, a smaller capital-output ratio, and less	
		consumption per person-year.	
7	Type I, Type II	Around the switch point, a lower rate of profits is	
		associated with a technique with a tractor lasting for	
		less time, a greater capital-output ratio, and more	
		consumption per person-year.	

Table 2: The Choice of Technique by Region in the Example

The analysis of the choice of technique is qualitatively invariant in each numbered region. Table 2 lists the cost-minimizing technique, in order of an increasing rate of profits, in each region. One technique is cost-minimizing, whatever the distribution of income, in regions 1 and 4. One switch point exists in regions 2 and 7. Double-switching occurs in regions 3 and 5. Finally, triple-switching occurs in region 6. Perturbations of the parameters for the example in section 2.1 can result in each type of tractor being cost-minimizing in two discrete ranges of the wage or the rate of profits. This partitioning is not unique to this model.

#### 2.3 Technological Progress in the Production of Type II Tractors

An analysis of technical change is an application of this partitioning of parameter space by fluke switch points. In this context, the change in properties of the wage frontier is the result of structural economic dynamics (Pasinetti 1993). A movement from the upper right to the lower left in Figure 2 reflects a specific kind of technical progress in producing tractors of type II. The quantity of type II tractors needed to manufacture a new type II tractor decreases with a movement to the left. The maximum rate of profits for the technique with type II tractors increases. The quantity of type II tractors needed to make a bushel of corn decreases with a downward movement. The maximum wage increases.

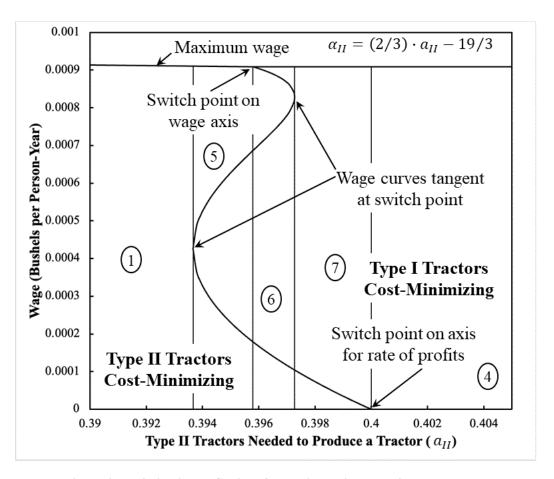


Figure 4: Variation in the Choice of Technique with Technical Progress

Figure 4 depicts<sup>11</sup> the variation in the analysis of the choice of technique along a specific line in Figure 2. The coefficients of production for inputs of type II tractors, in both industries, fall together in travelling from the upper-right to the lower-left in the graph. Switch points and the maximum wage are plotted. Four fluke switch points provide vertical divisions in the diagram. The cost-minimizing technique is labeled among ranges of the wage. From right to left, the diagram shows how technical progress in producing type II tractors results in the corresponding technique ultimately replacing the technique with type I tractors.

Region 6, with triple-switching, arises in the midst of this transition. It can be preceded by a region with a single switch point, as it is in this diagram. Or triple-switching can be preceded by region 3, in which double-switching occurs. The region with triple-switching can be followed by region 7, with a single switch point, or by region 5, with double-switching. A region with triple-switching might not occur at all, as in a path through regions 3, 2, and 5. The appendix provides an example in which double-switching can occur, but not triple-switching. Double-switching might also not occur at all, with a path from region 4, through region 2, and into region 1.

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<sup>&</sup>lt;sup>11</sup> Vienneau (2024b) provides other examples of this kind of diagram.

This example therefore suggests that a technology in which the corresponding prices of production exhibit triple-switching will appear only as a transient phenomenon, as one technique replaces another due to technical change. This conclusion also applies to double-switching. Presumably, the same result applies to multiple switching with four or more switch points.

How multiple switching manifests in market prices depends on many determinations not considered in this article. The speed at which coefficients of production evolve with technical change, compared to the speed at which market prices approach prices of production, if they do, seems of some importance. Robinson (1975) argues the latter process should be analyzed in historical time, not with a mechanical process in logical time. The stability of wages and the rate of profits is another issue. Accounting conventions for depreciation and for allocating overhead costs might impact these processes. The size of extra profits obtained by being first to adopt a new process or technique is another consideration. Nevertheless, no theoretical basis seems to exist for the idea, for example, that a rise in wages or a fall in the interest rate is associated with a drop in employment due to the adoption of a less labor-intensive or more capital-intensive technique of production, out of a given and known book of blueprints.

#### 3. Conclusion

Steedman, as in many of his papers, seems to be setting a homework problem for the advanced student:

"We therefore urge Sraffa-inspired authors to pay more attention to the analysis of fixed capital in *simple* models of production and hope that enough has been said here to provide a systematic basis for such further analysis." (Steedman 2019)

This article is my answer, with the solution extended to consider perturbations of coefficients of production and a kind of structural economic dynamics. It validates the claim that triple-switching can arise in a simple example of the corn-tractor model. The physical capital-intensity varies between industries for a type of tractor that lasts more than one production period in this example. It also validates the possibility of double-switching, even when, for each type of tractor, the physical capital-intensity is constant across industries. As Steedman notes, this result contradicts Samuelson (1962).

An illustration is given of how parameter spaces are partitioned with fluke switch points. The resulting qualitative structure of regions is claimed to be generic. The example illustrates that in a process of technical change, with one technique replacing another, parameters corresponding to cases of multiple switch points seem to only be transient<sup>12</sup>. The question of how prices of production relate to market prices is left unaddressed.

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<sup>&</sup>lt;sup>12</sup> Suppose technical progress occurs for the techniques associated with both types of tractors. If this progress is balanced, in some sense, the parameters can persist in a region, corresponding to region 6, in a higher-dimensional space for a long time.

#### Appendix A: An Example with Constant Physical Capital-Intensity

This appendix alters the example in Section 2 such that the physical capital intensity for tractors that last two years does not vary, for both types of tractors, between the tractor and corn industries. Table A-1 repeats the parameters from Table 1, with this modification. The parameters  $a_{II}$  and  $a_{II}$  in Table A-1 are such that the two switch points for the wage curves for Type I and Type II tractors are on the wage axis and the axis for the rate of profits.

Parameter	Type I Tractors	Type II Tractors
Tractor input per tractor (a)	$a_I \approx 0.3062258$	$a_{II} = 2/5$
Labor input per tractor (b)	$b_I \approx 233.6967$	$b_{II} = 20$
Years tractors last in tractor	$n_I = 1$	$n_{II}=2$
industry (n)		
Tractor input per bushel corn $(\alpha)$	$\alpha_I = 1$	$\alpha_{II} = 17.6$
Labor input per bushel corn $(\beta)$	$\beta_I = \alpha_I \cdot b_I / a_I$	$\beta_{II} = \alpha_{II} \cdot b_{II}/a_{II}$
Years tractors last in corn industry	$\nu_I = 1$	$v_{II}=2$
$(\nu)$		

Table A-1: Parameters for Technology in another Example for the Corn-Tractor Model

The invariance of physical capital-intensity between industries is manifested by the independence of the price of production of new tractors, of both types, from variations in the rate of profits. The price of production of one-year old type II tractors varies with the rate of profits. The wage curve for type II tractors is concave to the origin. The capital-output ratio for the technique using type II tractors increases with the rate of profits.

Fluke switch points partition the parameter space for this example (Figure A-1). Perturbations of the parameters  $a_{II}$  and  $\alpha_{II}$  are considered. Partitions are drawn for switch points on the wage axis, switch points on the axis for the rate of profits, and switch points at which the two wage curves are tangent. Each of these fluke switch points has a corresponding partition in Figure A-1. Regions are numbered as in Table 2. The equation for interceptions of wage curves is a quadratic. The first term in the cubic mentioned in the main text becomes zero. Triple-switching cannot occur.

A specific type of technical progress in the production of type II tractors can be depicted as a movement, in Figure A-1, from the upper right to the lower left. The figure is drawn under the assumption that  $\beta_{II}$  varies with  $a_{II}$  and  $a_{II}$  to ensure that the physical capital-intensities in the tractor and corn industries are identical. This assumption is quite artificial, especially when applied to technical change. Qualitative properties, however, of the partitioning of the parameter space by fluke switch points, when one region exists for double-switching, are generic<sup>13</sup>. Loci for fluke switch points on the wage axis, on the axis for the rate of profits, and for two repeated switch points at which the wage curves

<sup>&</sup>lt;sup>13</sup> Vienneau (2022) provides qualitatively similar partitions of a parameter space in a model of extensive rent.

are tangent, bound the region for parameters for which the double-switching occurs. The *locus* for repeated switch points is tangent at the points of intersections with the other two *loci*.

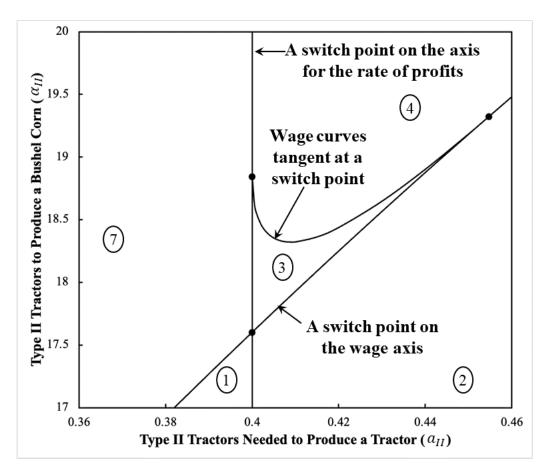


Figure A-1: Parameter Space for Example with Constant Capital-Intensity

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