WORKING PAPER
DIPARTIMENTO DI ECONOMIA PUBBLICA

Working Paper n.136

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A Simple Critical Introduction to Temporary General Equilibrium Theory

Roma, Dicembre 2010

SAPIENZA
UNIVERSITÀ DI ROMA

A Simple Critical Introduction to Temporary General Equilibrium Theory

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ABSTRACT. *During the last forty years, general equilibrium theorists have been especially concerned with the analysis of economies in which forward markets for commodities are limited in number or non-existent and trade takes place sequentially over time. Many distinguished scholars approached the study of such economies in the 1970s from the perspective of temporary equilibrium theory, which focuses on the behaviour of agents in a given period, stresses the dependence of agents' choices on their subjective expectations of future prices and discusses the existence of general equilibrium on current markets. Research in the field of temporary equilibrium theory was abandoned in the subsequent decade, however, and the work carried out in this area has since fallen into oblivion. The purpose of this paper is to provide an accessible exposition of temporary equilibrium theory and highlight the shortcomings that led to its abandonment in the conviction that basic knowledge of this area of research can prove conducive to correct appraisal of the current situation in general equilibrium analysis.*

JEL CLASSIFICATION: D 51, D 52

KEYWORDS: General equilibrium, sequential economies, temporary equilibrium

1. Introduction

By the end of the 1960s, the intertemporal model of Arrow and Debreu (cf. Debreu, 1959) was firmly established as the fundamental model of reference for general equilibrium analysis. The efforts of general equilibrium theorists were then directed towards overcoming the model's evident limitation, namely the assumption that the transactions associated with the future activities of economic agents are all regulated at the initial date on the basis of a complete system of forward markets for commodities. Many of these efforts drew inspiration during the 1970s from an analytical approach outlined in Hicks (1939) and gave rise to modern *temporary equilibrium theory*, the basic features of which can be summarised as follows. As in the Arrow-Debreu model, time is divided into a sequence of periods. It is, however, assumed with a view to the realistic representation of trading processes that spot markets for commodities are active in every period. It is further assumed that spot markets coexist with some asset

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markets, such as a *restricted set* of forward commodity markets. Within this framework, the theory focuses on the behaviour of agents in the initial period, stresses the dependence of agents' choices on their individual expectations as regards future prices, and discusses the existence of general equilibrium on current markets. A distinctive feature of the analysis is that no substantial restriction is placed *a priori* on the expectations held by agents at the beginning of the first period. Temporary equilibrium theory is thus ready to acknowledge that economic agents have limited predictive capabilities and may for this reason base their choices on *erroneous* expectations.

Research in the field of temporary equilibrium theory attracted many distinguished scholars during the 1970s but was gradually abandoned in the subsequent decade. The work carried out in the field has since fallen into a sort of oblivion, as attested by the fact that temporary equilibrium models are not even mentioned in recent textbooks. It is, however, our belief that basic knowledge of this area of research can still be of use today with respect to correct appraisal of the current situation in general equilibrium analysis. In accordance with this conviction, we shall endeavour to provide an accessible exposition of temporary equilibrium theory and highlight the analytical problems leading to its abandonment.

The paper is organised as follows. Temporary equilibrium theory is first illustrated in Section 2 with reference to a simple pure-exchange economy. The extension of the theory to the case of economies with production is then discussed in Section 3. Section 4 goes on to examine the application of the theory to the study of monetary economies. Finally, Section 5 draws conclusions concerning the reasons that led to the abandonment of temporary equilibrium theory and briefly comments on a related aspect of current general equilibrium analysis.

2. An introductory pure-exchange model

We shall begin our exposition of temporary equilibrium theory by focusing attention on the simplest analytical case. Consider a pure-exchange economy with H households (indexed by $h = 1, \ldots, H$) and $N \geq 2$ non-storable consumption goods (indexed by $n = 1, \ldots, N$) that is active for two periods of time, period 1 (the present) and period 2 (the future). At the beginning of period 1, by assumption, there are N distinct spot markets for the different consumption goods *and* a forward market for good 1, i.e. a market on which contracts can be traded for the delivery of physical units of good 1 at the beginning of the next period. In period 2, only the N spot markets for commodities are open. Given this market structure, we shall now introduce a further assumption informally and provide an initial, intuitive account of the behaviour of agents in the first period.

Assume that each household observes the prices quoted on the $N+1$ current markets at the beginning of period 1 and forms definite *expectations* as regards the future relative prices of commodities in terms of good 1. Under those circumstances the generic household h will calculate that by trading appropriately on the single forward market in existence, it can purchase or sell commodities for future delivery as freely as in the presence of a *complete system* of forward markets. To clarify this point, let us consider any of the $N-1$ commodities other than good 1, say 'grapes', and assume that household h thinks a unit of grapes will exchange in period 2 for three units of good 1. The household will then calculate that if it wishes to purchase in the present one unit of grapes for future delivery, it can obtain this result by buying forward three units of good 1 *in the anticipation* of exchanging them for the desired unit of grapes in period 2. Similarly, the household will calculate that if it wishes to sell in the present a unit of grapes to be delivered in the future, it can obtain this result by selling forward three units of good 1 *in the anticipation* of surrendering a unit of grapes in period 2 against three units of good 1 and then using those units to honour its forward sale.

The above example shows that for a household endowed with definite expectations as regards future relative prices, trading on the single forward market open in period 1 is essentially a way of transferring *purchasing power* across time. By buying forward units of good 1 at the current price, the household can thus transfer to period 2 the purchasing power (in terms of good 1) that it considers necessary in order to finance its desired future consumption. In the same way, by selling forward units of good 1 at the current price, the household can capitalise in the present the expected purchasing power (in terms of good 1) of any commodity or commodity bundle that it wishes to surrender in period 2. In order to highlight this aspect of the economy under consideration, we shall refer to the single forward market in existence as a market for *bonds* specified in terms of good 1, where the unit bond is defined as a promise to deliver a physical unit of good 1 at the beginning of period 2.

Given that the bond market allows for intertemporal transfers of purchasing power, it is reasonable to assume that households will simultaneously plan both their present *and* their future consumption at the beginning of period 1. We shall accordingly assume that each household trades commodities for present consumption and bonds at the initial date so as to attain the most preferred consumption stream over periods 1 and 2. By definition, a state of the economy in which all households trade in this way, and individual trades are such that all the $N+1$ current markets clear, is a *temporary equilibrium* of the exchange economy for period 1. In the remainder of this section the behaviour of households will be examined in detail with the aid of some formalisation. In order to simplify the exposition, it will be assumed that good 1 is the *numéraire* in terms of which both the current and the expected prices are measured.

2.1 The formal model

We shall first address the characteristics of the H households operating in the economy at the beginning of period 1. Let a *two-period consumption stream* of the generic household h be denoted by the vector $x_{12}^h = (x_1^h, x_2^h)$, where the sub-vector $x_t^h = (x_{1t}^h, \ldots, x_{Nt}^h)$ denotes a consumption bundle for period t (t = 1, 2). We assume that the set of admissible consumption streams, or *two-period consumption set*, of the generic household is $X_{12}^h = \Re_+^{2N}$. We further assume that the generic household knows its current commodity endowments $\omega_1^h = (\omega_{11}^h, \ldots, \omega_{N1}^h)$ at the initial date and takes it for granted that its future endowments will be $\omega_2^h = (\omega_{12}^h, \ldots, \omega_{N2}^h)$. Finally, in order to simplify the analysis, we introduce the following assumption concerning the households' preferences and endowments:

Assumption 2.1. (a) *The generic household h has a preference ordering over two-period consumption streams in* X_{12}^h *that can be represented by the continuous, strictly increasing and strictly quasi-concave utility function* $U^h(x_1^h, x_2^h)$;

(b) *for all* h, $\omega_1^h \gg 0$, $\omega_2^h \gg 0$.

As regards the prices guiding households' choices, we shall denote the prices in terms of good 1 ruling on current markets by the non-negative vector $p = (p_1, q_1)$, where the sub-vector $p_1 = (p_{11}, \dots, p_{N1})$ with $p_{11} = 1$ refers to the *N* spot markets for commodities and the scalar q_1 is the price of a unit bond. As regards the future spot prices expected by households at the initial date, we assume that individual price forecasts are both *subjective*, and therefore likely to differ among agents, and *certain*, in the sense that each household expects a definite price system to obtain in the future with probability 1. The system of future prices in terms of good 1 *as expected by the generic household h* will be accordingly denoted by the vector $p_2^h = (p_{12}^h, \dots, p_{N2}^h)$ with $p_{12}^h = 1$. In general, expected prices will depend both on the prices observed in the past and on those currently observed. For the moment, however, we shall assume that price forecasts are based exclusively on past prices and therefore independent of current prices (*fixed* expectations). We shall further assume that expected prices are strictly positive.

Assumption 2.2. (a) *The system of future prices* p_2^h *expected by the generic household h is given at the initial date independently of current prices;*

(b) *for all h,* $p_2^h \gg 0$.

We shall now go on to examine the behaviour of households at the opening of markets in period 1. To begin with, we assume that each household issues a quantity of bonds corresponding to the maximum it expects to be able to repay in the future. Given that the unit bond entitles the holder to the future delivery of one unit of numéraire, this means that the quantity of bonds issued by the generic household *h* coincides numerically with the value of the household's future endowments as anticipated by the household itself.

Assumption 2.3. *At the beginning of period 1 the generic household h issues a quantity of bonds* $\overline{b}_1^h$ *such that* $\overline{b}_1^h = p_2^h\, \omega_2^h$.

By issuing bonds in accordance with Ass. 2.3, the generic household *h* capitalises at the initial date the expected value of its future endowments. Since the receipts from this operation amount to ($q_1\, \bar{b}_1^h$) units of numéraire, the total wealth that the household can spend in period 1 on goods for present consumption and bonds is $W_1^h = p_1\, \omega_1^h + q_1\, \bar{b}_1^h$. The *first period budget constraint* of household *h* can therefore be written as

$$p_1\, x_1^h + q_1\, b_1^h = p_1\, \omega_1^h + q_1\, \bar{b}_1^h \tag{2.1}$$

where b_1^h denotes the quantity of bonds demanded. On the other hand, the household anticipates that it will have to surrender its entire endowment ω_2^h in period 2 in order to honour the bonds issued in period 1, and therefore calculates that the wealth it will be able to spend in the future on its own consumption is wholly determined by the repayment of the bonds purchased in the present. The *(expected) second period budget constraint* of household *h* thus reads as follows:

$$p_2^h\, x_2^h = b_1^h \tag{2.2}$$

The description of agents' behaviour at the beginning of period 1 can be finally completed by assuming that each household chooses its current consumption of goods, current demand for bonds and planned future consumption so as to attain a most preferred two-period consumption stream subject to budget constraints (2.1)-(2.2). It can be stated in formal terms that the choice of the generic household *h* at given current prices *p* and fixed expected prices p_2^h is a solution to the following maximisation problem:

[2.I] *Maximise* $U^h(x_1^h,\, x_2^h)$ *with respect to* $x_1^h \geq 0,\ b_1^h \geq 0,\ x_2^h \geq 0$
subject to constraints (2.1)-(2.2)

Let a solution to problem [2.I] be denoted by the triple (x_1^{h*}, b_1^{h*}, x_2^{h*}). It is clear that only the first two components will manifest themselves on current markets, in the form of demand for commodities to be consumed in the present and demand for bonds, while planned future consumption x_2^{h*} will remain, as it were, in the household's mind. It can therefore be stated that a solution to problem [2.I] identifies the *optimal action* $a^{h*} = (x_1^{h*}, b_1^{h*})$ taken by the generic household on period 1 markets.

The focusing of attention on budget constraints (2.1)-(2.2) will show how problem [2.I] can be solved. Note that by substituting for b_1^h and $\bar{b}_1^h$ in (2.1) according to (2.2) and Ass. 2.3 respectively, we obtain the equation

$$p_1\, x_1^h + q_1\, p_2^h\, x_2^h = p_1\, \omega_1^h + q_1\, p_2^h\, \omega_2^h$$

By adopting the convention $q^h = q_1\, p_2^h$, this can be written as

$$p_1\, x_1^h + q^h\, x_2^h = p_1\, \omega_1^h + q^h\, \omega_2^h \tag{2.3}$$

It thus emerges from the above manipulations that a solution to problem [2.I] is such that the corresponding consumption stream (x_1^{h*}, x_2^{h*}) fulfils equation (2.3), where the choice variable b_1^h does not appear. On the other hand, we know that the chosen demand for bonds b_1^{h*} must fulfil constraint (2.2). It follows that problem [2.I] can be solved in two successive steps. *In the first*, household *h* determines its optimal consumption stream $x_{12}^{h*} = (x_1^{h*}, x_2^{h*})$ by solving the problem

[2.II] *Maximise* $U^h(x_1^h, x_2^h)$ *with respect to* $x_1^h \geq 0,\ x_2^h \geq 0$
subject to constraint (2.3)

In the second, the household then determines through equation (2.2) the quantity of bonds to be purchased in order to finance planned future consumption, i.e. the quantity b_1^{h*} such that $b_1^{h*} = p_2^h\, x_2^{h*}$.

Closer examination of the first step shows that constraint (2.3) in problem [2.II] can be interpreted as *the single budget constraint* that household *h* faces when choosing its consumption stream at the initial date. To clarify this point, recall that in the presence of a bond market, the generic household *h* feels that it can trade goods for future delivery as freely as it could on a complete system of forward markets. Then note that from the viewpoint of household *h*, the *N* components of the vector q^h appearing in (2.3) are precisely *the prices* at which this intertemporal trade of commodities can be carried out in the present. In other words, they are precisely the 'present prices' for commodities to be delivered in the future. Examination of the first two components of q^h will suffice to show that this is so. In view of the convention adopted, the first component is $q_1^h = q_1 \ p_{12}^h = q_1$, i.e. precisely the 'present price' of a unit of good 1 for future delivery *as actually quoted* on the current bond market. On the other hand, the second component is $q_2^h = q_1 \ p_{22}^h$, where p_{22}^h is the future spot price of commodity 2 as expected by household *h*. Given the argument put forward at the beginning of this section, it becomes clear that q_2^h is indeed the 'present price' of a unit of good 2 for future delivery *as calculated by household h*, since it is both the price that the household would have to pay in the present in order to buy forward p_{22}^h units of numéraire to be exchanged in the future for a unit of good 2, *and* the price that the household calculates it could obtain in the present for a unit of good 2 to be delivered in the future.

On the above interpretation of q^h as a vector of 'present prices' for commodities to be delivered in period 2, it should be clear that the constraint (2.3) in problem [2.II] is the *intertemporal budget constraint* perceived by household *h* at the initial date. On the right-hand side we find the total wealth of the household, given by the value of current endowments plus the (expected) value of future endowments capitalised in the present, and on the left-hand side we find the household's current expenditure for both present consumption and planned future consumption. We therefore conclude that in the first step of the procedure, the choice of the optimal consumption stream at current prices $p = (p_1, q_1)$ and fixed expected prices p_2^h is *formally equivalent* to standard consumer choice under complete forward markets at prices $p' = (p_1, \ q^h)$, where $q^h = q_1 \ p_2^h$.

In the light of the abovementioned formal equivalence, the solution to problem [2.I] as emerging from the two-step procedure is easily discussed. Let us begin by examining the first step. Under Ass. 2.1 on households' characteristics and Ass. 2.2 of strictly positive expected prices, it is readily ascertained by analogy with basic consumer theory: (a) that the first step of the procedure *univocally determines* the consumption stream $x_{12}^{h*} = (x_1^{h*}, x_2^{h*})$ chosen by the generic household *h* at any given $p \in \Re_{++}^{N+1}$; and (b) that each component of x_{12}^{h*} changes continuously with *p* as the latter varies in $\Re_{++}^{N+1}$. This means that both the current and the planned future demand for consumption goods on the part of the generic household can be represented as *continuous functions* of period 1 prices, provided that the latter remain strictly positive. These demand functions will be denoted respectively by $x_1^h(p)$ and $x_2^h(p)$ from now on. It should now be recalled that the quantity of bonds demanded by household *h* is determined in the second step of the procedure by the condition $b_1^{h*} = p_2^h x_2^{h*}$. This means that the household's demand for bonds is also a continuous function of (strictly positive) current prices, that we shall denote by $b_1^h(p)$. In the light of the above considerations, it can finally be concluded that the continuous function $a^h(p) = (x_1^h(p), b_1^h(p))$ identifies the *optimal action* taken by the generic household at any given $p \in \Re_{++}^{N+1}$.

The ground has now been prepared for the formal definition of temporary equilibrium for the economy under examination. Let us restrict our analysis to strictly positive vectors of current prices and introduce the functions $z_1^h(p) = x_1^h(p) - \omega_1^h$ and $z_b^h(p) = b_1^h(p) - \bar{b}_1^h$ (where $\bar{b}_1^h$ is a given parameter in view of assumptions 2.3 and 2.2(a)). It should be clear that $z_1^h(p)$ represents the period 1 excess demand function for commodities of the generic household *h* and $z_b^h(p)$ the household's excess demand function for bonds. Summation over the *H* households then yields the corresponding *aggregate excess demand functions* $z_1(p)$ and $z_b(p)$, which are obviously continuous.[1] In this notation, a *temporary equilibrium of the exchange economy* for period 1 is finally defined as a system of current prices $p^* \in \Re_{++}^{N+1}$ and a corresponding set of

[1] It is easily proved that these aggregate excess demand functions are homogeneous of degree zero in *p* and fulfil Walras's Law.

optimal actions $\{a^1(p^*), \ldots, a^H(p^*)\}$ on the part of households such that the N+1 market clearing conditions $z_1(p^*) = 0$, $z_b(p^*) = 0$ are simultaneously fulfilled.

It can be proved that temporary equilibrium of the exchange economy exists under assumptions 2.1-2.3. Moreover, the existence of temporary equilibrium is preserved if it is assumed that individual expectations depend continuously on the current prices, i.e. if a continuous *expectation function* Ψ^h such that $p_2^h = \Psi^h(p)$ is introduced for each *h*. We shall refrain from substantiating these assertions, as the introductory model examined here is a particular specification of the temporary equilibrium model put forward by Arrow and Hahn (1971: Ch. 6), to which readers are referred for existence proofs. (See footnote 4 for the relationship between the introductory model and the Arrow-Hahn model.) We shall instead focus in the remainder of this section on the analytical scope of the introductory model**,** which has taken us quite comfortably from the Arrow-Debreu world with complete forward markets to the more realistic environment of temporary equilibrium theory. It will be argued that the model contains a hidden problem and is not really robust.

2.2 Discussion of the introductory model

The introductory model assumes that *a single* forward market is open in period 1 together with the spot markets for the *N* consumption goods. This is a restrictive assumption, however, as temporary equilibrium theory only postulates that the number of forward markets in existence is *lower than N*. It is therefore natural to wonder whether the model is susceptible of generalisation to economies with a larger set of forward markets. As we shall now see, unfortunately, even a slight increase in the number of forward markets in existence has serious consequences for temporary equilibrium analysis.

Let us modify the introductory model by assuming that $N > 2$ consumption goods are traded in the economy and, more importantly, that *two* distinct forward markets are open at the initial date, say the forward market for good 1 and the forward market for good 2. (We could equivalently state that two distinct *bond* markets are open, one for bonds specified in terms of good 1 and the other for bonds specified in terms of good 2.) This change in market structure necessitates some adjustment of the formal description of the economy. To begin with, the current prices in terms of good 1 will now be denoted by the non-negative vector $p = (p_1, q)$, where sub-vector p_1 refers to spot markets and sub-vector $q = (q_1, q_2)$ to forward markets. It will also be convenient to denote the quantities of goods that the generic household h trades on forward markets by the vector $b^h = (b_1^h, b_2^h)$, where by assumption $b_i^h > 0$ indicates a quantity of good i *demanded* and $b_i^h < 0$ a quantity of good i *supplied* by the household (i = 1, 2). In this notation, the *first period budget constraint* of the generic household h reads as

$$p_1 x_1^h + q\, b^h = p_1 \omega_1^h \tag{2.4}$$

and the household's *expected budget constraint for period 2* can be written as

$$p_2^h x_2^h = p_2^h \omega_2^h + p_2^{h'} b^h \tag{2.5}$$

where p_2^h denotes the future prices in terms of good 1 as anticipated by the household and $p_2^{h'} = (1, p_{22}^h)$ is the vector whose components coincide with the first two components of p_2^h.

Once these adjustments have been introduced, the economy can be described along the same lines as in the introductory model. We accordingly assume that given the current and expected prices, the generic household h chooses its current consumption, current trading on forward markets and planned future consumption at the initial date so as to maximise the utility function $U^h(x_1^h, x_2^h)$ subject to budget constraints (2.4)-(2.5). Provided that it is well-defined, this choice in turn identifies the *optimal action* $a^{h*} = (x_1^{h*}, b_1^{h*}, b_2^{h*})$ taken by the household on period 1 markets, where $a^{h*} \in \Re^{N+2}$. Within this framework, a *temporary equilibrium of the modified exchange economy* is finally defined as a system of current prices and a corresponding set of optimal actions on the part of the H households such that the N+2 current markets are simultaneously cleared.

It should be noted, however, that temporary equilibrium of the modified exchange economy will generally *not* exist under assumptions 2.1-2.2. As we shall see presently, the reason for this negative result is the fact that the introduction of an additional forward market has a substantial impact on trading opportunities and creates a problem as regards the determination of the behaviour of households.

The following example will help to clarify the nature of the problem. Assume that the price system ruling on forward markets at the beginning of period 1 is $\overline{q} = (\overline{q}_1, \overline{q}_2)$ such that $\overline{q}_2 / \overline{q}_1 = 2$. Assume further that at the initial date the generic household *h* believes that the future price of good 2 will be $\overline{p}^h_{22} > 2$. In these circumstances, household *h* has a strong incentive to trade on forward markets for *speculative* purposes. Consider, for example, how the household will evaluate an operation consisting of buying forward a unit of good 2 and simultaneously selling forward two units of good 1. On the one hand, the household will see that the total cost of the operation is zero under the assumed price conditions. On the other, household *h* will calculate that in period 2 it will be able to exchange the unit of good 2 delivered to it for a quantity of good 1 that *exceeds* the two units that the household has undertaken to deliver. In particular, the household will calculate that the operation in question ensures a future profit equal to ($\overline{p}^h_{22}$–2) units of numéraire. Household *h* will therefore conclude that by trading appropriately on forward markets *it can increase its wealth at no cost* or, to use a technical expression, that forward markets provide an opportunity for *profitable arbitrage operations*. It should now be recalled that the household is non-satiated in both present and future consumption (Ass. 2.1.(a)) and accordingly feels that any increase in its future wealth will make it better off. It is then clear that in these circumstances, household *h* will tend to increase *with no limit* the quantity of good 2 for future delivery demanded in the present and financed by selling forward good 1. This means that household's optimal action *is not determined*, however, and the possibility of the economy being in temporary equilibrium at the assumed price conditions must therefore be ruled out.

It is easy to show that the above argument can be repeated for all the states of the economy in which the prices quoted on current markets and those expected by the household *h* are such that $(q_2 / q_1) < p^h_{22}$. Moreover, a symmetrical argument shows that in the event of current and expected prices being such that $(q_2/q_1) > p_{22}^h$, household *h* would increase with no limit the quantity of good 1 for future delivery demanded in the present and financed by selling forward good 2. As a result, the household's optimal action would again not be determined and the possibility of the economy being in temporary equilibrium would again therefore be ruled out.

It thus emerges from the above considerations that the modified exchange economy can be in temporary equilibrium only if profitable arbitrage operations appear impossible to all households, i.e. only if the equality $(q_2/q_1) = p_{22}^h$ holds for all *h* (the *no-arbitrage condition*). It should be noted, however, that this necessary condition requires households to share *the same expectation* as regards the future price of good 2. In the presence of subjective expectations, the possibility therefore exists that the no-arbitrage condition may *not* be fulfilled at any admissible system of current prices. This is quite obvious in the case of fixed expectations, i.e. expectations that are independent of current prices as stated by Assumption 2.2(a). In this case, it is sufficient to imagine that just two households in the economy disagree over the future price of good 2 in order to be certain that the no-arbitrage condition will be violated at every system of current prices and that temporary equilibrium does not exist. The same problem also arises, however, under the assumption that expected prices are *continuous functions* of current prices, as individual expectation functions may well be such that two or more households disagree over the future price of good 2 at any system of current prices. For example, let us assume that three of the *N* households operating in the economy estimate the future price of good 2 as a weighted average of the price observed for that good in both the current period and the two previous periods. Let us further assume that the weights used to calculate the average differ among the three households. Under these assumptions, it will normally be found that at any given $p \geq 0$, at least two of the three households assign different values to the future price of good 2.

The problem that (perceived) arbitrage opportunities create for the existence of temporary equilibrium was pointed out by Green (1973) within the context of a pure-exchange economy similar to the one examined here. Green showed that the problem is reduced when the expectation functions attributed to households associate to each admissible system of current prices a *probability distribution* of future prices. At the same time, Green made it clear that this formulation of agents' predictions does not entirely eliminate the difficulty, as there must still be a substantial 'overlapping' of individual expectations in order to prevent unlimited arbitrage operations on forward markets.

3. Extension to the case of economies with production

We shall now see how the introductory model with a single forward market can be modified so as to transform it into a model of exchange *and* production. This extension of the model will provide an opportunity to point out the issues that arise in attempts to introduce production into the framework of temporary equilibrium analysis.

The first step towards the proposed extension consists of introducing the following basic changes in the model. To begin with, we assume that the N commodities traded in the economy include not only consumption goods but also goods and services susceptible of being used as production inputs. Second, we assume that a given number F of firms (indexed by $f = 1, \ldots, F$) are active in the economy. Third, as in the Arrow-Debreu model, we assume that the ownership of each firm is divided among households at the initial date in accordance with a given allocation of 'ownership shares'. The last basic change to be made is closely related to the third. It will be shown below that households are generally willing to *trade* their shares of ownership in firms within a temporary equilibrium framework. We therefore assume that F distinct markets for the shares in the different firms are active in period 1 in addition to the N spot markets for commodities and the market for bonds specified in terms of good 1. Having thus altered the structure of the economy, we shall now go on to analyse the behaviour of agents in period 1. As in the previous section, it will be assumed for simplicity that the consumption good listed as 'good 1' is the numéraire and that agents have fixed price expectations.

Let us begin with the productive sector of the economy. We assume that the production processes available to firms develop in cycles, i.e. that inputs are employed at the beginning of period 1 and the corresponding outputs emerge at the beginning of period 2. A *two-period production plan* of the generic firm f will accordingly be denoted by the vector $y_{12}^f = (y_1^f, y_2^f)$, where the sub-vector $y_1^f \in \Re_-^N$ denotes first period inputs and sub-vector $y_2^f \in \Re_+^N$ the associated future outputs. (Note that inputs are denoted by *negative* numbers.) The set of production plans that are technically feasible for firm *f* (the *production set* of the firm for short) will be denoted in turn by Y_{12}^f.

Due to the cyclical nature of production, the economy is endowed at the beginning of period 1 with given stocks of commodities derived from the activity of firms in the previous period. We assume that these stocks are entirely included in the initial endowments of households and that, for this reason, firms must finance their current input expenditure entirely by issuing bonds. We finally assume that each firm is run by a manager who is responsible for selecting the two-period production plan. Under these assumptions, the formation of production decisions can be described as follows.

At the beginning of period 1, the manager of the generic firm *f* is certain that the price system $p_2^f = (p_{12}^f, \ldots, p_{N2}^f)$ such that $p_2^f \geq 0$, $p_{12}^f = 1$ will obtain on future spot markets. Given the expected prices, the manager observes the prices $p = (p_1, q_1)$ quoted on current markets and assesses the profitability of the alternative plans in Y_{12}^f. In evaluating a hypothetical plan $y_{12}^f = (y_1^f, y_2^f)$, the manager realises that the firm would have to issue a quantity of bonds b^f such that $q_1\, b^f = -(p_1\, y_1^f)$ in order to finance its current input expenditure and would accordingly have to repay $b^f = -(1/q_1)(p_1\, y_1^f)$ units of numéraire at the beginning of period 2. At the same time, the manager anticipates that the plan would yield future receipts equal to $(p_2^f\, y_2^f)$ units of numéraire. According to the manager's subjective expectations, the hypothetical plan under consideration would therefore yield profits equal to $\pi_2^f = [p_2^f\, y_2^f + (1/q_1)(p_1\, y_1^f)]$ in period 2. In order to simplify the treatment of production decisions, however, it is convenient to introduce an alterative formulation of these expected profits. Given that a quantity π_2^f of the numéraire good for future delivery can be traded in the present on the bond market at the total price $\pi_1^f = (q_1\, \pi_2^f)$, we can say that the *present value* of the profits expected by the manager is $\pi_1^f = q_1 [p_2^f\, y_2^f +$

$(1/q_1)(p_1\, y_1^f)$]. By adopting the convention $q^f = q_1\, p_2^f$, the present value of expected profits can then be expressed in the equivalent form $\pi_1^f = (q^f\, y_2^f + p_1\, y_1^f)$, where, it should be noted, the components of vector q^f are precisely the 'present prices' of commodities for future delivery *as calculated by the manager of firm f*. Adopting this alternative formulation, we shall assume that the manager of the generic firm chooses the production plan so as to maximise the present value of expected profits.

Assumption 3.1. *Given the expected prices* p_2^f *and the current prices* p, *the manager of the generic firm f chooses a production plan that maximises the 'profit function'* $\pi_1^f(y_{12}^f) = (q^f\, y_2^f + p_1\, y_1^f)$, *where* $q^f = q_1\, p_2^f$, *subject to* $y_{12}^f \in Y_{12}^f$.

Under Ass. 3.1, the choice of the production plan at current prices $p = (p_1, q_1)$ and fixed expected prices p_2^f is *formally equivalent* to standard producer choice under complete forward markets at prices p" $= (p_1, q^f)$ and could be analysed in the same way. If we now use $y_{12}^{f*} = (y_1^{f*}, y_2^{f*})$ to denote the plan chosen by the manager of the firm *f* in accordance with Ass. 3.1, it is clear the manager's choice identifies both the firm's current demand for inputs and the current supply of bonds, where the latter is given by $b^{f*} = -(1/q_1)(p_1\, y_1^{f*})$, and therefore determines the *optimal action* $a^{f*} = (y_1^{f*}, b^{f*})$ taken by the firm on period 1 markets.

Now let us go on to examine the household sector. As previously assumed, households are endowed at the initial date with given 'shares of ownership' in the different firms. We shall denote the share endowment of the generic household *h* by the vector $\overline{\theta}^h = (\overline{\theta}_1^h, \ldots, \overline{\theta}_F^h)$ and assume that $\overline{\theta}^h \geq 0$ for all *h*, $\sum_h \overline{\theta}_f^h = 1$ for all *f*. It should be clear from the description of firms' behaviour that the possession of an ownership share in a firm throughout period 1 entitles the holder to the same proportion of the profits accruing to the firm at the beginning of period 2. It should be noted, however, that when the firms' plans are announced at the initial date, households will estimate the associated receipts according to their individual expectations and will thus typically form *different opinions* concerning the amount of profit to be earned by holding shares in any given firm. In the presence of those different opinions, it is natural to assume that households will find it advantageous to *trade* shares on the corresponding F markets in existence. Taking this aspect of the economy into account, we shall now examine the behaviour of households on period 1 markets *after* the announcement of the production plans selected by managers.

As regards trading on share markets, we shall drastically simplify our analysis by assuming that the shares of each firm are automatically transferred to the household (or group of households) expecting the highest amount of profit from the firm's plan, at a price exactly equal to the present value of those expected profits.[2] This assumption can be formally stated as follows. Define the present value of the profits that household h expects from the plan y_{12}^{f*} announced by firm f as $\pi_1^{hf} = q_1 [p_2^h y_2^{f*} + (1/q_1)(p_1 y_1^{f*})]$ and consider the equivalent formulation $\pi_1^{hf} = (q^h y_2^{f*} + p_1 y_1^{f*})$, where vector

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[2] It should be noted that this assumption is not totally *ad hoc*, as it can be justified for particular constellations of individual expectations. To clarify this point, let us examine the demand for the shares of the generic firm f on the part of the generic household h at different prices. To begin with, let us assume that the price for the whole of firm f's shares coincides with ($q_1\ \pi_2^{hf}$), i.e. with the present value of the amount of future profits π_2^{hf} that h expects from the plan announced by the firm. It is readily ascertained that in these circumstances, the question of whether to purchase the whole of firm f's shares or invest ($q_1\ \pi_2^{hf}$) units of numéraire on the bond market will be a matter of indifference to h. It follows that in the event of the price for 100% of the firm's shares being *higher* than ($q_1\ \pi_2^{hf}$), the household's demand for shares in firm f would be zero, as it would prefer to invest its savings in bonds. Finally it should be noted that in the event of the price for the whole of the firm's shares being *lower* than ($q_1\ \pi_2^{hf}$), household h would have an incentive to buy the firm outright and finance the purchase by borrowing on the bond market, because in the household's opinion the operation would ensure a positive profit in period 2 at no cost. Having established these preliminary results, let us assume for simplicity that there are only three households in the economy (h = 1, 2, 3) and that individual price expectations are such that $\pi_2^{1f} \geq \pi_2^{2f} \geq \pi_2^{3f}$. In those circumstances, it can be argued (a) that the equilibrium price for the whole of firm f's shares cannot be higher than ($q_1\ \pi_2^{1f}$) and (b) that the equilibrium price cannot be lower than ($q_1\ \pi_2^{2f}$), since at a price lower than ($q_1\ \pi_2^{1f}$) at least two households would be interested in purchasing the whole of firm f's shares and an aggregate excess demand would accordingly appear on the market for those shares. It can thus be concluded that the equilibrium price for 100% of the firm's shares must lie in the interval [($q_1\ \pi_2^{1f}$), ($q_1\ \pi_2^{2f}$)]. This in turn means that the assumption introduced in the text concerning the price for the shares of the generic firm can be justified in practice when the difference between π_2^{1f} and π_2^{2f} is negligible, and can be fully justified when two or more households have the most optimistic expectation as regards the firm's profits (i.e. in the particular case in which $\pi_2^{1f} = \pi_2^{2f} \geq \pi_2^{3f}$).

$q^h = q_1 \, p_2^h$ denotes the 'present prices' of commodities for future delivery *as calculated by household h*. Then denote by v^f the current price for the whole of firm *f*'s shares, or *market value of the firm* for short. Finally, denote by θ_f^h the share in firm *f* transferred to household *h* after the announcement of production plans. The following assumption then holds:

Assumption 3.2. (a) *The market value of the generic firm f in period 1 is*

$$v^f = \underset{h}{Max} \; \pi_1^{hf} = \underset{h}{Max} \; (q^h \, y_2^{f*} + p_1 \, y_1^{f*});$$

(b) *for all h and all f,* $\theta_f^h \geq 0$;

(c) *for all h and all f,* $\theta_f^h > 0$ *if and only if* $\pi_1^{hf} = v^f$;

(d) *for all f,* $\sum_h \theta_f^h = \sum_h \bar{\theta}_f^h = 1$.

As regards the market for bonds, we assume that each household issues bonds so as to capitalise its expected future wealth, which is given in the present context by the expected value of future endowments *plus* the household's share of expected profits from firms.

Assumption 3.3. *At the beginning of period 1 the generic household h issues a quantity of bonds* $\bar{b}_1^h$ *such that* $q_1 \, \bar{b}_1^h = q^h \omega_2^h + \sum_f \theta_f^h \, (q^h \, y_2^{f*} + p_1 \, y_1^{f*})$.

Under Ass. 3.3 the current wealth of the generic household is $W_1^h = p_1 \, \omega_1^h + \sum_f \bar{\theta}_f^h v^f + q_1 \, \bar{b}_1^h$. Part of this wealth is used to pay for the share transfers carried out in accordance with Ass. 3.2 and the remainder is spent on commodities for current consumption and bonds. The *first period budget constraint* of household *h* is therefore given by the equation

$$p_1 \, x_1^h + \sum_f \theta_f^h \, v^f + q_1 b_1^h = p_1 \, \omega_1^h + \sum_f \bar{\theta}_f^h \, v^f + q_1 \, \bar{b}_1^h \tag{3.1}$$

which, by substituting for $q_1\,\bar{b}_1^h$ according to Ass. 3.3 and taking Ass. 3.2 into account, can be written[3]

$$p_1\, x_1^h + q_1 b_1^h \;=\; p_1\, \omega_1^h \;+\; q^h \omega_2^h \;+\; \sum_f \bar{\theta}_f^h\; v^f \qquad (3.1')$$

On the other hand, the household anticipates that in period 2 it will have to surrender both its commodity endowments and its share of firms' profits in order to repay the bonds issued in accordance with Ass. 3.3. The household's *(expected) budget constraint for period 2* is therefore

$$p_2^h\, x_2^h \;=\; b_1^h \qquad (3.2)$$

Comparison of budget constraints (3.1')–(3.2) and budget constraints (2.1)–(2.2) of section 2 shows that once the firms' plans have been announced and share transfers have taken place, households are fundamentally in the same position as in the introductory pure-exchange economy. We therefore assume that in these circumstances, the generic household h will choose its current consumption, current demand for bonds and planned future consumption so as to maximise the utility function $U^h(x_1^h,\; x_2^h)$ subject to constraints (3.1')–(3.2). As in the introductory model, this choice will in turn determine the *optimal action* $a^{h*} = (x_1^{h*},\; b_1^{h*})$ taken by the household on period 1 markets.

The description of agents' behaviour at given current prices and fixed price expectations is now complete. Given that share markets are 'automatically cleared' in view of Ass. 3.2, a *temporary equilibrium of exchange and production* can be accordingly defined as a system of current prices, a corresponding set of F optimal

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[3] By substituting for $q_1\,\bar{b}_1^h$ as indicated in the text, the right-hand side of equation (3.1) becomes

$$p_1\,\omega_1^h + \sum_f \bar{\theta}_f^h\; v^f + q^h\omega_2^h + \sum_f \theta_f^h\,(q^h\; y_2^{f*} + p_1\; y_1^{f*}).$$

As θ_f^h is strictly positive if $\pi_1^{hf} = (q^h\; y_2^{f*} + p_1\; y_1^{f*}) = v^f$ and must otherwise be zero (Ass. 3.2 (b)-(c)), the right-hand side can be rewritten in the equivalent form

$$p_1\,\omega_1^h + \sum_f \bar{\theta}_f^h\; v^f + q^h\omega_2^h + \sum_f \theta_f^h\; v^f\,.$$

Once the right-hand side of equation (3.1) has been reformulated in this way, elimination of the total expenditure for shares $\sum_f \theta_f^h\; v^f$ from both sides yields equation (3.1').

actions on the part of firms, and a corresponding set of H optimal actions on the part of households such that the N spot markets and the market for bonds are simultaneously cleared in period 1.

It must be stated at this point that the model outlined in this section is not new but corresponds essentially to the temporary equilibrium model with production put forward by Arrow and Hahn (1971: Ch. 6).[4] As regards the *existence* of temporary equilibrium we can therefore take advantage of the results obtained by those authors, who prove in this connection that temporary equilibrium of exchange and production exists under standard assumptions on preferences and productions sets. They also show that this result holds not only in the case of fixed expectations but also under the assumption that individual price expectations are continuous functions of current prices. Having thus briefly dealt with the question of existence, we shall now go on to closer examination of the assumptions concerning production decisions made in the extended model. It will be argued that they are more problematic than they may appear.

[4] Our repeated reference to the contribution of these authors calls for some clarification as regards the link that can be established between the temporary equilibrium model with production of Arrow and Hahn (1971) and the models presented in this section and section 2.1 respectively. To start with the model outlined in this section, even though all the assumptions concerning the behaviour of agents are either borrowed from the Arrow-Hahn model or compatible with it, there are two differences in the formulation adopted. As readers can check, in the model of Arrow and Hahn prices are expressed in terms of a fictitious currency of account ('bancors') and the unit bond is defined as a promise to pay a unit *of that currency* in period 2. These differences are, however, immaterial. To substantiate this assertion, consider a version of the Arrow-Hahn model in which all agents expect that the future price of good 1 in terms of 'bancors' will be equal to 1. In these circumstances, which are fully compatible with Arrow and Hahn's formal treatment of expectations, the market for bonds specified in 'bancors' becomes the same thing as a market for bonds *specified in terms of good 1.* As a result, the version of the Arrow-Hahn model under consideration coincides with the extended model outlined in this section except for the numéraire adopted. Given that the behaviour of agents in Arrow and Hahn's contribution is independent of the numéraire measuring current prices, however, we can safely modify that version by taking good 1 as numéraire. Having thus established that the model with production presented in this section is simply a version of the Arrow-Hahn model, we shall now show that further specification of that version makes it possible to obtain precisely the pure-exchange model of section 2. Assume that there is only one firm in the economy (F=1) and that its production set is $Y^1_{12} = -\Re^{2N}_+$. The last part of the assumption states that the only processes the firm can operate are *free disposal* processes, through which any good available in any of the two periods is instantaneously destroyed by using no other input than the good itself. Under this particular specification of the productive sector, which is compatible with Arrow and Hahn's formal model despite its *ad hoc* nature, the single firm in existence will remain totally *inactive* in period 1 at every non-negative vector of current prices. As a result, the particular 'production economy' under consideration coincides in fact with the pure-exchange economy of section 2.1.

3.2. *Discussion of the extended model* [5]

In the intertemporal model of Arrow and Debreu, the existence of complete forward markets for commodities allows simple treatment of production decisions within firms. Let us consider, within that model, the position of the households holding ownership shares in a generic firm at the initial date. On the one hand, each household is interested in receiving the highest amount of profit from the firm, as any increase in profit would correspondingly increase the household's initial wealth and therefore improve the household's consumption opportunities. On the other, the profitability of the alternative production plans that are feasible for the firm can be assessed *objectively* on the basis of the prices observable on the current system of spot and forward markets. It follows from these considerations that the households sharing the ownership of a generic firm at the initial date will *unanimously approve* the choice of a production plan that maximises profits calculated at the currently observed prices.[6]

By contrast, the treatment of production decisions encounters considerable complications in a temporary equilibrium framework. In order to discuss the main issues that arise, let us return to the Arrow-Hahn model as presented in the first part of this section and focus on the position of households at the initial date. Jointly considered, budget constraints (3.1')-(3.2) show that the utility a household can plan to obtain by trading on current markets increases with the value of its period 1 wealth, which in turn depends partly on the value of the household's initial endowment of shares. This means that any household holding an initial share in the generic firm *f* will favour the choice of the production plan that receives the highest evaluation on the market for the firm's shares, i.e. the choice of the plan *that maximises the market value of the firm* v^f. According to Ass. 3.1, however, the manager of the generic firm will select the plan to which he *individually* attaches the greatest present value, so that he does not even try in general to act in the interest of the firm's initial owners. An unsatisfactory feature of the model is therefore that the criterion of choice attributed to managers has no clear rationale. We shall now show that this shortcoming is not easily remedied, as it is a symptom of an authentic analytical problem.

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[5] This part is based on Ravagnani (1989, 2000).

Suppose for the sake of argument that the manager of the generic firm, in an effort to serve the interests of the initial owners, forms a definite opinion as regards the production plan that will generate the highest market value of the firm and then announces that he intends to implement precisely this project. Since the manager's opinion is necessarily subjective, the firm's initial owners may happen to have a different opinion and wish to alter the manager's decision. Moreover, the initial owners may well have *conflicting opinions* as regards which plan will ensure maximisation of the firm's market value. In these circumstances, no production plan could be unanimously approved by the initial owners and a sort of *social choice* problem would therefore arise within the constituency of the firm's owners. While this problem could be tackled in principle by assuming that some *institutional rule* leading to a definite production decision is at work within the firm, the fact that a variety of such rules can be conceived (e.g. different voting schemes) makes it hard to see how that assumption should be precisely specified.

On the other hand, it is possible to adopt a pragmatic attitude and argue that the assumption that managers choose production plans according to their own evaluation of future receipts provides a realistic representation of where control over firms actually resides (see, for example, Bliss, 1976: 194–195). This attitude may explain why that assumption has been commonly adopted in temporary equilibrium models with production. As discussion of a further shortcoming of the Arrow-Hahn model will presently show, however, the assumption of production plans autonomously chosen by managers is hardly tenable in a temporary equilibrium framework.

The aspect we shall now discuss concerns the *financing* of the production plans selected by managers in accordance with their personal expectations of future receipts. As shown above, Arrow and Hahn assume that firms finance those plans by selling bonds on a *single* market where the securities issued by different agents are traded at the same price and therefore treated as perfect substitutes. It is highly doubtful, however, that rational households would be generally willing to trade on that single bond market. A simple example will clarify this point.

[6] This argument presupposes that the owners of the generic firm are 'price takers', i.e. they believe that current prices are not appreciably altered by changes in the firm's production plan.

Consider an economy with only two firms and assume that the manager of each firm selects a plan that maximises the present value of profits calculated on the basis of his individual price expectations. Then assume that when the manager of firm 1 announces the chosen plan, all the other agents in the economy expect that the future price of planned output will be so low as to generate *negative* profits for the firm in period 2. Finally, assume that all households expect positive profits from the plan announced by firm 2. In such circumstances, the entire ownership of firm 1 would be transferred to the firm's manager when the markets open at the initial date. Moreover, the following situation would occur on the bond market. Except for the optimistic manager of firm 1, all households in the economy would calculate that firm 1 is going to issue bonds that cannot be repaid out of the firm's future receipts – and since they do not know whether the future wealth of the firm's new owner will be sufficient to guarantee repayment, those households would have to regard the bonds floated by firm 1 as *risky* assets. At the same time, they would regard the bonds issued by firm 2 as perfectly safe. The announcement of the production plans independently chosen by managers would thus signal to households that in the overall supply of bonds risky assets may coexist with others whose repayment is beyond doubt. In this situation it is unreasonable to suppose, as the Arrow-Hahn model implicitly does, that households may be disposed to purchase bonds on a single, 'anonymous' market where risky securities cannot be distinguished from safe ones.[7]

[7] Problematic situations such as the one described in the text may also arise if the Arrow-Hahn model is modified by assuming that managers endeavour to select production plans that maximise the market value of their respective firms. For example**,** consider an economy with two firms, A and B, that can produce two different qualities of wine by employing grape must as the only input. Assume that each firm can produce any combination of wines by operating two independent processes defined by the production functions $y_{12} = (-y_1)^{1/2}$ for wine of type 1 and $y_{22} = 2(-y_1)^{1/2}$ for wine of type 2, where y_1 (a negative number) denotes the quantity of must employed and y_{i2} the output of wine of type i (i = 1, 2). Assume further that there are four households in the economy characterised by the following fixed expectations. Household 1, which includes only the manager of firm A, expects that the price for wine of type 1 will be $\hat{p}_{12} > 0$ and that the price for wine of type 2 will be zero. Household 2 has the same expectations as household 1. Household 3, which includes only the manager of firm B, expects that the price for wine of type 1 will be zero and that the price for wine of type 2 will be $\overline{p}_{22}$ = 1/2 $\hat{p}_{12}$. Finally, household 4 has the same expectations of household 3. Now recall that in the Arrow-Hahn model, the market value of each firm coincides with the present value of the profits that the most optimistic household (or group of households) expects from the firm's plan (Ass. 3.2). Taking this assumption into account, we can readily see that at any given positive price for grape must, there are

In order to avoid the abovementioned shortcoming, the model would have to be reformulated so as to enable potential lenders to identify the agents issuing bonds and learn how they plan to repay their debts. This could be done by introducing a *separate market* for the bonds issued by any individual agent, but then the hypothesis that managers autonomously select production plans could hardly be retained. For example, suppose that the manager of the generic firm *f* selected a definite plan $\bar{y}_{12}^{f}$ with the intention of covering the input cost through the sale of a sufficient quantity of bonds at price $\bar{p}_{b}^{f}$. When the plan is announced, households would evaluate future output according to their own price expectations (as well as the future wealth of the firm's owners, if the latter are legally responsible for the firm's debt) in order to assess the amount that could be paid back to lenders, and would thus form an opinion about the rate of return that could actually be obtained on the bonds supplied by firm *f*. If this largely subjective rate of return proved to be lower than that expected on the bonds of some other firm, however, households would not buy firm *f*'s securities. It would then be impossible to implement the plan chosen by the manager, and the theory would have to explain how the original project is to be revised.

Discussion of the Arrow-Hahn model thus shows that in the presence of subjective price expectations, it is not reasonable to assume that managers can raise funds freely on capital markets. If the hypothesis that managers determine production choices independently is to be maintained, temporary equilibrium theory would therefore have to introduce financing processes that do not depend on borrowing. It

always two distinct production plans that ensure maximisation of the market value of the generic firm in the economy under consideration. The first involves producing only wine of type 1 in the quantity that maximises the present value of profits calculated at the positive price expected for that wine by households 1 and 2. The second involves producing only wine of type 2 in the quantity that maximises the present value of profits calculated at the positive price expected for that wine by households 3 and 4. Having established this point, assume that managers seek to maximise the market value of their respective firms and that if two or more plans ensuring this result are identified, each manager will choose the one that he thinks will yield the highest amount of profits (reasonable behaviour). Finally, assume for the sake of argument that both managers can correctly predict how individual households will evaluate any feasible production plan. Under these assumptions, each manager will be able to identify the pair of plans that ensure maximisation of the market value of his firm when markets open in period 1. Moreover, the manager of firm A will choose and announce the plan that involves producing only wine of type 1, while the manager of firm B will opt for and announce the plan that involves producing only wine of type 2. On the other hand, every household will calculate that one of the announced plans will yield positive profits while the other is bound to bring about losses. The announcement of production plans will thus signal to households that risky bonds may coexist with safe ones in the overall supply.

should be noted in this connection that an alternative has been suggested in temporary equilibrium literature, notably by Grandmont and Laroque (1976). This rests essentially on two assumptions. The first is that the stocks of produced commodities available in the economy at the initial date are not in the hands of households, as postulated by Arrow and Hahn, but constitute the *initial endowments of firms*. This means that at the current prices each firm is endowed with well-defined initial wealth. The second assumption is that each firm must finance its current input expenditure entirely out of its wealth. Let us now consider whether the hypothesis that managers choose production plans independently is immune to problems under these alternative assumptions. The following example prompts a negative answer.

Assume that the manager of the generic firm *f*, guided by his personal evaluation of future receipts, chooses a production plan that involves using the whole of the firm's initial wealth to finance input expenditure. Assume further that when the manager's decision is announced, all the households in the economy (except for the manager's) anticipate that the firm's planned output will have negligible value in the future. In these circumstances, it is reasonable to imagine that the current price for the whole of the firm's ownership shares would be very close to zero. Assume that this is indeed the case and consider the position of the initial owners of firm *f*. Apart from the negligible price they could receive from the sale of their shares in the firm, these owners would calculate that the manager's decision requires them to give up some of their potential period 1 wealth (corresponding to the value of the firm's commodity endowment) in order to finance a project that they regard as a sheer waste of resources. At the same time, each owner would calculate that he would be better off if the firm were instructed to close down, as then he could regain his share of the firm's initial wealth and improve his consumption opportunities. Even though there may be disagreements concerning the 'optimal' plan to put into operation, all the initial owners would thus prefer the firm not to engage in production, and in the presence of this *unanimously preferred option* it is paradoxical to suppose that they would passively agree to finance the manager's project.

The considerations put forward thus far indicate that the assumption that managers select production plans according to their personal anticipations of future revenues should be avoided in a temporary equilibrium framework, as *the divergence of* *individual expectations* makes it difficult to assume that managers would then be free to finance the chosen plans either by borrowing on capital markets or by using the wealth of their respective firms. On the contrary, temporary equilibrium theory should admit that managers' decisions are subject to the ultimate judgement of savers, who may refuse to supply the required funds and thus force *revision* of the original projects. In this situation it would appear more appropriate to assume that managers, when selecting production plans, take into account the opinion of the agents who provide funds to the productive sector. This assumption gives rise to a new problem, however, because in order to develop a plausible notion of temporary equilibrium, the theory would have to explain how managers can succeed in *correctly* interpreting the private opinions of the potential financiers of firms.

4. Temporary equilibrium in economies with 'money'

The models discussed in the previous sections fail to capture one aspect of real-world trading processes, namely the fact that economic agents wish to keep stocks of a special good – *money* – that has no intrinsic value and is used essentially in exchange against physical goods. It should be noted, however, that much of the research carried out by temporary equilibrium theorists had the precise aim of incorporating money into modern general equilibrium analysis. In this section we shall therefore illustrate some basic results emerging from that specific application of temporary equilibrium theory.[8] This will be done through reference to a simple model drawn from Grandmont (1983), whose basic features are summarised below.

The model regards an exchange economy in which spot markets are active in each period, no forward market exists, and agents can transfer wealth from one period to the next only by holding a particular asset, 'money', which is available in the system in a constant amount. By assumption, the existing stock of this asset is made up entirely of *outside* (i.e. paper) money and can therefore be seen as part of the households' net wealth. The model is exclusively concerned with the store-of-value function of the asset and does not consider the other services performed by money in real-world economies (e.g. as a medium of exchange). Moreover, money is taken as numéraire and the behaviour of agents is analysed under the condition of strictly positive monetary prices. It should be noted that this choice of numéraire is incompatible with states of the economy characterised by aggregate excess supply of money, as the exchange value of money in terms of any commodity would clearly be zero in such circumstances. The main issue addressed by the model is therefore whether a temporary equilibrium for period 1 exists in which households are willing to hold the whole stock of money in circulation. Let us now go on to develop a detailed formal exposition.

As in Section 2, we shall refer to an economy with *H* households and *N* non-storable consumption goods that is active for two periods of time.[9] At the beginning of period 1, the generic household *h* has both a commodity endowment ω_1^h and an endowment of money $\overline{m}^h$ stemming from its past saving decisions. It is also certain that its future commodity endowment will be ω_2^h. The household observes the *monetary* prices $p_1 \in \Re_{++}^N$ quoted on current spot markets and expects the system of *monetary* prices p_2^h to obtain in period 2. (We shall continue to denote prices as in the previous sections even though they are now expressed in money for the sake of economy of notation.) Unlike the arguments developed in sections 2 and 3, we do not regard the vector p_2^h as fixed but assume that expected prices depend on current prices. To be more precise, we assume that $p_2^h = \Psi^h(p_1)$, where the expectation function Ψ^h can include past prices among its parameters. Finally we introduce the following assumption concerning the characteristics and expectations of households:

Assumption 4.1. (a) *The generic household h has a preference ordering over two-period consumption streams in the set* $X_{12}^h = \Re_+^{2N}$ *that can be represented by the continuous, strictly increasing and strictly quasi-concave utility function* $U^h(x_1^h, x_2^h)$*;*

[8] For an extensive treatment of the monetary issues addressed by temporary equilibrium theorists – which include the validity of the quantitative theory, the possibility of monetary authorities to manipulate the interest rate, and the existence of a 'liquidity trap' – the reader is referred to Grandmont (1983).

[9] The analysis that follows can, however, be readily extended to economies in which markets are active for more than two periods and households formulate their plans accordingly (cf. Grandmont, 1983, Ch. 1).

(b) *for all h,* $\omega_1^h >> 0$, $\omega_2^h >> 0$*;*

(c) $\overline{m}^h \geq 0$ *for all h,* $\Sigma_h \, \overline{m}^h = M > 0$;

(d) *for all h, the expectation function* Ψ^h *is continuous and such that* $\Psi^h(p_1) \in \Re^N_{++}$ *for every* $p_1 \in \Re^N_{++}$.

.

Note that by postulating that households expect strictly positive but finite monetary prices for period 2, part (d) rules out two circumstances under which there is no reason to transfer money to that period, namely the case in which households think that the future money prices of all commodities will be zero and the case in which they are certain that future commodity prices in terms of money will be infinite (i.e. that money will have no exchange value in period 2).

Let us now examine the behaviour of households at the beginning of period 1. Given the ruling prices and the associated expected prices, the generic household *h* must choose a most preferred two-period consumption stream out of those it believes it can attain in view both of the value of its commodity endowments and of the possibility of transferring money to period 2. It can be stated in formal terms that at any given price system $p_1 \in \Re^N_{++}$, the generic household *h* must solve the following problem:

[4.I] *Maximise* $U^h(x_1^h, x_2^h)$ *with respect to* $x_1^h \geq 0$, $m_1^h \geq 0$, $x_2^h \geq 0$,

subject to the current and expected budget constraints:

$$p_1 \, x_1^h + m_1^h = p_1 \, \omega_1^h + \overline{m}^h \tag{4.1}$$

$$\Psi^h(p_1) \, x_2^h = \Psi^h(p_1) \, \omega_2^h + m_1^h \tag{4.2}$$

where the choice variable m_1^h denotes the amount of money demanded in the present and carried over to period 2 in order to finance future consumption. Note that m_1^h must be non-negative because, by assumption, the household cannot borrow money in period 1. Note also that the household does not plan to demand money in period 2, as it is aware that economic activity is going to cease at the end of that period.[10]

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[10] It should be noted that the lack of incentives to demand money in period 2 creates a problem, because if the generic household thought that all the other agents would also abstain from demanding money in that period, it could not reasonably expect money to have a positive exchange value in the future as

Discussion of the solution to problem [4.I] will be facilitated by focusing on budget constraints (4.1)–(4.2). To begin with, it should be noted that the outcome of the problem remains the same if those constraints are modified by replacing the equality signs with inequality signs, since U^h is strictly increasing. We can therefore consider the modified budget constraints

$$p_1\, x_1^h + m_1^h \leq p_1\, \omega_1^h + \overline{m}^h$$

$$\Psi^h(p_1)\, x_2^h \leq \Psi^h(p_1)\, \omega_2^h + m_1^h$$

On adding up the modified constraints and eliminating m_1^h, it becomes clear that the consumption stream chosen by the household must fulfil the inequality

$$p_1\, x_1^h + \Psi^h(p_1)\, x_2^h \leq p_1\, \omega_1^h + \Psi^h(p_1)\, \omega_2^h + \overline{m}^h \qquad (4.1')$$

which we shall call the *intertemporal budget constraint* of household *h*. On the other hand, we know that m_1^h must be non-negative, i.e. that the household cannot borrow money in period 1. This means that the consumption stream chosen by the household must also fulfil the inequality

stated by Ass. 1(d). Moreover, it can be argued that the same problem arises when the two-period model put forward in the text is extended to economies that are active for a higher but *finite* number of periods. To clarify this point in intuitive terms, assume (a) that economic activity comes to an end in an arbitrarily given period $T > 2$ and (b) that all agents are aware of that future event. Then consider a generic household operating in the economy at the beginning of period 1. Under assumptions (a)-(b), the household would calculate that no agent will want to hold money balances in the terminal period *T* and that, for this reason, money will have no exchange value in that period. Moreover, the household would calculate that at the beginning of period *T*-1 all agents in the economy will similarly realise that money is going to be worthless in the terminal period. The household would accordingly conclude that no agent will want to hold money balances in period *T*-1 and that, as a result, money will be worthless in that period too. By further pursuing this line of reasoning, the generic household would eventually conclude that money will be worthless in every future period. In order to avoid the problem under discussion, the temporary equilibrium model with 'money' should therefore be modified by assuming that economic activity extends *indefinitely* over time. Within that context, the fact that human life has limited duration could be taken into account by assuming that two generations of households co-exist in the economy in every period of time, an 'older' generation initially endowed with the whole money stock and a 'younger' generation demanding money in the belief that the new younger generation will do the same in the subsequent period (cf., for example, Grandmont and Laroque, 1973). The structure of the temporary equilibrium model with 'money' would thus become more complex, since the maximisation problem attributed to the younger generation should be neatly distinguished from that attributed to the older one. There is no need to introduce this complex construction for our purposes, however, as the conditions ensuring the existence of temporary monetary equilibrium would remain essentially the same as those emerging from the simple model examined in this section.

$$p_1\, x_1^h \;\leq\; p_1\, \omega_1^h \;+\; \overline{m}^{\,h} \qquad (4.2')$$

which we shall call the *liquidity constraint* of household *h*.[11] It can be stated in the light of these considerations that problem [4.I] can be solved in two steps. In the first, the generic household determines its optimal consumption stream $x_{12}^{h*} = (\, x_1^{h*} ,\; x_2^{h*} \,)$ by solving the problem

[4.II] *Maximise* $U^h\,(\, x_1^h ,\; x_2^h \,)$ *with respect to* $x_1^h \geq 0,\; x_2^h \geq 0,$

subject to constraints (4.1')–(4.2')

In the second, it determines its optimal demand of money m_1^{h*} through the condition $m_1^{h*} \;=\; p_1\, \omega_1^h + \overline{m}^{\,h} - p_1\, x_1^{h*}$.

Let us focus on problem [4.II] and define the *opportunity set* of household *h* as the set of two-period consumption streams in $X_{12}^h \;=\; \Re_+^{2N}$ that fulfil both the constraints (4.1')-(4.2'). It is easily proved that this set is compact and convex under the assumption that both the current and the expected prices are strictly positive.[12] Given that U^h is strictly quasi-concave, it follows from the properties of the opportunity set that problem [4.II] *uniquely determines* the consumption stream $x_{12}^{h*} \;=\; (\, x_1^{h*} ,\; x_2^{h*} \,)$ chosen by household *h* at any given $p_1 \in \Re_{++}^N$. In these circumstances, the amount of money demanded by the household is itself uniquely determined in the second step of the procedure. We therefore conclude that both the household's current consumption demand x_1^{h*} and its money demand m_1^{h*} can be represented as functions of (strictly positive) vectors of current prices, which we shall denote by $x_1^h\,(p_1)$ and $m_1^h\,(p_1)$

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[11] Constraints (4.1')-(4.2') can be interpreted in economic terms as follows. Assume for the moment that household *h* is not only able to transfer money balances to period 2 but also to borrow money at no interest in period 1 within the limit set by its expected future wealth. It is easy to ascertain that in these circumstances, the household will only be subject to the *intertemporal budget constraint* (4.1'). Since we are assuming that the amount of money the household can actually borrow is zero, however, the *liquidity constraint* (4.2') on current consumption expenditure must also be introduced.

[12] Key: under the assumption mentioned in the text, the opportunity set is the intersection of two convex and compact sets.

respectively. In this notation, the *optimal action* $a^{h*} = (x_1^{h*}, m_1^{h*})$ taken by household h at any given $p_1 \in \Re^N_{++}$ is univocally identified by the function $a^h(p_1) = (x_1^h(p_1), m_1^h(p_1))$.

Let us now consider the first period *excess demand function* of the generic household h, defined as $z_1^h(p_1) = x_1^h(p_1) - \omega_1^h$, and the household's money demand function $m_1^h(p_1)$. It can be proved that they are both *continuous* functions (Grandmont, 1983: App. B, p. 165). It should also be noted that since the household's optimal choice must fulfil budget constraint (4.1), the equality $p_1 z_1^h(p_1) + m_1^h(p_1) = \overline{m}^h$ necessarily holds at every strictly positive vector of current prices. It follows from this last consideration that first period *aggregate* excess demands satisfy *Walras's Law*:

$$p_1 \Sigma_h z_1^h(p_1) + \Sigma_h m_1^h(p_1) = \Sigma_h \overline{m}^h = M \qquad \textit{for every } p_1 \in \Re^N_{++} \tag{4.3}$$

Given the above formal description of the behaviour of households, a *temporary monetary equilibrium* of the exchange economy for period 1 can be finally defined as a system of monetary prices $p_1^* \in \Re^N_{++}$, and a corresponding set of optimal actions on the part of households, such that the following market-clearing conditions are simultaneously satisfied:

$$\Sigma_h z_1^h(p_1^*) = 0\,, \quad \Sigma_h m_1^h(p_1^*) = \Sigma_h \overline{m}^h = M \tag{4.4}$$

Let us now address the question of the *existence* of temporary monetary equilibrium. It must be stated in this connection that existence is *not* guaranteed under Ass. 4.1. To clarify this point, we shall argue in steps by focusing on the simplified case of an exchange economy with a single consumption good ($N = 1$). In this case, the opportunity set of the generic household h at an arbitrary strictly positive price $\overline{p}_1$ for the consumption good can be represented as in Figure 1.

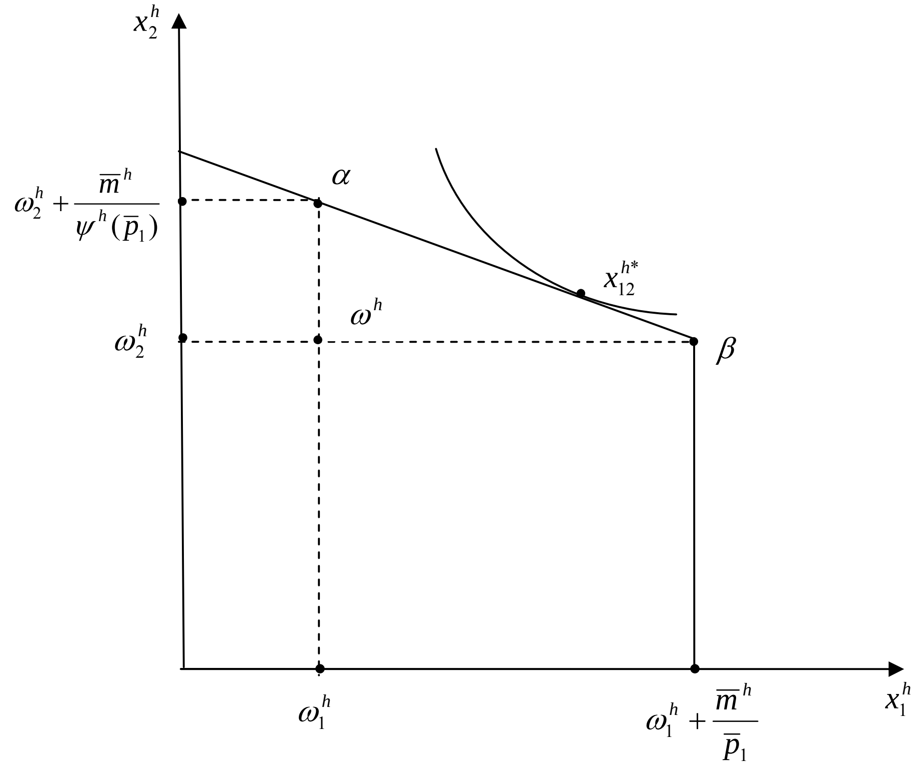

Figure 1

By examining constraints (4.1')-(4.2') taken for $N=1$, it is easy to ascertain that the line going through points α and β, whose slope is $\overline{p}_1/\Psi^h(\overline{p}_1)$, represents the *intertemporal budget constraint*, while the vertical half-line going through β represents the *liquidity constraint*. The optimal choice of household *h* therefore corresponds to point x_{12}^{h*}, which in turn identifies both the household's current excess demand for the consumption good and the household's demand for money balances. We shall now use this graphic device to analyse how the optimal choice of the generic household changes as the current price for the consumption good changes from $\overline{p}_1$. We shall only deal with a rise in the price, as the analysis that follows is easily adapted to the case of a fall.

Let us first assume that the household has *unit elastic price expectations*, i.e. that $\Psi^h(\lambda\, p_1) = \lambda\, \Psi^h(p_1)$ for every positive value of p_1 and every positive number λ. In this case, an increase in the current price of the consumption good from $\overline{p}_1$ to $\lambda\,\overline{p}_1$, $\lambda > 1$, causes both the intertemporal budget line and the liquidity line to move to the left, without however altering the slope of the former. What basically happens is that the price rise proportionately reduces the purchasing power of the household's money endowment while leaving the 'relative price' of present consumption in terms of future consumption unchanged at the initial level $\bar{p}_1 / \Psi^h(\bar{p}_1)$. The rise in the current price thus generates a *real balance effect* that will in turn normally reduce the current demand for the consumption good (Figure 2).

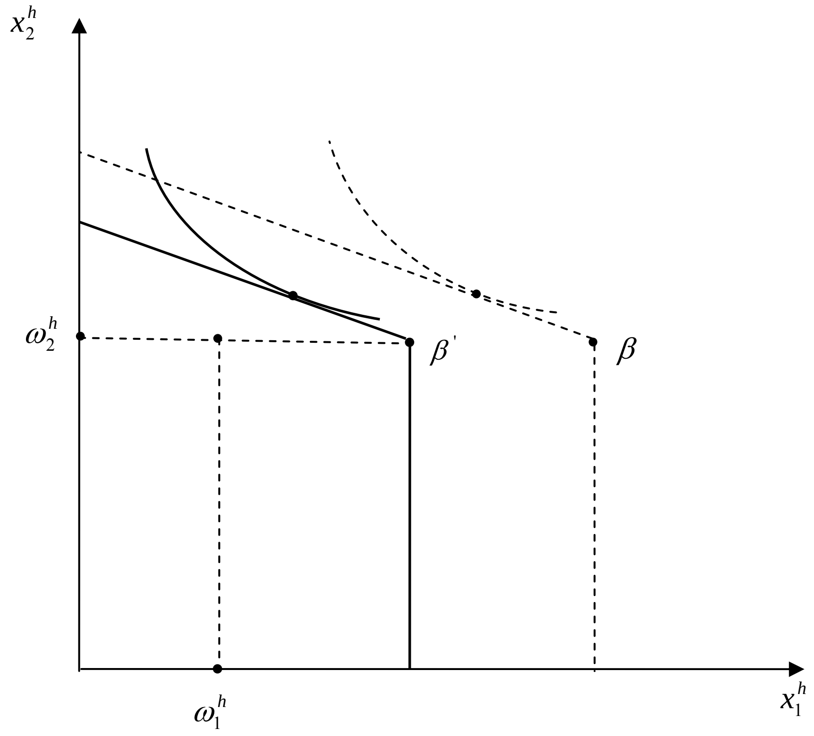

Figure 2

Let us now assume instead that the household's expectations are not unit elastic, i.e. that $\Psi^h(\lambda p_1) \neq \lambda \Psi^h(p_1)$ for every $p_1 > 0$ and every $\lambda > 0$. In these circumstances, the change in the opportunity set generated by an increase in the current price from $\bar{p}_1$ to $\lambda \bar{p}_1$, $\lambda > 1$, can be broken down into two 'successive' changes. The first is the shift to the left of both the intertemporal budget line and the liquidity line that would take place *if* expectations were unit elastic. This is precisely the real balance effect mentioned above. The second is the *rotation* of the intertemporal budget line around the new point β' due to the fact that the 'relative price' of present consumption in terms of future consumption must now change with respect to its initial level. This change in the relative price will further affect the household's choice by giving rise to an *intertemporal substitution effect*. If the elasticity of expectations is lower than 1 ($\Psi^h(\lambda \bar{p}_1) < \lambda \Psi^h(\bar{p}_1)$), the intertemporal budget line rotates upward and the intertemporal substitution effect is likely to reduce present consumption, thereby reinforcing the real balance effect (Figure 3). If elasticity is higher than 1, the intertemporal substitution effect is likely to act in the opposite direction and the overall effect of the price rise on the household's choice cannot be assessed *a priori*.

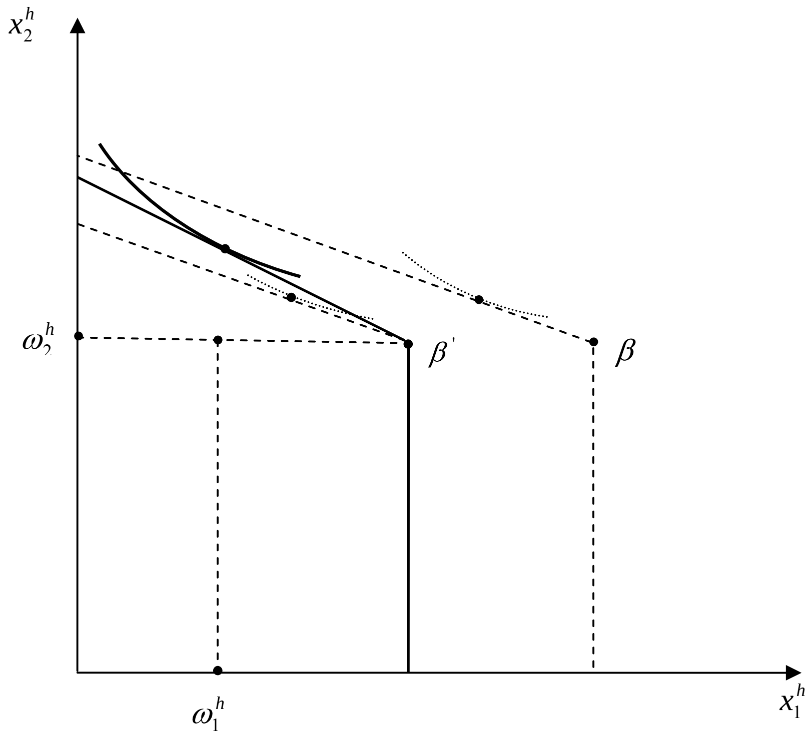

Figure 3

In the light of the above analysis, we can now discuss the existence of temporary monetary equilibrium for the one-commodity exchange economy. We shall show first of all that existence is *not* guaranteed when expectations 'depend too much' on the currently observed price. This will be done by means of two examples taken from Grandmont (1983: 22-24), in which it is implicitly assumed that $\bar{m}^h > 0$ holds for all h.

Example 1. A preliminary remark is called for in connection with the first example. Recall that the preferences of households can be represented by strictly increasing and strictly quasi-concave utility functions. It should thus be clear from Figure 4 that at any given $\overline{p}_1 > 0$, the current consumption demand of the generic household *h* will exceed the endowment ω_1^h if and only if the 'relative price' $\overline{p}_1 / \Psi^h(\overline{p}_1)$ is lower than the household's marginal rate of substitution evaluated at point $\alpha = (\omega_1^h,\ \omega_2^h + \overline{m}^h / \Psi^h(\overline{p}_1))$.

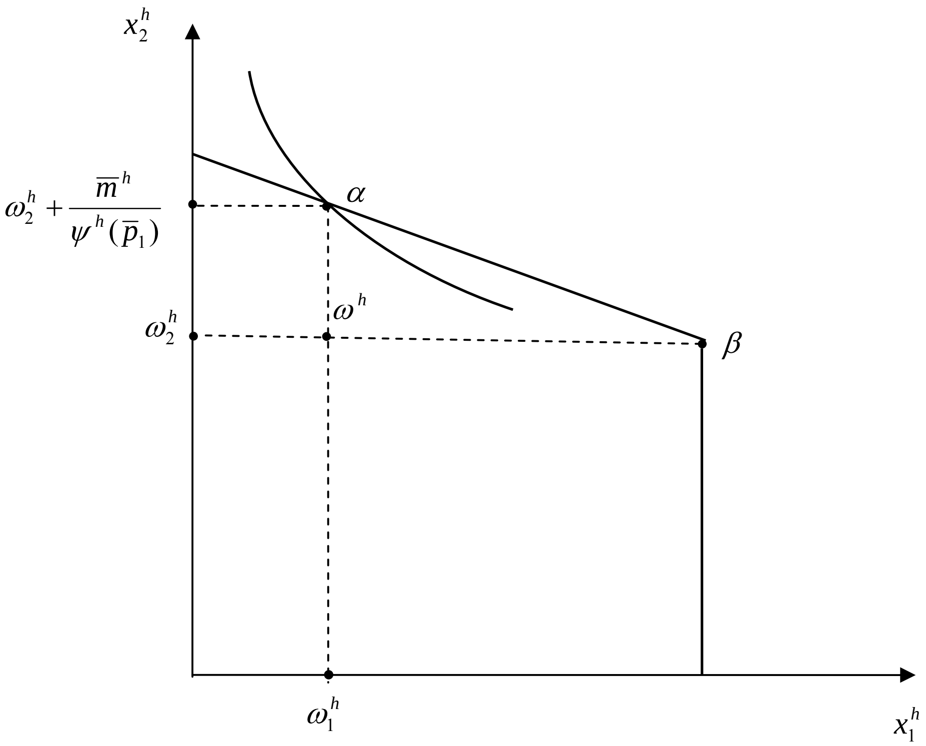

Figure 4

Having established this preliminary result, let us assume that the households' utility functions can be written $w(x_1^h) + \delta^h w(x_2^h)$, $h = 1, \ldots, H$, where $w(\ .)$ is strictly concave and differentiable and $0 < \delta^h < 1$ for all *h*. Let us further assume that the expectation function of household *h* is such that the following condition holds:

$$\frac{p_1}{\Psi^h(p_1)} \leq \frac{w'(\omega_1^h)}{\delta^h w'(\omega_2^h)} \qquad \textit{for every } p_1 > 0 \tag{4.5}$$

where the term on the right-hand side of the inequality is the household's marginal rate of substitution evaluated at the endowment point $\omega^h = (\omega_1^h,\ \omega_2^h)$. Given that the marginal rate of substitution increases as we move upward along the vertical half-line with origin (ω_1^h, 0), since $w(\,.)$ is strictly concave, it becomes clear that at any positive value of the current price, the household will be precisely in the position depicted in Figure 4, and will therefore manifest an excess demand for the consumption good in period 1. If we finally assume that expectation functions are such that condition (4.5) holds *for all h*, it is clear that at every $p_1 > 0$, there will be an aggregate excess demand on the current commodity market, which will be accompanied in accordance with Walras's law by a corresponding aggregate excess supply of money. This means that no temporary equilibrium exists for period 1 in which households are willing to hold the whole stock of money in circulation. In particular, the phenomenon described will occur when expectation functions are *unit elastic* and such that, for all *h*, the (constant) 'relative price' $p_1 / \Psi^h(p_1)$ fulfils condition (4.5).

Example 2. Let us assume that preferences are such that, for all *h*, the value of the marginal rate of substitution along the vertical half-line with origin (ω_1^h, 0) is bounded above by a strictly positive number v^h. Let us further assume that expectations are such that the following condition holds for all *h*:

$$\frac{p_1}{\Psi^h(p_1)} > v^h \qquad \textit{for every } p_1 > 0 \tag{4.6}$$

Under these assumptions, each household will be in a situation opposite to the one shown in Figure 4 at any given $\overline{p}_1 > 0$. As a result, $z_1^h(p_1) < 0$, $m_1^h(p_1) > \overline{m}^h$ will hold for all *h* at every $p_1 > 0$ and temporary monetary equilibrium will not exist. In particular, this phenomenon will occur when expectations are *unit elastic* and such that for all *h*, the (constant) 'relative price' $p_1 / \Psi^h(p_1)$ fulfils condition (4.6).

Similar examples could be constructed for economies with a variety of consumption goods (cf. Grandmont, 1983: 25). The simple examples put forward are, however, sufficient to develop the relevant economic considerations. To begin with, let us consider Example 1 on the hypothesis of unit elastic expectations. As we have seen, at an arbitrarily chosen $\overline{p}_1 > 0$ there is aggregate excess demand on the period 1 commodity market. We also know that an increase in the current price from $\overline{p}_1$ would generate a real balance effect that is likely to reduce that excess but cannot eliminate it completely. Similarly, on the hypothesis of unit elastic expectations, Example 2 shows that the real balance effect resulting from a fall in the current price may be not strong enough to compensate fully for an initial excess supply on the current commodity market. A negative conclusion therefore emerges from the temporary equilibrium model with 'money' as regards the effectiveness of the real balance effect as a mechanism capable of regulating the market. This negative conclusion attracted a great deal of attention when the original version of the model was published (Grandmont, 1974), as its was commonly held among neoclassical economists at the time that the real balance effect would normally ensure market clearing in economies endowed with outside money (cf., for example, Patinkin, 1965).

It should further be noted that conditions (4.5) and (4.6) in the examples presented are constraints on the variability of the 'relative price' $p_1/\Psi^h(p_1)$ and therefore impose limits on the strength of the intertemporal substitution effects that can be generated by changes in p_1. It may accordingly be conjectured that the introduction of restrictions on expectations capable of ensuring high variability of this 'relative price' could allow the intertemporal substitution effects engendered by changes in p_1 to become strong enough to reinforce the real balance effect and eliminate disequilibrium on current markets. The following argument indicates that this is a reasonable conjecture.

Assume that in the single-commodity exchange economy with 'money' there is a household $\overline{h}$ whose expectations are 'insensitive' to large changes in p_1, in the sense that two numbers $\varepsilon > 0$, $\eta > 0$ exist such that $\varepsilon \leq \Psi^{\overline{h}}(p_1) \leq \eta$ for every $p_1 > 0$ (*bounded* expectations). Under this assumption, it can be stated (a) that for p_1 large enough an aggregate excess supply appears on the period 1 commodity market and (b) that for p_1 low enough an aggregate excess demand appears on that market. By continuity, a price $p_1^* > 0$ should then exist that leads to equilibrium on the current commodity market and, in view of Walras's Law, on the money market as well.

In the above argument, statement (a) can be justified on the grounds that when p_1 rises indefinitely, point β of the 'insensitive' household's opportunity set tends to the endowment point $\omega^{\bar{h}}$ and the slope of the intertemporal budget line rises with no limit. As a result, the household's planned demand for future consumption tends to infinity together with the household's current demand for money. Since by assumption the money demand of every household is bounded below by zero, this means that as p_1 progressively increases, an aggregate excess demand for money must eventually appear in period 1 with a corresponding aggregate excess supply on the current commodity market. As regards statement (b), note that when p_1 falls progressively towards zero, point β in the insensitive household's opportunity set shifts indefinitely to the right, while the slope of the intertemporal budget line tends to zero, thus giving rise to a strong substitution effect in favour of current consumption. As a result, the insensitive household's demand for present consumption tends to infinity. Since the current consumption demand of the generic household *h* is bounded below by $-\omega_1^h$, this means that with the progressive fall in p_1, an aggregate excess demand must eventually appear on the current commodity market.

The above heuristic argument indicating that the assumption of *bounded expectations* ensures the existence of temporary monetary equilibrium can be rigorously formulated and generalised to exchange economies with any finite number of goods. The following theorem has indeed been proved for $N \geq 1$:

Theorem. Let Assumption 4.1 *hold in the exchange economy with 'money'. Assume further that there is at least one household* $\bar{h}$, *with* $\overline{m}^{\bar{h}} > 0$, *whose expectations are bounded in the sense that two vectors* $\varepsilon \in \Re^N_{++}$, $\eta \in \Re^N_{++}$ *exist such that* $\varepsilon \leq \Psi^{\bar{h}}(p_1) \leq \eta$ *for every* $p_1 \in \Re^N_{++}$. *Then a temporary monetary equilibrium exists.*

Proof: cf. Grandmont (1983: Appendix B).

The assumption of *bounded expectations* has been generalised to the case of monetary economies in which agents' predictions take the form of probability distributions over future prices (cf., for example, Grandmont, 1974). But is this really a *plausible* assumption? As noted, it postulates that price expectations are 'rigid' with respect to large variations in current prices and in particular that the price expected for any commodity remains practically unchanged when the current price keeps rising (falling) beyond a sufficiently high (low) level. It is quite doubtful, however, that expectations would normally display this property. As an expert in the field has pointed out, '[p]rice forecasts are indeed somewhat volatile, and are presumably quite sensitive to the level of current prices' (Grandmont, 1983: 26). On the other hand, the examples presented in this section show that temporary monetary equilibrium may not exist under such circumstances. The conclusion to be drawn is therefore that 'the existence of a [temporary] equilibrium in which money has positive value is somewhat problematic' (Grandmont, 1983: 27).

5. Conclusions

The studies in the field of temporary equilibrium theory carried out in the 1970s and early 1980s endeavoured to overcome the limitations of the Arrow-Debreu model by focusing analysis on economies in which forward markets are limited in number or non-existent and trade takes place sequentially over time. As we have seen, the models put forward in that period examine the behaviour of economic agents in an arbitrarily chosen 'initial period', stress the dependence of agents' decisions on their subjective price expectations, and analyse the conditions ensuring the existence of general equilibrium on current markets. According to the scholars active in the field, the analysis concerning this isolated period should be seen as the first step of a more extensive research programme, whose ultimate goal is to model the evolution of the economy as a *sequence* of temporary equilibria (Grandmont, 1977: 542–543; 1989: 299).

The simplified exposition presented in this paper indicates, however, that the very first step of the programme pursued by temporary equilibrium theorists gives rise to serious problems. In particular, the discussion developed in the previous sections shows that a major source of difficulties for the treatment of temporary equilibrium in a single market period is precisely the central role attributed to the subjective price expectations of economic agents. This point is first illustrated with reference to economies with a numéraire commodity. Section 2 focuses on the case of pure exchange economies and shows that substantial difficulties arise in the determination of households' behaviour if individual expectations are not sufficiently uniform. Section 3 addresses the case of economies with production and argues that the divergence of individual expectations creates additional difficulties in the treatment both of the formation of production decisions within firms and of the financing of production plans. Finally, section 4 goes on to consider monetary economies and shows that temporary equilibrium may not exist in those economies if expectations are overly sensitive to the level of current prices.

The above analytical difficulties may help to explain why research in the field of temporary equilibrium theory was abandoned about twenty-five years ago. More importantly, they afford some insight into why general equilibrium theorists have since chosen to study economies with sequential trade under the assumption of *correct* (or *self-fulfilling*) price expectations, for example along the lines indicated by Radner (1972). This assumption is normally taken for granted in advanced textbooks nowadays (cf., for example, Mas-Colell *et al.*, 1995: 696), even though it presupposes that economic agents have extraordinarily strong predictive capabilities. In this situation, basic knowledge of temporary equilibrium models makes us aware of the fact that modern general equilibrium theory can hardly dispense with that special assumption.

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Pubblicato in proprio
Dipartimento di Economia Pubblica
Facoltà di Economia
Università degli Studi di Roma “La Sapienza”
Via del Castro Laurenziano 9 – 00161 Roma

ISSN 1974-2940

Working Paper del Dipartimento di Economia Pubblica
Università degli studi di Roma "La Sapienza"
Via del Castro Laurenziano 9 – 00161 Roma

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