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Wicksell Effects, Demand for Capital and Stability

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I.1 Introduction

The dichotomic conception of capital in neo-classical theory:

i) **AGGREGATE CAPITAL** is expressed in value terms;

ii) **TECHNICAL CAPITAL** is expressed in physical terms.

Since capital enters into the technical conditions of production in a form different from that in which the demand for capital is conceived, then there is no guarantee about the direction of the following relationships:

A) between interest rate and demand for capital per worker in value terms;

B) between interest rate and net product per worker (in physical or value terms).

The inverse direction of the relationships (A) and (B) has been viewed as a possible cause of equilibrium instability.

I.2 Introduction

Some representative essays of **Garegnani** about capital and equilibrium instability:

[1] 'Heterogeneous Capital, the Production Function and the Theory of Distribution', *The Review of Economic Studies*, 37, 407-436. 1970.

[2] 'Notes on Consumption, Investment and Effective Demand, I and II', *Cambridge Journal of Economics*, 2, 335-53 and 3, 63-82. 1978 and 1979.

[3] 'Quantity of Capital', in J. Eatwell, M. Milgate and P. Newman (eds), *Capital Theory*, London: Macmillan. 1990.

[4] 'Savings, Investment and Capital in a System of General Intertemporal Equilibrium', in F. Hahn and F. Petri (eds), *General Equilibrium: Problems and Prospects*, London: Routledge. 2003. [Reprinted, with corrections, in R. Ciccone, C. Gehrke and G. Mongiovi (eds) *Sraffa and Modern Economics*, Oxon and New York: Routledge. 2011.]

II.1 The neo-classical theory of the rate of interest

The equilibrium considered here is the one Keynes described as follows in discussing the neo-classical theory of the rate of interest:

> this tradition has regarded the rate of interest as the factor which brings the demand for investment and the willingness to save into equilibrium with one another. Investment represents the demand for investible resources and saving represents the supply, whilst the rate of interest is the 'price' of investible resources at which the two are equated. Just as the price of a commodity is necessarily fixed at that point where the demand for it is equal to the supply, so the rate of interest necessarily comes to rest under the play of market forces at the point where the amount of investment at that rate of interest is equal to the amount of saving at that rate. (Keynes 1973 [1936]: 175)

II.2 The neo-classical theory of the rate of interest

Stated in formal terms, both the (gross) investments and the (gross) savings per worker are regarded as functions of the rate of interest, respectively: $v(r)$ and $v^s(r)$.

Because of the usual market mechanism, the rate of interest r is assumed to increase when $v(r) > v^s(r)$ and to decrease when the opposite is true. Therefore, given a sign-preserving function h(.), we have the following differential equation:

$$\frac{dr}{dt} = h\left[v(r) - v^s(r)\right] \tag{1}$$

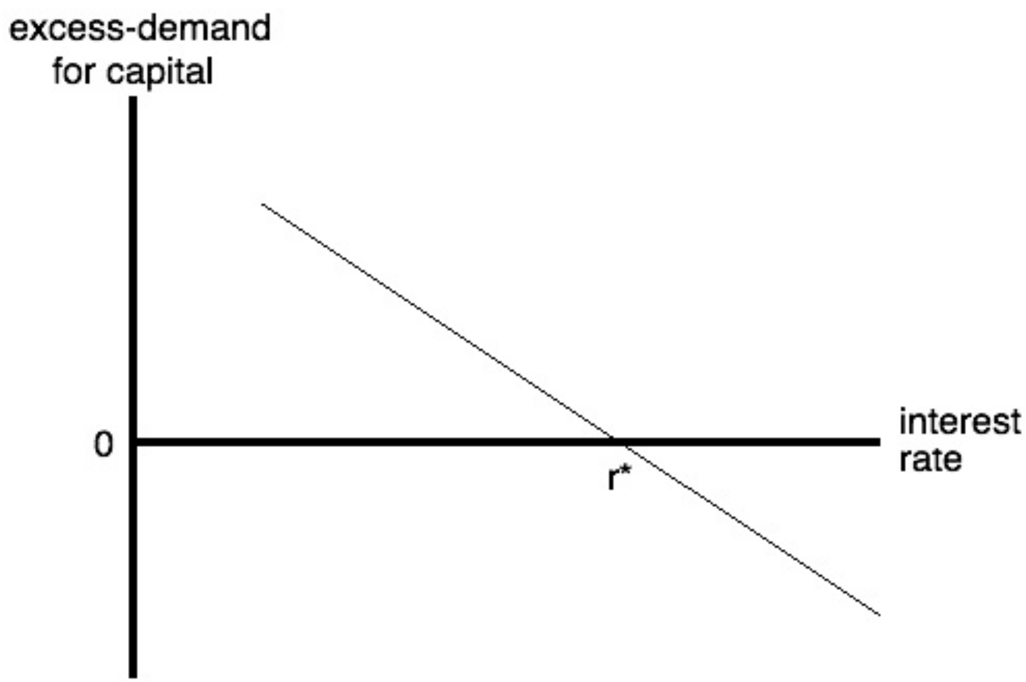
Given the above equation, in accordance with classical mechanics, we shall call equilibrium an interest rate level r^* such that $dr/dt = 0$, i.e. $v(r^*) = v^s(r^*)$.

II.3 The neo-classical theory of the rate of interest

Let r* be an equilibrium – i.e. $v(r^*) = v^s(r^*)$ – following the standard arguments we say that r* is locally (asymptotically) stable if the excess demand for capital and the interest rate vary in opposite directions in a neighbourhood of r*.

Local stability:

$$\left.\frac{dv}{dr}\right|_{r^*} - \left.\frac{dv^s}{dr}\right|_{r^*} < 0$$



III.1 Production and Investment Decisions

We consider an economy with n products. The commodity labelled [1] is both a consumption good and a circulating capital good, while the other commodities – labelled [2], [3], ..., [n] – are pure (circulating) capital goods.

A continuum of possible techniques of production is available. Each technique is characterized by an n×n matrix $A(\theta)$ and an n-vector $\ell(\theta)$, for every $\theta \in \Theta$, with $\Theta = \{\theta \in R: 0 \leq \theta \leq 1\}$, such that $a_{ij}(\theta) \geq 0$ and $\ell_i(\theta) > 0$ are, respectively, the quantity of commodity [j] and the amount of labour employed in the production of one unit of commodity [i].

Assumption 1. *The functions* $a_{ij}(\theta)$ *and* $\ell_i(\theta)$ *are continuous and at least twice differentiable on the set* S, *with* $S = \{\theta \in R: 0 < \theta < 1\}$.

III.2 Production and Investment Decisions

For each technique θ, there exist a scalar y(θ) and a (row) vector q(θ) that are respectively the net product of commodity [1] per worker and the vector of activity levels generating it. These can be obtained by solving the following equations:

$$y(\theta)\cdot e_1 = q(\theta)\cdot[I - A(\theta)] \quad (2)$$

$$1 = q(\theta)\cdot \ell(\theta) \quad (3)$$

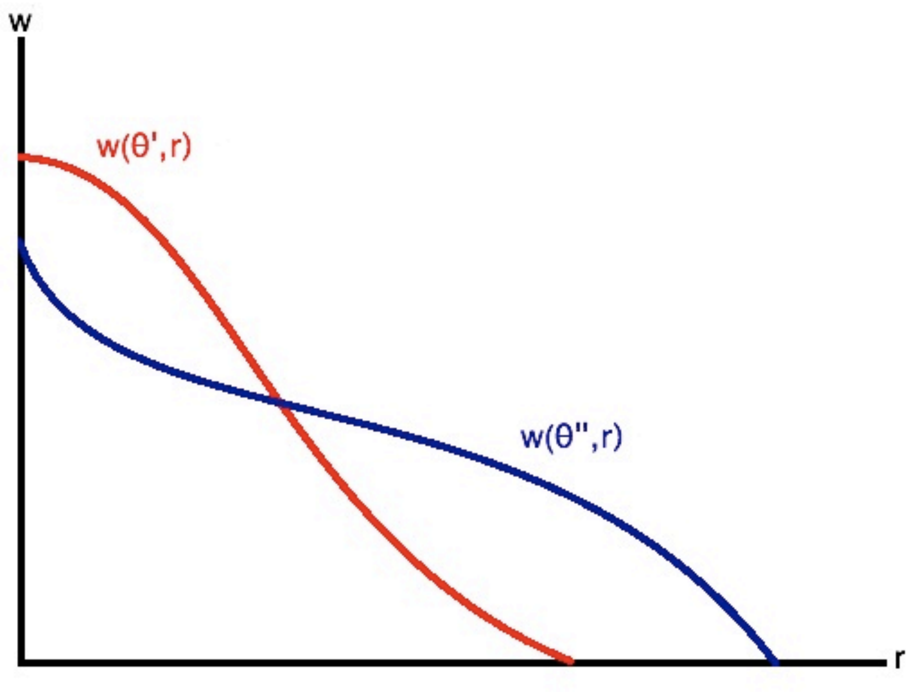
Assumption 2. No technique in Θ is dominated. That is, for every technique θ∈Θ, there exists no other technique θ' such that y(θ') > y(θ), q(θ')•A(θ') ≤ q(θ)•A(θ) and q(θ')•ℓ(θ') ≤ q(θ)•ℓ(θ).

Assumption 3. The techniques in Θ are labelled in such a way that y(θ'') > y(θ') whenever θ'' > θ'.

III.3 Production and Investment Decisions

Let us assume commodity [1] as the numéraire and denote with p(θ,r) and w(θ,r) respectively the price vector and the wage rate that make the unit cost vector equal to the price vector with the technique θ and the interest rate r.

The function w(θ,r) is known as the wage-interest function or curve for technique θ.



III.4 Production and Investment Decisions

According to a well-known result, given an interest rate r (taken within a certain interval), the technique $\theta°$ is optimal if and only if $w(\theta°,r) \geq w(\theta,r)$ for every $\theta \in \Theta$.

Therefore, solving the following maximisation problem:

$$\begin{cases} \max\limits_{\theta} \quad w(\theta,r) \\ \text{s.t.:} \ \theta \in \Theta \end{cases} \tag{4}$$

with r considered parametrically between 0 and a certain maximum, makes it possible to express the optimal technique as a function of the rate of interest: $\theta° = \theta(r)$.

III.5 Production and Investment Decisions

Once the function $\theta° = \theta(r)$ is known, all the magnitudes that were formerly functions of the technique in use now become functions of the rate of interest.

While $y(\theta)$, $q(\theta)$, $w(\theta,r)$ and $p(\theta,r)$ are the net and gross product, the wage rate and the price system with a possible technique θ, $y(r) = y[\theta(r)]$, $q(r) = q[\theta(r)]$, $w(r) = w[\theta(r),r]$ and $p(r) = p[\theta(r),r]$ are the corresponding variables with the optimal technique at the rate r.

➔Vector of capital goods per worker with a technique θ: $k(\theta) = q(\theta) \bullet A(\theta)$

➔Vector of capital goods per worker with the optimal technique: $k(r) = k[\theta(r)]$

III.6 Production and Investment Decisions

Since k(r) is the vector of the employment of (circulating) capital goods per worker with the optimal technique for the interest rate r and p(r) is the price vector with this technique and rate of interest, the (gross) investment decisions (per worker) can be therefore defined as follows:

$$v(r) = k(r) \cdot p(r) \tag{5}$$

The change in the investment decisions per worker due to a variation of the interest rate can be decomposed into a *real Wicksell effect* and a *price Wicksell effect*:

$$\frac{dv}{dr} = \underbrace{\frac{dk}{dr} \cdot p(r)}_{\text{real effect}} + \underbrace{k(r) \cdot \frac{dp}{dr}}_{\text{price effect}} \tag{6}$$

IV.1 Reswitching and Real Wicksell Effect

Reswitching can occur: it is possible for a technique to be optimal for two different levels of the interest rate but not for the levels between them.

Reswitching implies a non-monotonic shape of the function y(r). This is particularly evident in our model, because there is a one-to-one correspondence between the technique in use and the amount of net product per worker (assumption 3).

When $r = 0$, the optimal technique is $\theta = 1$, i.e. the one with the greatest net product per worker. This implies that an increase in the rate of interest initially entails a decrease in the net product per worker.

After the initial decreasing stretch, the pattern of the function y(r) becomes unpredictable. Since every technique can be brought back into use several times, the function y(r) can exhibit alternatively decreasing and increasing stretches.

IV.2 Reswitching and Real Wicksell Effect

The sign of the variation in the net product per worker due to a rise in the rate of interest is closely linked to that of the real Wicksell effect. This can be easily proved.

Let us consider an interest rate r and denote by θ° the corresponding optimal technique, while y° and k° are the net product and the vector of capital goods, both per worker, when θ° is in use. Since technique θ° entails zero (extra)profits and is profit maximizing for r, p (r) and w(r), then:

$$y^\circ - r \cdot [k^\circ \cdot p(r)] - w(r) = 0 \tag{7}$$

$$\left(y^\circ + \frac{dy}{d\theta}\right) - r \cdot \left[\left(k^\circ + \frac{dk}{d\theta}\right) \cdot p(r)\right] - w(r) = 0 \tag{8}$$

IV.3 Reswitching and Real Wicksell Effect

Therefore, substituting equation (7) in equation (8), we have:

$$\frac{dy}{d\theta} - r \cdot \frac{dk}{d\theta} \cdot p(r) = 0 \qquad (9)$$

Bearing in mind that y(r) = y[θ(r)] and k(r) = k[θ(r)], this clearly implies:

$$\frac{dy / d\theta}{d\theta / dr} = r \cdot \frac{dk / d\theta}{d\theta / dr} \cdot p(r) \qquad (10)$$

Therefore, for $r > 0$, the real Wicksell effect is negative if and only if $dy/dr < 0$.

As a result, the reswitching of techniques entails a positive, i.e. anti-neo-classical, real Wicksell effect.

V.1 Saving Decisions

We consider an overlapping-generation model with identical individuals whose life lasts for two periods.

During the first period of life, each individual is a worker and inelastically supplies one unit of labour. As a consequence, the individual’s income in this period is equal to the wage rate.

In the second period, the individual becomes unable to work, and therefore her/his consumption depends on the part of the wage rate saved during the first period.

Therefore, the wage rate is the only individual intertemporal income.

V.2 Saving Decisions

Denoting by $u(x_1, x_2)$ a consumer utility function with the customary properties, we have the following utility maximisation problem:

$$\begin{cases} \max \;\; u(x_1, x_2) \\ \text{s.t.:} \;\; w = x_1 + \dfrac{x_2}{1+r} \end{cases} \tag{11}$$

whose solution is $x_1(w,r)$ and $x_2(w,r)$.

The optimal saving decision per worker is therefore:

$$v^s(r) = w(r) - x_1[w(r),r] \tag{12}$$

V.3 Saving Decisions

What can we say about the saving function $v^s(r) = w(r) – x_1[w(r),r]$ *?*

Because of the usual substitution effect, an increase in the rate of interest (with a fixed w) should cause the consumption of the first period x_1 to decrease with respect to the consumption of the second period x_2, but since there is also an income effect, x_1 may very well increase when the rate of interest increases.

Moreover, the wage rate is inversely proportional to the rate of interest, and an increase in the latter will therefore decrease the income out of which savings are made.

VI.1 Equilibrium and Stability

As already stated at the beginning, given the dynamic process initially described by equation (1), an equilibrium is an interest rate r^* such that: $v(r^*) = v^s(r^*)$.

Let us assume that at least an equilibrium r* exists. We say that it is locally (asymptotically) stable if:

$$\left.\frac{dv}{dr}\right|_{r^*} - \left.\frac{dv^s}{dr}\right|_{r^*} < 0 \qquad (13)$$

The equilibrium r* is locally stable if the excess demand for capital and r vary in opposite directions in a neighbourhood of r*.

VI.2 Equilibrium and Stability

The investment function derivative can be always decomposed into a real Wicksell effect and a price Wicksell effect:

$$\left.\frac{dv}{dr}\right|_{r^*} = \underbrace{\left.\frac{dk}{dr}\right|_{r^*} \cdot p(r^*)}_{\text{real effect}} + \underbrace{k^* \cdot \left.\frac{dp}{dr}\right|_{r^*}}_{\text{price effect}} \tag{6bis}$$

The two effects can be either positive or negative and the same or opposite in sign, so that dv/dr can be negative even in a case with a positive real Wicksell effect (cf. Fratini 2010 for an example).

VI.3 Equilibrium and Stability

On the contrary, savings manifest themselves as a pure amount of value with no specified physical shape, and can thus take every possible form.

When an equilibrium is reached, however, savings are and must be converted into a precise system of (real) assets: the equilibrium vector of capital goods k* = k(r*). Therefore, in a small neighbourhood of r* only, we can therefore decompose the variation of savings into an “asset value effect” and a “residuum” z:

$$\left.\frac{dv^s}{dr}\right|_{r^*} = \underbrace{k^* \cdot \left.\frac{dp}{dr}\right|_{r^*}}_{\text{asset value effect}} + z \qquad (14)$$

VI.4 Equilibrium and Stability

The local equilibrium stability is affected by the real Wicksell effect and by the residuum only, while the price Wicksell effect has no relevance because it is compensated for exactly by the asset value effect:

$$\left.\frac{dv}{dr}\right|_{r^*} - \left.\frac{dv^s}{dr}\right|_{r^*} = \underbrace{\left.\frac{dk}{dr}\right|_{r^*} \cdot p(r^*)}_{\text{real effect}} + \underbrace{k^* \cdot \left.\frac{dp}{dr}\right|_{r^*}}_{\text{price effect}} - \underbrace{k^* \cdot \left.\frac{dp}{dr}\right|_{r^*}}_{\text{asset value effect}} - z = \qquad (11)$$

$$= \underbrace{\left.\frac{dk}{dr}\right|_{r^*} \cdot p(r^*)}_{\text{real effect}} - z$$

Even though stability depends on the real Wicksell effect, a positive real Wicksell effect is in general neither a necessary nor a sufficient condition for instability. This is due to the presence of the residuum z, which can determine stability or instability independently of the real Wicksell effect.

VII.1 Real Wicksell Effect and Stability

As already proved, in the model we are considering, the real Wicksell effect can be positive if and only if re-switching occurs.

The argument can be summed up as follow:

1) Sign $\frac{dk}{dr} \cdot p(r)$ = sign $\frac{dy}{dr}$. Intuitively, an increase of the employment of technical capital per worker involves an increase of the net product per worker.

2) In the model we are considering (thanks to ass. 3), there is a one-to-one correspondence between the technique in use and the amount of net product per worker.

3) Starting from r = 0, an increase in the rate of interest initially entails a decrease in the net product per worker, and therefore dy/dr > 0 (i.e. real effect > 0) iff re-switching occurs.

VII.2 Real Wicksell Effect and Stability

A positive real Wicksell effect is in general neither a necessary nor a sufficient condition for instability. The relevance of the real Wicksell effect for equilibrium stability can, however, be studied by imposing some restrictions on the residuum.

In particular, if we assume $z \geq 0$ (which means that the change in savings due to an increase in r is equal to or greater than the asset value effect) then the positive real Wicksell effect becomes a necessary, but not sufficient, condition for instability.

If we assume $z = 0$ (i.e. that savings vary exactly as much as the value of the equilibrium vector of assets, at least in a small neighbourhood of r*), then the equilibrium can be locally unstable if and only if the real Wicksell effect is positive.

VII.3 Real Wicksell Effect and Stability

The stability of the equilibrium between investments and savings depends on the real Wicksell effect and on the residuum. The shape of the investments or demand-for-capital curve appears instead irrelevant for stability.

In particular:

- a monotonically decreasing investment function can coexists with a positive real Wicksell effect (cf. Bidard 2010 and Fratini 2010);
- a positive real Wicksell effect can be cause of instability;
- instability due to re-switching is possible even in the case of a monotonically decreasing investment function.

VIII.1 Conclusions

The possibility of an increasing demand for capital schedule, at least in a certain stretch, which emerged as a result of the capital debate of the 1970s, has been viewed as a possible cause of instability for the equilibrium on the capital market.

This argument was mainly presented in terms of Wicksell's theory, where the exogenously given supply of capital is misleading for two reasons:

i) the property of stability seems to depend on the shape of the demand-for-capital curve alone rather than the shape of the excess-demand curve, as is usually the case.

ii) the value of any bundle of commodities cannot be consistently taken as given before income distribution and relative prices are determined and, as a result, numerical solutions of Wicksell's equations cannot be regarded as economically meaningful equilibria.

VIII.2 Conclusions

We have tried to put the possibility of instability arising from capital paradoxes on a different basis.

1) We used a particular device: we decomposed the change in savings due to a change in the interest rate into an “asset value effect” and a residuum, and since the asset value effect and the price Wicksell effect cancel out each other, we arrived at the conclusion that the local stability of the equilibrium ultimately depends on the real Wicksell effect and on the residuum.

2) In cases where the residuum is well-behaved (or even negligible), instability is therefore possible only if the real Wicksell effect is positive, as happens in the case of re-switching.

GRAZIE

THANK YOU

OBRIGADO

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GRACIAS

DANKE