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The supermultiplier as a vertically hyper-integrated sector

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Abstract
This paper tries to shed new light on demand-led growth from a disaggregated multiplier-accelerator model (*the supermultiplier*, so to speak). This mechanism was presented as an answer to Harrod's instability puzzles (Serrano, 1995; Bortis, 1997, Dejuán, 2005). The solution, however, was greeted with suspicion even among Sraffian and postKeynesian economists (Palumbo & Trezzini, 2003; Lavoie, 2010; Smith, 2013). A multisectoral setting of the supermultiplier model, as a variant of Pasinetti's vertically hyper-integrated sectors, may help to understand its meaning and the mechanisms through which it shapes the structure of the economy until the warranted rate of growth matches the autonomous trend. It also clarifies the limits of the autonomous trend and the conditions for stability in the lines opened by White (2009) and Allain (2013).

Keywords: Demand-led growth, multiplier-accelerator models, postKeynesianism

JEL Codes: B51, E22, O41

1. Introduction

The first attempt to dynamize Keynes' General Theory was led by (Harrod, 1939). He combined the multiplier and the acceleration in the simplest macroeconomic model where there is only induced consumption ($C_t=c_y \cdot Y_t$) and expansionary investment ($I_t=k_y \cdot g_d \cdot Y_t$). ($Y$ stands for income; c_y is the propensity to consume; k_y, the optimal "capital/final output" ratio; g_d, the expected growth of aggregate final demand). Harrod found the equilibrium rate of growth that ensured, year after year, the investment of full capacity savings ($g^*=(1-c_y)/k_y$) and labelled it "the warranted rate". If the expected growth of demand turned out to be equal to g^*, firms had warranted the sale of full capacity output. Otherwise, centrifugal forces would lead the economy to a collapse, either by explosion or implosion. How could a model leading to such results be credible?

(Serrano, 1995) found that the model was stable provided a non-capacity creating autonomous demand (Z_t) was included and it integrated the multiplier and accelerator mechanisms into the *supermultiplier*. The macro model becomes: $Y_t=C_t+I_t+Z_t = c_y \cdot Y_t+k_y \cdot (g_d \cdot Y_t)+Z_t = [1/(1-c_y-k_y \cdot g_d)] \cdot Z_t$. The term in the square brackets is the supermultiplier. An upward shift in the autonomous trend (g_z) is supposed to reshape the structure of the economy until demand expectations and the warranted rate adjust to the autonomous trend ($g_d=g_z=g^*$). This adjustment requires a structural change that amounts to an increase in the share of investment in final output (i_y) at the expense of the share of autonomous demand (z_y).

Bortis (1997), Dejuán (2005), Dejuán (2013), White, (2006) and Freitas & Dweck (2013) support this idea and use the supermultiplier for different purposes. Yet one should recognize that even among Sraffian and Kaleckian economists the supermultiplier model has provoked more suspicion than enthusiasm Palumbo & Trezzini (2003), Lavoie (2010), Smith (2013). Only recently, some of them have started to recognize the validity and advantages of the supermultiplier approach Cesaratto (2012), Allain (2013), Lavoie (2013).

Probably, the macroeconomic setting was not the best scenario to tell the story of the supermultiplier. It conceals the mechanisms that make possible structural changes. And it requires a number of heroic assumptions. A dynamic macroeconomic model can only raise questions whose answer implies a balanced growth of the economy. Otherwise we have to assume that all industries share the same technology. Suppose, to illustrate the point, that the basket of goods that defines autonomous demand consists of luxury cars for domestic (autonomous) consumption and food for exports. The traditional Keynesian multiplier allows us to compute the impact of the same percentage increase in both goods; not the increase in luxury cars on its own. The use of the aggregate supermultiplier requires, in addition, that this general increase coincides with the warranted rate of growth corresponding to the initial conditions. Otherwise we have to assume that all industries share the same technology. A heroic assumption, indeed!

This paper attempts to shed new light on demand-led growth from a multisectoral approach. It reinforces the main conclusions of the aggregate supermultiplier, namely the endogeneity of the warranted rate which adjust to the autonomous trend. And it does so in the proper framework: a multisectoral economy where changes in the industry composition are possible and plausible. The stability of the process, the stumbling block of the supermultiplier theory, is also clarified when we separate the impacts in different sectors. Our methodological contribution consists in differentiating the main economy from the complementary one. The first is the vertically hyper-integrated sector corresponding to autonomous demand and governed by the supermultiplier. The complementary part is the vertically integrated sector which produced the extra equipment required to shift to the new fully adjusted path of growth. We shall show that both subsystems are stable in their own and there is no feedback between the second and the first one.

Multisectoral models have a long tradition in growth theory, at least in the heterodox strands (Fel'dman (1928/1964), Leontief (1970/1986) and Löwe (1976)) are classical works in the field. From different perspectives they clarify the key role of the industry producing equipment and the need to expand its relative weight when the economy shifts to a faster path of growth. The supermultiplier, presented here as a variant of Pasinetti's vertically hyper-integrated sectors (Pasinetti, 1988), will help to trace the adjustments towards the exogenously altered growth path[1]. This is done in section 3.

Neumann (1945-46) is the classical reference in the literature of multisectoral growth. A reformulation of von Neumann's supply side model will help us to understand the limits of the autonomous trend in a demand-led growth model. He does not clarify, however, the stable convergence after a shock. To prove such stability we have relied on recent findings by White (2009) and Allain (2013). They show the importance of distinguishing between the expected growth of autonomous demand (what we have called the "autonomous trend", g_z), the expected growth of aggregate demand (g_d) and the expected grow of demand for the specific output of the industry we are considering (g_{dj}). From different methodologies they try to specify the range of the parameters that render stable a first order differential equations system. The economic rationale of such ranges is not evident. A disaggregated model, like the one we are using here, will show the economic rationale of the limits of autonomous growth and the conditions for stable growth. This will be done in section 4.

Prior to this (in section 2) we present the exogenous data, the implicit hypotheses and the basic equations of our disaggregated model. It could be labelled "CLAKESCH" because it combines the Classical theory of value that grants the autonomy of distribution (Sraffa, 1960), the Keynesian theory of output based on the principle of effective demand and the multiplier (Keynes, 1936; Kalecki, 1971) and the Schumpeterian emphasis on innovative entrepreneurs who are able to attract credit for their new projects (Schumpeter, 1912).

The numerical Appendix illustrates the models and their tools. It may help to follow the impacts of an acceleration of the autonomous trend to which we refer in the theoretical part of this paper. It also illustrates the downwards adjustment after a deceleration of the autonomous trend that it is not completely symmetrical.

2. Key hypothesis and relationships of the model.

2.1 General picture of the economy represented by its input-output table

We are going to analyze an economy that has been growing along a fully adjusted path of growth, i.e. with normal capacity utilization and constant proportions. All the relevant variables are growing at the warranted rate (g^*) corresponding to the initial conditions of the economy regarding technology and expenditure patterns.

[1] The process of adjustment to a different path of growth has been called "the traverse" by Hicks (1950). He was also the first to talk about the "supermultiplier".

Our multisectoral economy can be arranged in an input output table (IOT) with four industries producing four different commodities or baskets of commodities[2]. The vertical reading of the IOT informs about the process of production: $\boldsymbol{T}+\boldsymbol{v}=\boldsymbol{q}$. The horizontal reading, about the allocation of the produced commodities: $\boldsymbol{T}+\boldsymbol{y}=\boldsymbol{q}$. Here $\boldsymbol{q}$ is the column vector of total output; $\boldsymbol{y}$ is the column vector of final net output, the typical variable of macroeconomic models[3]; $\boldsymbol{v}$, the row vector of value added; $\boldsymbol{T}$ is the matrix of inter-industry transactions where we also include fixed capital consumption. We also know the labour employed in each industry (row vector $\boldsymbol{L}$) and the value of the fixed capital installed in each industry (matrix $\boldsymbol{KI}$ that can be represented by a row vector since we assume that fixed capital consists of the commodity called "equipment").

To be more precise, the sectoral composition of the economy is as follows:

- Industry 1 produces intermediate goods and capital goods for replacement. They are purchased by firms and depend on the actual level of output in each industry. They fill the first row of the inter-industry transaction matrix. Actually it is the only row of the original $\boldsymbol{T}$ with positive figures.
- Industry 2 produces commodities devoted to household's final consumption that is paid out of wages: $C=W$. This is induced consumption, the basis of the multiplier.
- Industry 3 produces traditional equipment that is purchased by firms willing to expand capacity. This is expansionary investment (I), the basis of the accelerator.
- Industry 4 produces for autonomous demand (Z). It encompasses commodities devoted to autonomous consumption, modernization investment, real public expenditure and exports. In this paper we shall usually identify it with modernization investment aimed at transforming productive capacity without enlarging it. Think, for instance, of wind turbines that will provide cleaner and cheaper sources of energy in the future[4]. We take as given the level of autonomous demand at the base year (Z_o) and its expected rate of growth (the autonomous trend, g_z). In our multiplier-accelerator model Z will be considered as the locomotive of the economy.

2.2 Technology and capacity utilization.

Competition compels firms, willing to maximize long-term profits, to install the best available technology and to operate it in the best conditions, i.e. at the normal rate of capacity utilization[5].

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[2] This is a simplification to make our presentation easier. In a multisectoral approach there is no limit to the number of industries to be considered and the possible rearrangements. It is even possible that the same commodity could appear in different baskets. Cars purchased by households would belong to the consumption industry; those purchased by firms to the equipment industry.

[3] "Total gross output" (q) is the typical variable of models disaggregated in an IOT. The passage from total to final magnitudes is immediate: $\boldsymbol{y}=\boldsymbol{q}(\boldsymbol{I}-\boldsymbol{A})$, where $\boldsymbol{A}$ is the matrix of technical coefficients and $\boldsymbol{I}$, the identity matrix.

[4] Once more, this is a simplification to make our presentation easier, especially regarding the finance of autonomous demand. In a multisectoral model there is no limit to the number of industries that may supply autonomous demand. Aggregating them into one means we are fixing the internal composition and/or that we are assuming the same technology across industries.

[5] Notice the different treatment of capacity and employment. Since the latter does not impinge upon profits, a "normal rate of employment" is not a condition for equilibrium either in the short or in the long term. Normal capacity, on the contrary, is a long run condition of equilibrium. The tendency towards a

Technology is given by three sets of coefficients that can be directly derived from the IOT: ***A*, *k*, *l***. Here $\boldsymbol{A}=\boldsymbol{T}\cdot<\boldsymbol{q}>^{-1}$ is the matrix of technical coefficients. Due to our aggregation procedure, only the first row of ***A*** has positive numbers. $\boldsymbol{k}=(\boldsymbol{KI})\cdot<\boldsymbol{q}>^{-1}$ is the row vector of fixed capital *directly* required per unit of output. $\boldsymbol{l}=\boldsymbol{L}\cdot<\boldsymbol{q}>^{-1}$ is the row vector of *direct* labour per unit of output.

For certain purposes it is convenient to operate with vertically integrated sectors. The coefficients corresponding to them can be derived by means of the Leontief inverse: $(\boldsymbol{I}-\boldsymbol{A})^{-1}$. Total labour per unit of final output is: $\boldsymbol{l}_v=\boldsymbol{l}\cdot(\boldsymbol{I}-\boldsymbol{A})^{-1}$. Total capital per unit of final output is: $\boldsymbol{k}_v=\boldsymbol{k}\cdot(\boldsymbol{I}-\boldsymbol{A})^{-1}$.

To adjust production to demand changes, the installed capacity can be operated for more or less hours per day. There are obvious limits. For sure, equipment cannot be used more than 24 hours a day. Engineers will set the technical limit some hours below ($u^{\wedge}$). Economists talk about a normal or optimal threshold: this is u^*, that we normalize at 1. It is associated to the maximum rate of profit free from the risk of losing customers when installed capacity is operating at $u^\wedge$ and there is a new burst of demand. It plays the role of a gravity centre because firms are interested in reverting to it in order to maximize profits in the long run[6].

Figure 1 illustrates these ideas that are the source of many controversies in demand-led growth models, even among postKeynesian strands. The actual rate of capacity utilization may be defined in different ways.

$$[1] \qquad u_j=\frac{KR_j}{KI_j}=\frac{q_j}{q_j^*}=\frac{h_j}{h^*}$$

The first expression computes the rate of capacity utilization in industry j in period t by the ratio between "required capacity (KR_j)" and "installed capacity (KI_j)". The second, by the ratio "current output (q_j)" and "capacity output (q_j^*)". The third, by the ratio between the hours per day that installed capacity is actually operated (h_j) and the desired hours that we assume equal across industries (h^*).

In the economy represented in figure 1, to which we shall turn later, the normal rate is 16 hours a day, five days a week (Monday to Friday). In the short run, to match peaks of demand without burning the machines out, firms can use 4 additional night hours. This implies a maximum rate per day u^=20/16=1,25. In this paper we shall reserve extra hours at the weekends to satisfy "transient demand". Our point is that a type of

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normal rate of utilization can be observed empirically. Dejuán (2013) illustrates, for OECD countries, two facts that reinforce the hypothesis defended here. (1) The difference between fast growing countries and slow ones is not reflected so much in the rate of utilization but in the share of investment. (2) The standard deviation of the rate of utilization is rather low. Overutilization occurs at the beginning of each boom but it fades away as soon as extra capacity is added.

[6] In the late eighties there was an interesting debate on "normal capacity utilization" between Kurz and Ciccone (Kurz, 1986; Ciccone, 1986, Ciccone, 1987). Here we rely on Kurz's idea that introduces the selection of normal capacity as part of the long run maximization procedure. Ciccone is right in assuming that firms plan investment on the basis of the regular fluctuations in demand which allows for the definition of the normal rate as the average of the cycle. There is no reason to accelerate investment in the peak of the cycle when u_t>1, since this possibility has already been taken into account in the investment decision. This conclusion does not hold, however, after permanent increases in demand like those which result from an acceleration of autonomous trend.

overutilization does not speed expansionary investment up, because it is considered a permanent flow.

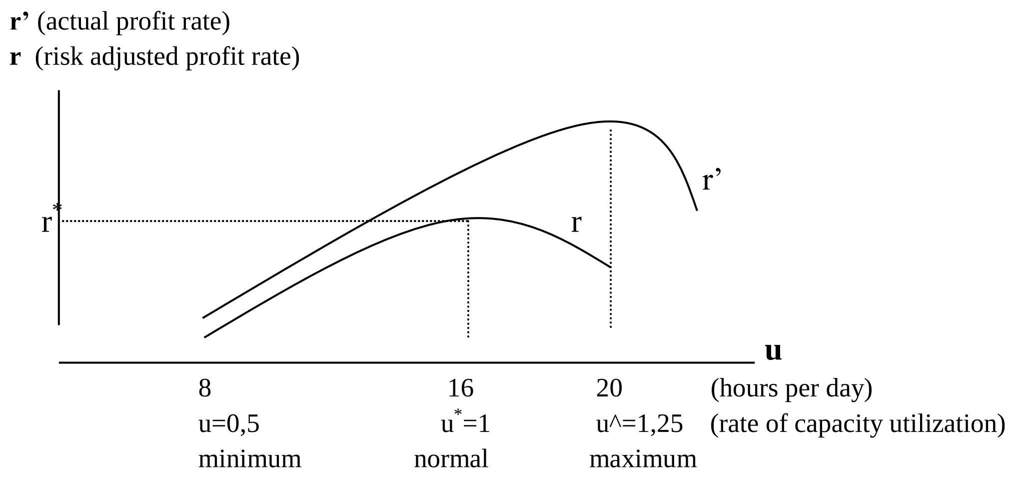


Figure 1. Setting the normal degree of capacity utilization.

2.3 Distribution, expenditure patterns and finance.

We take as given the real wage per worker inherited from the past (w). In principle it will increase in parallel to technical progress, although here we assume constant productivity. Given technology and the real wage, the Sraffian equations determine the prices of commodities in terms of the chosen *numeraire* (in our case, it will be the traditional equipment sector, p_3=1) (Sraffa, 1960)[7]. They also determine the *normal* (and uniform) rate of profit (r^*). To get the actual rate of profit in period t we have to multiply the normal one by the actual degree of capacity utilization: $r_t=r^* \cdot u_t$.

Given technology and the distributive variables, we can get the shares in total output of the wages and profits paid in each industry j: $\omega_j=w \cdot l_j$; $\beta_j=r \cdot k_j$. This result can be directly derived from the IOT, after dividing wages and profits by total output.

Although different paths of growth may entail changes in distribution in this paper we are interested in showing that the economy may shift to a faster path of growth with a constant distributive variables (w, r^*) and constant income shares (ω, β).

Expenditure patterns will also be kept constant. To simplify our exposition we shall rely on the extreme classical assumption: all wages are consumed, all profits are saved. Then, propensities to consume and save out of wages (w) and profits (r) are: c_w=1; c_r=0; s_w=0; s_r=1.

In our Keynesian model, aggregate profits and savings are determined by total investment, here identified with the production of ordinary equipment and modern equipment: $R=S=(I+Z)$. If demand expectations are confirmed, at the end of the period a part of profits (σR) will finance expansionary investment. To simplify, and following

[7] Sraffian prices of production require keeping prices and quantities apart in the base year. When this is not possible we shall use Leontief's prices that set all prices equal to one in the base period. Then, what becomes unknown is the physical measure of each commodity. $p_3=1$ might mean that 3 machines are worth one million euros.

Shaikh (2009) we can assume that it is financed with retained profits: $R_i=S_i=I$. The remaining part of profits is distributed to shareholders and will finance modernization investment via bank loans: $R_z=S_z=Z$.
The "financial lever" (σ) is positively related to the expected growth of demand. During the traverse it may differ across sectors. In the final equilibrium it will be uniform and coincide with the ratio between the warranted rate (at which autonomous demand and aggregate demand grow) and the uniform rate of profit.

[2] $$\sigma = \frac{R_i}{R} = \frac{I}{R} = \frac{I/K}{R/K} = \frac{g^*}{r^*}$$

The point to be emphasized at this moment is that firms are able to adapt investment flows to new paths of growth by raising the financial lever.

2.4 Aggregate demand and its endogeneization in a closed Leontief system.
In our original IOT only the first row of the transaction table has positive figures; they correspond to intermediate consumption and fixed capital consumption. The remaining industries produce goods for final demand (C, I, and Z). We can endogeneize them to get a *closed Leontief system*. Our expenditure assumptions simplify the task. Final consumption will be allocated in the cells of row 2 in proportion to the wages paid in each industry. In cell j we get: $C_j=C\cdot(W_j/W)$. Expansionary investment will be allocated in row 3 in proportion to the fixed capital allocated in each industry or, to what amounts to the same, in proportion to sectoral profits: $I_j = I\cdot(K_j/K)\cdot\sigma_j = I\cdot(R_j/R)\cdot\sigma_j$. The rest of value added in each industry will be allocated in row 4 which adds up to the value of modernization investment.
In the presentation of a growth model, expansionary investment requires special attention. According to the acceleration principle firms try to match expected demand in an efficient way, i.e. at normal capacity. They will expand capacity when they expect a permanent increase in demand. Expansionary investment in industry j can be computed by the difference between the required capacity for the next and future years ($KR_{j(t+1)}$) and the capacity installed from the beginning of year t ($KI_{(t)}$). The purchase of equipment at the end of period t will be[8]:

[3] $$I_{j(t)} = KR_{j(t+1)} - KI_{j(t)} = k_j \cdot g_{dj(t)} \cdot D_{jt} + KT_{j(t)}$$

The first part of the last expression corresponds to *ex-ante* or *planned* investment, i.e. to the production of capital goods from the first day of the period of production in order to match the expected demand for them at the end of (t). It will be captured by the supermultiplier that we shall study soon. In a fully adjusted path of growth, *ex-post* investment expenditure at the end of the period coincides with the planned one. In the general case we have to add the term $KT_{j(t)}$ which stands for the shortages of capacity in industry j in period t.

[8] $I_{j(t)} = KR_{j(t+1)}-KI_{j(t)} = k_j\cdot D_{j(t)}\cdot(1+g_d)-KI_{j(t)} = k_j\cdot D_{j(t)}\cdot g_{dj(t)}+[k_j\cdot D_{j(t)}-KI_{j(t)}] = k_j\cdot g_{dj(t)}\cdot D_{j(t)}+KT_{j(t)}$.

After dividing each column of the new "inter-industry" transactions table by total output we get matrix $\boldsymbol{A}_{ciz}$ whose columns add up to one. Each column j has four elements. The first one corresponds to the technical coefficient, the a_{1j} of $\boldsymbol{A}$. The other three elements correspond to the endogeneized final demand and can be computed in different ways.

[4] $$c_j = \left[c_w \cdot (w \cdot l_j)\right] = \omega_j$$

[5] $$i_j = \left[k_j \cdot g_{dj}\right] = \sigma_j \cdot \beta_j$$

[6] $$z_j = Z_j/q_j = \left(1-\sigma_j\right)\beta_j$$

The central terms (in square brackets) explain the determinants of each category of demand. Terms on the right indicate the funding of each type of demand. The share of induced consumption is the propensity to consume out of wages (c_w=1) times the unit wage paid in industry j ($w{\cdot}l_j$). Given our expenditure patterns it coincides with the share of wages (ω_j). The share of expansionary investment is the expected growth of demand times the optimal capital/output ratio ($g_{dj}{\cdot}k_j$). It is financed with a part of retained profits ($\sigma_j{\cdot}\beta_j$) [9]. The share of autonomous demand in industry j is the remaining part: $z_j = Z_j/q_j = 1-a_j-c_j-i_j$. In our example, autonomous demand is financed with bank loans that accrue from distributed profits $(1-\sigma_j)\beta_j$.

The different issues we are going to treat in this paper may require starting from different technological matrices. We already know the composition of: $\boldsymbol{A}$ and $\boldsymbol{A}_{(ciz)}$. Other variants result from the endogeneization of only a part of final demand. Each coefficient should bear two sub-indexes to show the origin and destiny of the flow. Since the first one is identified by the letter we only write the second one.

$\boldsymbol{A}_{(c)}$			
a_1	a_2	a_3	a_4
c_1	c_2	c_3	c_4
0	0	0	0
0	0	0	0

$\boldsymbol{A}_{(ci)}$			
a_1	a_2	a_3	a_4
c_1	c_2	c_3	c_4
i_1	i_2	i_3	i_4
0	0	0	0

$\boldsymbol{A}_{(cz)}$			
a_1	a_2	a_3	a_4
c_1	c_2	c_3	c_4
0	0	0	0
z_1	z_2	z_3	z_4

3. The dynamics of the economy as a vertically hyper-integrated sector

3.1 The tools: multiplier and supermultiplier

We are now prepared to compute the equilibrium level of output in a given period (t) by means of the multiplier and/or the supermultiplier applied to a given autonomous demand vector defined in a broad or narrow sense.

[9] In the general case, where shortages of capacity are possible, we get the following expression for *ex-post* investment after dividing [3] by total output: $i_{j(t)} = k_j{\cdot}g_{dj(t)}+u''_{j(t)}$. The last term derives from the following definition: $KT_{j(t)} = KR_{j(t)} - KI_{j(t)} = (KR_{j(t)}/KI_{j(t)}) -1 = u_{j(t)} -1 = u'_{j(t)}$. Now we divide by q_j to get $u''_{j(t)}$.

The multiplier of *total* output is the Leontief inverse of the extended matrix of coefficients ($\boldsymbol{A}_c$) that shows the coefficients of the social accounting matrix (SAM) resulting from the endogeneization of induced consumption[10].

[7] $$[\boldsymbol{\mu}] = [\boldsymbol{I} - \boldsymbol{A}_c]^{-1}$$

The supermultiplier of total output will be the Leontief inverse of an extended SAM that includes both induced consumption and expansionary investment. Notice that the supermultiplier is an expression of the vertically hyper-integrated sector corresponding to a (unit) vector of autonomous demand (VHIS-4 is producing one unit of output)[11].

[8] $$[\boldsymbol{\mu}^*] = [\boldsymbol{I} - \boldsymbol{A}_{ci}]^{-1}$$

The supermultiplier will help us to analyze the dynamics of the economy both in equilibrium and disequilibrium situations.

3.2 Fully adjusted path of growth.

Suppose that until the base year (0), the economy has been following its fully adjusted path of growth driven by modernization investment ($Z_{(o)}$) growing at g_z=g^*. If demand expectations (g_d, that are implicit in the supermultiplier) advance at the same rhythm, we can compute the output in any period (t) by the following equations.

[9]
$$\begin{aligned}
\boldsymbol{q}_{(1)} &= [\boldsymbol{\mu}^*] \cdot \boldsymbol{Z}_{(0)}(1 + g^*) \\
\boldsymbol{q}_{(2)} &= [\boldsymbol{\mu}^*] \cdot \boldsymbol{Z}_{(0)}(1 + g^*)^2 \\
&\dots \\
\boldsymbol{q}_{(t)} &= [\boldsymbol{\mu}^*] \cdot \boldsymbol{Z}_{(0)}(1 + g^*)^t
\end{aligned}$$

The warranted rate implies a steady expansion of industries. The shares in total output of intermediate consumption (a), final consumption (c), expansionary investment (i) and modernization investment (z) remain constant.

3.3 Traverse with perfect foresight.

Imagine now a twin economy where, from t=1 on, the autonomous trend rises to (g') > (g^*=$g_{z(0)}$). As a first approximation, let us make the heroic assumption that firms know about this change in advance and on the last day of period (0) install the equipment necessary to start efficiently the new path of growth. We can imagine that in (0) the Government passed a law obliging firms to accelerate the shift towards wind power

[10] Alternatively we can start from the closed Leontief system, at the unit level, and set to zero rows 3 and 4. Pre-multiplying [7] by the value added vector ($\boldsymbol{\nu}$) we get the *income* multiplier. It is a disaggregated presentation of the traditional Keynesian multiplier: $\mu = 1/(1\text{-}c_y) = 1/s_y$.

[11] Again, to obtain the supermultiplier of income we should pre-multiply this expression by $\boldsymbol{\nu}$. The concept of VHIS was introduced by Pasinetti (1986) and applied recently by Garbellini & Wirkierman, (2013) in their analysis of the labour market. It implies the endogeneization of expansionary investment. Our supermultiplier is broader because it also endogeneizes final consumption.

electricity and banks approved credit lines to finance the production and purchase of wind turbines. In addition we have to assume that the stock of inventories in industry 3 was large enough to cope with the extra demand for the new equipment required in all sectors to start a faster path of growth. The level of output in the twin economy during period (1) can be split into two subsystems. The main sector corresponds to the vertically hyperintegrated sector corresponding to autonomous demand (VHIS-4). It produces the wind turbines plus the intermediate goods directly and indirectly required, plus the consumption goods demanded by the workers directly and indirectly required, plus the equipment demanded by firms willing to grow at the expected rate. It produces $\boldsymbol{q}'_{(1)}$ from Monday to Friday using extra time during the night if necessary. At the weekends, the secondary sector produces the extra equipment demanded by firms to adjust, once and for all, to the new expected rate of growth. It amounts to $\boldsymbol{q}''_{(1)}$.

[10] $$\boldsymbol{q}_{(1)} = \boldsymbol{q}'_{(1)} + \boldsymbol{q}''_{(1)} = [\boldsymbol{\mu}^{*\prime}] \cdot \boldsymbol{Z}_{(\mathbf{0})}(1+g') + [\boldsymbol{\mu}] \cdot (b \cdot \boldsymbol{KT}_{(0)})$$

$[\boldsymbol{\mu}^{*\prime}]$ is the new supermultiplier, after introducing the new autonomous trend in the third row of matrix $\boldsymbol{A}_{(ci)}$: $i_j' = g' \cdot k_{dj}$. It multiplies the autonomous demand that is g' times larger than in the previous year. $\boldsymbol{KT}_{(0)}$ stands for the shortages of equipment that were filled from the stock of inventories and must now be reproduced, at least partially ($0 < b \leq 1$). The span of the adjustment will depend on this parameter. If b=0, because the stock of inventories had already been prepared for such shocks, the adjustment will be immediate. If b=1 the adjustment may require several periods when it is not possible to produce $KT_{(0)}$ in the 40 hours available each weekend of period 1. The production of $KT_{(0)}$ has the usual multiplier effects on consumption; not the acceleration effects on investment because firms are aware that this is a transient demand that does not require a permanent expansion of capacity.

3.4 Traverse without perfect foresight.

Consider, as a second more realistic approximation, that firms start period (1) with their traditional expectations of demand growth: $(g_{dj}=g_{z(0)}) < g'$; and the traditional stock of capital: $(KI_{(1)} = [KI_{(0)}+I_{(0)}]) < KR_{(1)}$. Output in period (1) will be:

[11] $$\boldsymbol{q}_{(1)} = [\boldsymbol{\mu}^{*}] \cdot \boldsymbol{Z}_{(\mathbf{0})}(1+g')$$

Autonomous demand is growing at the new (higher) rate, g'. But h expectations on demand growth, implicit in the supermultiplier, continue to be ($g_{dj}<g'$). Such a dis-adjustment obliges firms to use capacity during night hours ($u'_{(1)}$>1) and triggers two responses: (1) The need to expand capacity to adapt to the new path of growth; (b) The need to adjust the expected growth of demand if overutilization persists despite the increase in capacity above the level suggested by the pure acceleration mechanism.

Suppose that, in year 2, firms in sector j make $g_{dj(2)}=g'$. Their output can be computed then by the first part of the following expression that embeds a larger supermultiplier. In addition, firms in VIS-3 produce during the weekends the extra capacity required by firms willing to follow efficiently the new path of growth.

[12] $$\boldsymbol{q}_{(2)} = [\boldsymbol{\mu}^{*'}] \cdot \boldsymbol{Z}_{(\boldsymbol{0})}(1+g')^2 + [\boldsymbol{\mu}] \cdot \left(b \cdot \boldsymbol{KT}_{(1)}\right)$$

Cautious entrepreneurs will adjust g_{di} to g' in a smoother way. While $g_{dj}<g'$, firms are obliged to use capacity in night hours. This will encourage them to revise upwards their demand expectations. This is the important point highlighted by Allain (2013).

Quickly or slowly, the adjustment to the higher autonomous trend is bound to reshape the structure of the economy if it lasts long enough. In the final equilibrium, the share of industry 3 (K_3/K or q_3/q or I/q) increases at the expense of industry 4 that serves autonomous demand[12]. The first one amounts to: $i'_j = g' \cdot \Sigma k_j \cdot = \sigma' \cdot \beta_j$. A rise in the expected rate of growth is bound to raise the aggregate financial lever (up to σ') and the aggregate investment share (up to i'). A multisectoral model helps to visualize such structural changes. In the new equilibrium path, all industries grow at g'. During the traverse all of them accelerate investment to adapt to the new autonomous trend. But industry 3 (the producer of equipment) is the one which experiences the highest burst of demand, the one that bears the highest overutilization and, consequently, the one that expands faster and more.

The preceding structural adjustment alters the warranted rate of growth until it coincides with the autonomous trend. Originally all the economic variables (autonomous demand, aggregate demand, output and capital) were growing at the warranted rate: $g_z = g_d = g_y = g_k = g^* = i/k$. After the rise in the autonomous rate to g_z', the share of investment rises until $g^{*'} = i'/k = g_z'$. This allowed Serrano (1995) to state the endogeneity of the warranted rate in the sense that it adapts to the new autonomous trend despite constant technology and distribution. Here, in a disaggregated model, we have seen that, although the coefficients of each industry remain fixed, aggregate measures are altered after the structural movements of the traverse[13].

4. Stability and limits of a demand-led growth.

In this section we are going to analyze the limits on the autonomous trend imposed from the supply side and the conditions for stability of the dynamics we have just presented. For the first purpose it is convenient to develop a von Neumann type model. We start from the matrix $\boldsymbol{A}_{(cz)}$, after endogeneizing the elements of final demand that do not expand capacity. From this we obtain the matrix of capital coefficients in VIS terms: $\boldsymbol{k}^v_{(cz)} = \boldsymbol{k} \cdot (\boldsymbol{I} - \boldsymbol{A}_{(cz)})^{-1}$. As we know it only has positive numbers in row 3 (the equipment industry). The maximum eigenvalue of $\boldsymbol{k}^v_{(cz)}$ coincides with the capital coefficient of

[12] Notice the paradox. The traverse starts after an acceleration of the autonomous trend ($g'_z > g_z$) and concludes with a fall in the autonomous share ($z' < z$). As a matter of fact autonomous demand has grown and is growing faster than before. Yet, during the traverse, investment and output have increased even faster.

[13] If VIS-3 is capital intensive, the aggregate coefficient k will be higher when the autonomous trend rises from g_z to g'_z.

VIS-3: $k^v_{(cz)3}$. Its inverse gives the potential rate of growth for a given technology and expenditure patterns: g^*. It also gives the ratio "retained profits / total capital" that would ensure the expansion of the economy at g^* if retained profits are invested. The potential rate of growth could be computed in a macro model by the expression on the right hand side of the following equation[14].

[13] $$g^* = \frac{1}{k^v_{(cz)3}} = \frac{1-c_y-z_y}{k_y}$$

Consumption type expenditures can decrease to a certain threshold that corresponds to subsistence consumption (in a historical sense). This threshold is identified in our model with workers consumption appearing in the second row of $\boldsymbol{A}_{(cz)}$. On the contrary, autonomous demand, in row 4, can fall down to zero. The maximum rate of growth corresponding to this economy is the inverse of the maximum eigenvalue of matrix $\boldsymbol{k}^v_{(c)}$ associated to $\boldsymbol{A}_{(c)}$. Its macroeconomic equivalent is Harrod's warranted rate, computed for an economy made off induced consumption and expansionary investment

[14] $$\hat{g} = \frac{1}{k^v_{(c)3}} = \frac{1-c_y}{k_y}$$

More valuable information can be taken out from a von Neumann type model. The right-hand-side eigenvector corresponding to the $\boldsymbol{k}^v_{(cz)}$ and to scalar g^* is a column vector of quantities with this structure: (0; 0; 1; 0). The interpretation is the following one: since only q_3 is suitable for accumulation, the 100% of "surplus" is bound to appear in this commodity. When "1" is replaced by $i=g^* \cdot \Sigma k^v_{3j}$, the model yields matrix $\boldsymbol{A}_{(cz)}$. If we introduce a higher (exogenous) rate of growth ($g'>g^*$) the level of investment will increase at the expense of z_j that it is the only element of matrix $\boldsymbol{A}_{(cz)}$ than can fall. The limit to the autonomous trend is achieved when z=0, as indicated in equation [14].

By means of Brouwer's fixed point theorem, von Neumann proves that there is a unique solution to the eigenvalue system derived from matrix $A_{(cz)}$ (in our case, from matrix $k^v_{(cz)}$). It is a saddle point that attracts the variables in a typical *minimax* game. This equilibrium will be reached (and recovered after a shock) if we are in a viable and competitive system. Competition forces firms to produce at minimum costs that will determine relative prices. Simultaneously competition forces firms to produce as much as possible with given resources and technologies. Since there are no problems of effective demand in this economy, firms are supposed to invest the entire surplus (*Y-C-Z*).

The same conclusion is reached in a demand-constrained system, where (1) an exogenous expenditure, financed via credit, is growing at the rate g_z; (2) a proportion of the incomes generated in the process of production is systematically consumed either by firms (intermediate consumption) or by households (final consumption); and (3) firms invest to match the expected growth of demand (g_{dj}). If such an autonomous trend lasts

[14] The subindex "y" indicates that the variable has been divided by net income (instead of total output).

long enough, it will influence the expected growth of demand. When both rates coincide in all sectors the economy will achieve its fully adjusted path of growth. But, how do these forces operate? Cann they not destabilize the system as Harrod feared? Von Neumann does not have any answer for these questions.

White (2009) and Allain (2013) show that the key to stability lies in the formations of expectations about autonomous demand (g_z), sectoral demand (g_{dj}) and output (g_q). Although their models are different, both of them tackle the instability problems by fencing the values of the first order differential equation resulting from a multiplier-accelerator model. The economic rationale for such thresholds is not clear. Aware of this flaw, Allain (2013) is bound to conclude: “Of course, since it depends on the parameters, this solution to the Harrod knife-edge problem remains fragile. But it opens a door that has never been opened before”.

Our proof of stability, already advanced in sections 3.3 and 3.4, is based on the superposition of two independent subsystems each one stable on its own. For a time, at the weekends, VIS-3 produces the extra equipment required everywhere to grow efficiently at the new (higher) rate. This has the ordinary multiplier effects on household consumption. Not the acceleration effects on business investment because firms in VIS-3 are aware that this is a transient demand. Instability cannot arise in this subsystem based on the simple multiplier and cut off from the main economy.

The main system, governed by the supermultiplier, is represented by the vertically hyper-integrated sector corresponding to the autonomous demand (VHIS-4). From Monday to Friday it produces the goods demanded on a regular basis. This system is stable provided autonomous trend falls below its technological limit ($g'_z < (1/k^v_{(c)3})$. It will converge to the autonomous trend if it lasts long enough and if the expected growth of sectoral demand adjusts to the autonomous trend ($g_{di} \rightarrow g'_z$). A trial and error process is involved in the adjustment. At the beginning firms respond to the burst of demand operating capacity at night hours during the five working days (u'>1). Later, when they realize that the increases in demand are permanent, they send a once and for all order of equipment to the VIS-3 that operates at the weekends. This entails a new type of overutilization that does not require further investment. If overutilization in VHIS-4 persists despite the increases in capacity, firms will suspect that demand is growing faster and they will adjust their expectations upwards. When $g_{dj}=g'_z$ the main system resumes a fully adjusted path of growth. As soon as VIS-3 has produced the extra capacity, the dynamics of the economy can be reproduced by the supermultiplier.

5. Conclusions

This paper reinforces the Keynes-Kalecki principle of effective demand by extending it to the long period, the realm of growth theories. It reinforces the hypothesis that capitalism is a demand constrained system; that growth, in capitalist economies, is demand-led.

The dynamics of growth in a fully adjusted path and the traverse to a new one can be better understood using the multiplier and acceleration mechanisms in a multisectoral setting. The supermultiplier can be represented by a vertically hyper-integrated sector

(VHIS-4) associated to the industries producing goods for autonomous demand. It captures the impact of the expansion of these industries on the intermediate consumption of firms, final consumption of households and expansionary investment.

If the autonomous trend is above the expected growth of demand implicit in the supermultiplier, firms are bound to overuse capacity during night hours to match demand. If they perceive that the new trend is permanent, they will install more equipment taken from the stock of inventories that they will reproduce later (either totally or partially). For a better visualization of the adjustment process, we have imagined that the VIS-3 producing this extra equipment operates at the weekends. It has the usual multiplier effects on consumption. It does not accelerate investment, however, since firms are aware that this is a transient demand that does not justify permanent expansion of capacity.

The process ends when all the relevant variables grow at the autonomous trend, the independent variable of the dynamic system. This conveys structural changes in the composition of output shaped by the supermultiplier. Installed capacity grows everywhere but especially in the industry producing equipment goods which bore the highest increases in demand and in the rate of utilization. After the adjustment, the share of equipment in output (or investment in demand) will increase at the expense of the share of autonomous demand in output.

Contrary to the dominant opinion, we have emphasized the stability of a demand-led growth based on the multiplier-accelerator mechanism. The conditions required for stability are the following ones:

(1) The autonomous trend lies below the threshold compatible with technology and expenditure patterns. After a reformulation of von Neumann’s model we have identified it with the inverse of the maximum eigenvalue of the relevant capital coefficient matrix. It is equivalent to Harrod’s warranted rate determined at the macro level in an economy without proper autonomous demand.

(2) The autonomous trend is truly autonomous and lasts long enough to reshape the economy. This trend is not influenced by the ups and downs of income and the rate of utilization.

(3) The expected growth of sectoral demand is influenced by permanent exogenous pressures on capacity utilization. If overutilization persists despite new investments, firms will revise upwards their growth expectations. The adjustment is completed when the expected growth of sectoral demand coincides with the new autonomous trend and the share of investment in output is enough to sustain this rate of growth.

(4) Firm are able to distinguish permanent from transient increases in demand. The purchase of equipment, to adapt to a faster path of growth, belongs to the second group and does not accelerate investment. This justifies the isolation of VIS-3 from the main economy (VHIS-4).

(5) Firms react to capacity deviations in a sensible way, avoiding any form of overreaction. After an increase in the autonomous trend firms in all industries are supposed to invest just enough to fill the capacity gap; not more. All or part of the inventories used to fill this gap will be reproduced, not more.

(6) There are physical limits to the rate of capacity utilization that may delay the process of adjustment but render the system more stable. Such limits prevent the typical abrupt movements of the accelerator that could confuse investors.

Our paper does not preclude the stability of capitalist economies. Certainly they are not stable. We are simply suggesting that the instability we observe in real life does not arise from the multiplier – accelerator interaction but from the evolution of the autonomous trend. Part of the criticisms made against the supermultiplier could be answered by separating proper autonomous demand, from expansionary investment and from induced consumption. Palumbo & Trezzini (2003) are right when they state the impossibility of elaborating a closed theory to explain the dynamics of the autonomous trend, the locomotive of the economy. We can say, however, that the train's wagons will eventually circulate at the speed set by the locomotive. These wagons give due account of the induced demand captured by the multiplier and the accelerator, the bulk of total demand in modern economies.

APPENDIX

A) Input-output tables

(A1) Original IOT in the base year (period 0)

	1	2	3	4	C	I	Z	***Y***	***Q***	Sum
1	5	10	5	10	0	0	0	***0***	***30***	
2	0	0	0	0	30	0	0	***30***	***30***	
3	0	0	0	0	0	10	0	***10***	***10***	
4	0	0	0	0	0	0	30	***30***	***30***	
W	10	10	2	8						*30*
R	15	10	3	12						*40*
VA	***25***	***20***	***5***	***20***						***70***
Q	***30***	***30***	***10***	***30***						***100***
Sum					*30*	*10*	*30*	***70***	***100***	
L(0)	10	10	2	8						*30*
KI(0)	150	100	30	120						*400*
I(0)=kgq	3,75	2,5	0,75	3						*10*
KI(1)	153,75	102,5	30,75	123						*410*

L(0): labour employed in t0; *KI(0)*= fixed capital installed at the beginning of the period; it is being used at the normal rate; *I(0)*: investment per industries at the end of period (0). This allows to compute the capital installed at the beginning of next period: *KI(1)*.

(A2) Closed Leontief system

We endogeneize final consumption ($C_j=W_j$) and expansionary investment ($I_j=g_{dj}\cdot k_j\cdot q_j$). The remaining output corresponds to autonomous demand (*Z*) that is added to row 4. From this transaction matrix we shall derive later technological matrices corresponding to different levels of endogeneization.

	1	2	3	4	*Q*
1 (*T*)	5	10	5	10	30
2(*C*)	10	10	2	8	30
3(*I*)	3,75	2,5	0,75	3	10
4(*Z*)	11,25	7,5	2,25	9	30
Q	30	30	10	30	

B) Technology: direct and total coefficients from the supply side

$\boldsymbol{A=T\cdot<Q>^{-1}}$					$\boldsymbol{(I-A)^{-1}}$				
	0,16	0,33	0,5	0,33		1,2	0,4	0,6	0,4
	0	0	0	0		0	1	0	0
	0	0	0	0		0	0	1	0
	0	0	0	0		0	0	0	1
$\boldsymbol{v=VA\cdot<Q^{-1}>}$	0,83	0,66	0,5	0,66					
$\boldsymbol{l=L\cdot<Q^{-1}>}$	0,33	0,33	0,2	0,26	$\boldsymbol{l^v=l\cdot(I-A)^{-1}}$	0,4	0,46	0,4	0,4
$\boldsymbol{k=K\cdot<Q^{-1}>}$	5	3,33	3	4	$\boldsymbol{k^v=k\cdot(I-A)^{-1}}$	6	5,33	6	6

v: value added per unit of output in industries; ***l***: direct labour coefficients; ***k***: direct capital coefficients. $\boldsymbol{l}_v$: total labour coefficients (in vertically integrated sectors, VIS). $\boldsymbol{k}_v$: total capital coefficients in VIS.

C) Technology and maximum rates of growth in a von Neumann type of model
The following matrices derive from the closed Leontief system setting row 3 = 0

$\boldsymbol{A}_{(cz)}$				$(\boldsymbol{I}\text{-}\boldsymbol{A})^{-1}{}_{(cz)}$			
0,16	0,33	0,5	0,33	3,6	2,85	3	2,8
0,33	0,33	0,2	0,26	3	4,125	3	3
0	0	0	0	0	0	1	0
0,375	0,25	0,225	0,3	3	3	3	4

As above, we can derive from $A_{(cz)}$ the vector of capital coefficients in VIS terms: $\boldsymbol{k}^{v}{}_{(cz)}=\boldsymbol{k}\cdot\boldsymbol{IL}_{(cz)}$.
The inverse of the coefficient corresponding to sector 3 is the potential rate of growth of an economy with a given technology and expenditure patters: $g^{*}=1/k^{v}{}_{(cz)3}$ = 1/40 = 0,025.
From $\boldsymbol{A}_{(c)}$ and $\boldsymbol{k}^{v}{}_{(c)}$ (below) we derive the maximum rate of growth corresponding to a given technology and consumption pattern supposing that autonomous demand is negligible: g^ $=1/k^{v}{}_{(c)3}$ = 1/10 = 0,1.
From $\boldsymbol{A}$ and $\boldsymbol{k}^{v}$ (above) we derive the maximum rate of growth corresponding to a given technology, supposing that the entire surplus can be accumulated: g^^ $=1/k^{v}{}_{3}$=1/6=0,16.

D) Multiplier (after the endogeneization of final consumption)

$\boldsymbol{A}_{(c)}$				$(\boldsymbol{I}\text{-}\boldsymbol{A})^{-1}{}_{(c)}$ = M = total output multiplier			
0,16	0,33	0,5	0,33	1,5	0,75	0,9	0,7
0,33	0,33	0,2	0,26	0,75	1,875	0,75	0,75
0	0	0	0	0	0	1	0
0	0	0	0	0	0	0	1
				2,25	*2,625*	*2,65*	*2,45*
				income multiplier = $\boldsymbol{v}\cdot(\boldsymbol{I}\text{-}\boldsymbol{A})^{-1}{}_{(c)}$			
				1,75	1,875	1,75	1,75

E) Supermultiplier (after the endogeneization of final consumption and expansionary investment)

Supermultiplier = $\boldsymbol{SM} = (\boldsymbol{I}\text{-}\boldsymbol{A})^{-1}{}_{(ci)}$. Investment per unit of output (row 3 of $\boldsymbol{A}_{(ci)}$) and the supermultiplier will change with the expected growth of demand (it is indicated in brackets)

$\boldsymbol{A}_{(ci)}$ (g=0,025)				$\boldsymbol{SM}$ (g=0,025)			
0,17	0,33	0,50	0,33	1,80	1,05	1,20	1,00
0,33	0,33	0,20	0,27	1,00	2,13	1,00	1,00
0,13	0,08	0,08	0,10	0,33	0,33	1,33	0,33
0	0	0	0	0	0	0	1
				3,13	*3,51*	*3,53*	*3,33*
				supermultiplier of income: $\boldsymbol{v}\cdot(\boldsymbol{I}\text{-}\boldsymbol{A})^{-1}{}_{(ci)}$			
				2,33	2,46	2,33	2,33

$\boldsymbol{A}_{(ci)}$ (g=0,03)				$\boldsymbol{SM}$ (g=0,03)			
0,17	0,33	0,50	0,33	1,89	1,14	1,29	1,09
0,33	0,33	0,20	0,27	1,07	2,20	1,07	1,07
0,15	0,10	0,09	0,12	0,43	0,43	1,43	0,43
0	0	0	0	0	0	0	1
				3,38	*3,76*	*3,79*	*3,59*

$\boldsymbol{A}_{(ci)}$ (g=0,035)			
0,17	0,33	0,50	0,33
0,33	0,33	0,20	0,27
0,18	0,12	0,11	0,14
0	0	0	0

SM (g=0,035)			
1,98	1,23	1,38	1,18
1,15	2,28	1,15	1,15
0,54	0,54	1,54	0,54
0	0	0	1
3,68	*4,05*	*4,08*	*3,88*

$\boldsymbol{A}_{(ci)}$ (g=0,0175)			
0,17	0,33	0,50	0,33
0,33	0,33	0,20	0,27
0,09	0,06	0,05	0,07
0	0	0	0

SM (g=0,0175)			
1,69	0,94	1,09	0,89
0,91	2,03	0,91	0,91
0,21	0,21	1,21	0,21
0	0	0	1
2,81	*3,19*	*3,21*	*3,01*

$\boldsymbol{A}_{(ci)}$ (g=0,01)			
0,16	0,33	0,5	0,33
0,33	0,33	0,2	0,26
0,05	0,03	0,03	0,04
0	0	0	0

SM (g=0,01)			
1,6	0,85	1	0,8
0,83	1,95	0,83	0,83
0,11	0,11	1,11	0,11
0	0	0	1
2,54	*2,92*	*2,94*	*2,74*

F) Sectoral dynamics.

The temporal index is omitted in the following list of symbols. An apostrophe before the letter refers to the value in the previous period. Bold variables without sectoral sub-indexes refer to matrices or vectors.

- $Z_j = {}'Z_j\cdot(1+g_z)$: autonomous demand that fills the "multiplicand". It grows at rate g_z.
- $\boldsymbol{q=q'+q''}$ total output.
- $\boldsymbol{q'} = \boldsymbol{SM\cdot Z}$**:** total output of the VHIS-4 corresponding to autonomous demand. The key difference in the following tables refers to the expected growth of demand (g_d) embedded in the supermultiplier (*SM*)
- $\boldsymbol{q''} = \boldsymbol{M}\cdot(\boldsymbol{'KT})$: total output of VIS-3 (to produce the shortages of capacity, *KT*). (***M*** is the income multiplier).
- q_j/q = share of sectoral output in total output. When $g_d=g_z$, the supermultiplier renders the equilibrium composition of output in VHIS-4.
- $KR_j = k_j\cdot q_j$: required capital
- $KI_j = ('KI_j+{}'KT_j)+{}'I_j = ('KI_j+{}'KT_j)\cdot(1+g_d)$: installed capacity at the beginning of period t.
- $KT_j = KR_j-KI_j$.: shortages of capacity. All or part of them will be reproduced in one or several periods with the usual multiplier effects
- $u'_j= KR_j/KI_j$ capacity utilization rate in the main economy (VIS-4).
- $u_j = (q'_j+q''_j)/q^*_j$: Capacity utilization rate for the entire economy. It includes the complementary VIS-3 that operates at the weekends to produce q''. Here $q^*_j = KI_j/k_j$: capacity output.
- g_z: autonomous trend; g_{dj}: expected growth of demand in each industry j;

(F1) Fully adjusted path of growth ($g_z = g_d = g^* = 0{,}025$)

	Z	*q'*	*(q'j/q)*	*KR*	*KI*	*KT*	*u'*	*Z*	*q'*	*(q'j/q)*	*KR*	*KI*	*KT*	*u'*
	t1: g_z=0,025; g_d=0,025							t2: g_z=0,025; g_d=0,025						
Ind-1	0	30,75	0,3	153,8	153,8	0	1	0	31,5	0,3	157,6	157,6	0	1
Ind-2	0	30,75	0,3	102,5	102,5	0	1	0	31,5	0,3	105,1	105,1	0	1
Ind-3	0	10,25	0,1	30,7	30,7	0	1	0	10,5	0,1	31,52	31,52	0	1
Ind-4	30,7	30,7	0,3	123,0	123,0	0	1	31,5	31,5	0,3	126,1	126,1	0	1
Ag	30,7	102,5	1	410,0	410,0	0	1	31,5	105,1	1	420,3	420,3	0	1

The SM ensures that all industries expand proportionally and with normal capacity utilization (*u*=1; *KT*=0). This is steady growth: $g_{dj} = g_z = g^* = 0{,}025$. The equilibrium shares are: 0,3; 0,3; 0,1; 0,3. They remain constant over time.

(F2) Traverse with perfect foresight (g_z = 0,025 → 0,035; g_d = 0,025 → 0,035)

	Z	*q'*	*q'j/q*	*KR*	*KI*	*KT*	*u'*	*q"*	*Z*	*q'*	*q'j/q*	*KR*	*KI*	*KT*	*u*	*q"*
	t1: g_z=0,035; g_d=0,035								t2: g_z=0,035; g_d=0,035							
1	0	36,8	0,305	183,9	153,7	30,2	1,20	38	0	38,1	0,305	190,3	190,3	0,0	1	38
2	0	35,8	0,297	119,4	102,5	16,9	1,17	35	0	37,1	0,297	123,6	123,6	0,0	1	35
3	0	16,7	0,139	50,16	30,7	19,5	1,63	10	0	17,3	0,139	51,9	51,9	0,0	1	10
4	31	31,1	0,258	124,2	123,0	1,2	1,01	1	32,1	32,1	0,258	128,5	128,5	0,0	1	1
Ag	31	120,4	1	477,7	410,0	67,7	1,17	83	32,1	124,6	1	494,4	494,4	0,0	1	83

At the end of t0 firms are informed that autonomous demand (and the economy as a whole) will grow faster (at g'_z=0,035). They purchase the equipment required to enter in the new path of growth efficiently. This equipment is taken from the stock of inventories and is reproduced in the future: $q''=M{\cdot}KT$= (76, 70, 20, 2). Half of it is reproduced in the weekends of t1; the other half in t2. The overuse of capacity is higher in industry 3, justifying a faster expansion. The equilibrium share of output is already achieved in t1 because $g_{dj}=g_z$. The share of industry 3 rises at the expense of 4 (from 0,1 to 0,139). The fact that industry 3 is material intensive, causes a small increase in the share of industry 1 (from 0,3 to 0,305). Capacity utilization returns to 1 in the main economy (VHIS-4) in t2 (*u'*=1). In t3 there would be no extra-production at the weekends (*'KT*=0; *q"*=0) so the rate of capacity utilization for the whole economy would be the normal one (*u*=1)

(F3) Traverse without perfect foresight (g_z = 0,025 → 0,035; g_d = 0,025 → 0,030 →0,035)

	Z	*q'*	*q'j/q*	*KR*	*KI*	*KT*	*u'*	*q"*	*Z*	*q'*	*q'j/q*	*KR*	*KI*	*KT*	*u'*	*q"*
	t1: g_z=0,035; g_d=0,025								t2: g_z=0,035; g_d=0,03							
1	0	31,1	0,3	155,3	153,8	1,5	1,01	0	0	34,9	0,303	174,5	159,9	14,5	1,09	4
2	0	31,1	0,3	103,5	102,5	1,0	1,01	0	0	34,4	0,299	114,8	106,6	8,2	1,08	4
3	0	10,4	0,1	31,1	30,8	0,3	1,01	0	0	13,8	0,120	41,3	32,9	9,3	1,29	0
4	31	31,1	0,3	124,2	123,0	1,2	1,01	0	32,1	32,1	0,279	128,5	127,9	0,6	1,00	1
	31	103	1	414	410	4,0	1,01	0	32,1	115	1	459,1	426,4	32,7	1,08	10
	t3: g_z=0,035; g_d=0,035								t4: g_z=0,035; g_d=0,035							
1	0	39,4	0,306	197,0	180,6	16,4	1,09	37	0	40,8	0,306	203,9	203,9	0	1	41
2	0	38,4	0,298	127,9	118,8	9,1	1,08	34	0	39,7	0,298	132,4	132,4	0	1	38
3	0	17,9	0,139	53,7	42,8	11,0	1,26	9	0	18,5	0,139	55,6	55,6	0	1	11
4	33,3	33,3	0,258	133,0	133,0	0,0	1,00	1	34	34,4	0,258	137,7	137,7	0	1	0
	33,3	129	1	512	475	36,6	1,08	80	34	133	1	530	530	0	1	90

In t1 innovative entrepreneurs of industry 4 start growing at g_z'=0,035 and continue doing so in the future. Firms in other industries don't realize the change at once. In t1 their demand expectations incorporated in the supermultiplier continue to be the previous ones (g=0,025). The excess of capacity in t1 encourages firms to revise their demand expectations upwards step by step. In t2, g_d=0,03; in t3, g_d=0,035. Probably the revision will start later because, as suggested in the main text, firms revise their expectations upwards when they realize that overcapacity persists despite their investment efforts.

(F4) Traverse towards a lower autonomous trend (g_z =0,025 → g'_z= 0,01; g_d= 0,025 → 0,0175 → 0,01)

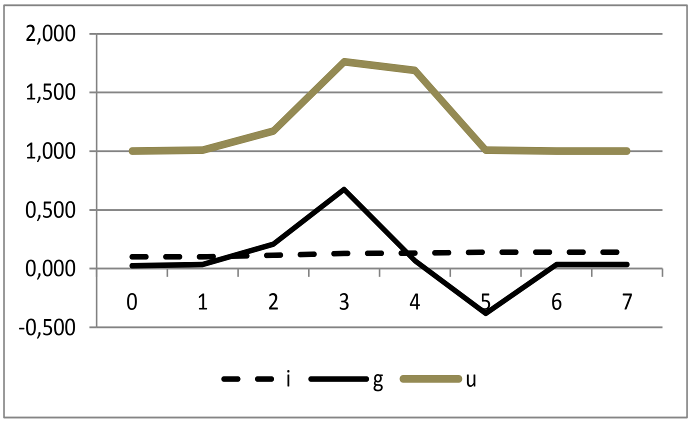
	Z	*q'*	*q'ⱼ/q*	*KR*	*KI*	*KT*	*u'*	*Z*	*q'*	*q'ⱼ/q*	*KR*	*KI*	*KT*	*u'*
t1:				g_z=0,01; g_d=0,025				t2:	g_z=0,01; g_d=0,0175					
Ind-1	0	30,3	0,3	151,5	153,8	-2,3	0,98	0	27,3	0,296	136,3	154,2	-17,8	0,88
Ind-2	0	30,3	0,3	101,0	102,5	-1,5	0,98	0	27,8	0,302	92,7	102,8	-10,0	0,90
Ind-3	0	10,1	0,1	30,3	30,8	-0,5	0,98	0	6,5	0,070	19,5	30,8	-11,4	0,63
Ind-4	30,3	30,3	0,3	121,2	123,0	-1,8	0,98	30,6	30,6	0,332	122,4	123,0	-0,91	0,99
Ag	30,3	101	1	404,0	410,0	-6,0	0,98	30,6	92,2	1	370,9	408,0	-40,1	0,91
t3:		g_z=0,01; g_d=0,01						t4:	g_z=0,01; g_d=0,01					
Ind-1	0	24,7	0,291	123,6	137,7	-14,0	0,90	0	25,0	0,291	124,9	124,9	0	1
Ind-2	0	25,8	0,304	85,9	93,7	-7,8	0,92	0	26,0	0,304	86,7	86,7	0	1
Ind-3	0	3,4	0,040	10,3	19,7	-9,4	0,52	0	3,50	0,040	10,4	10,4	0	1
Ind-4	30,9	30,9	0,364	123,6	123,6	0	1	31,2	31,2	0,364	124,9	124,9	0	1
Ag	30,9	84,8	1	343,4	374,7	31,2	0,92	31,2	85,7	1,010	346,9	346,9	0	1

The accelerator is an asymmetric mechanism. The adjustment is easier downwards. The excesses of capacity resulting from a fall in the autonomous trend become idle capacity. As a result, *ex-post* investment will be lower than the one derived from the pure accelerator (planned investment). In the previous cases we added to planned investment the shortages of capacity (positive *KT*) which compels firms to reproduce part or all of depleted inventories. Now (after a fall in g_z) we subtract from planned investment the excesses of capacity (negative *KT*). Yet "negative production" is not possible. If underutilization persists despite the investment cuts, firms will reduce their expectations on demand growth. Here, in period 2 they fall from 0,025 to 0,0175. They are cut again in period 3 down to 0,01. Once g_{dj}=g'=0,01, the shares of industries in VHIS-4 get their equilibrium position.

G) Aggregate dynamics (*g = (q-'q)/'q) = growth of total output*)

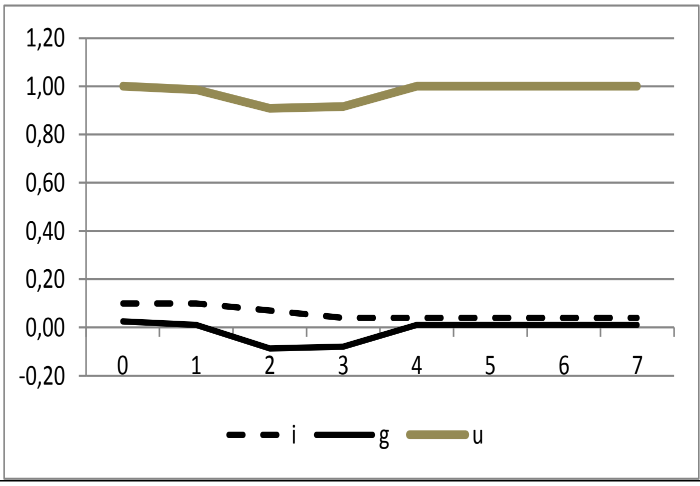
(G1) Upwards adjustment (g_z = 0,025 → 0,035; g_d → 0,025 → 0,03 → 0,035)

	Z	*q*	*i*	*u*	*g*
0	30,0	100,0	0,10	1,00	0,025
1	31,1	103,5	0,10	1,01	0,035
2	32,1	125,0	0,11	1,17	0,207
3	33,3	209,4	0,13	1,76	0,676
4	34,4	223,5	0,13	1,69	0,067
5	35,6	138,1	0,14	1,01	-0,382
6	36,9	143,0	0,14	1,00	0,035
7	38,2	148,0	0,14	1,00	0,035



(G2) Downwards adjustment (g_z = 0,025 → 0,01; g_d = 0,025 → 0,0175 → 0,01)

	Z	*q*	*i*	*u*	*g*
0	30,0	100,0	0,10	1,00	0,025
1	30,3	101,0	0,10	0,99	0,010
2	30,6	92,2	0,07	0,91	-0,087
3	30,9	84,8	0,04	0,92	-0,080
4	31,2	85,7	0,04	1,00	0,010
5	31,5	86,5	0,04	1,00	0,010
6	31,8	87,4	0,04	1,00	0,010
7	32,2	88,3	0,04	1,00	0,010



All the variables refer to the total output of the entire economy: (VHIS-4) + (VIS-3). g_d adjusts step by step as in F3 and F4. We observe that the downward adjustment is faster because it is not necessary to reproduce the commodities filling (negative) shortages of capacity.

References

Allain O. (2013). Tackling the Instability of Growth: A Kaleckian Model with Autonomous Demand Expenditures. *Documents de Travail du Centre d'Economie de la Sorbonne*, 2013-26, 1-22.

Bortis H. (1997). *Institutions, Behaviour and Economic Theory*. Cambridge, UK: Cambridge University Press.

Cesaratto S. (2012). Neo-Kaleckian and Sraffian Controversies on Accumulation Theory. *Quaderni del Dipartimento di Economia Politica e Statistica (Università degli Studi di Siena)* 650, 1-39.

Ciccone R. (1986). Accumulation and Capacity Utilization: some Critical Considerations on Joan Robinson's Theory of Distribution. *Political Economy, Studies in the Surplus Approach*, 2, 17-36.

Ciccone R. (1987). Accumulation, Capacity Utilization and distribution: a Reply. *Political Economy: Studies in the Surplus Approach*, 3:1, 97-111.

Dejuán Ó. (2005). Paths of Accumulation and Growth: towards a Keynesian Long-Period Theory of Output. *Review of Political Economy*, 17:2, 231-252.

Dejuán Ó. (2013). Normal paths of growth shaped by the supermultiplier. In E.S. Levrero, A. Palumbo and A. Stirati (eds), *Sraffa and the Reconstruction of Economic Theory, Volume Two: Aggregate Demand, Policy Analysis and Growth*, 139-157, New York: Palgrave-MacMillan.

Fel'dman G. A. (1928 [1964). *Foundations of Soviet Strategy for Economic Growth*. Bloomington, Indiana: Spulber.

Freitas F. and Dweck E. (2013). The Pattern of Economic Growth of the Brazilian Economy 1970-2005: A Demand-led Growth Perspective. In E.S. Levrero, A. Palumbo and A. Stirati (eds), *Sraffa and the Reconstruction of Economic Theory, Volume Two: Aggregate Demand, Policy Analysis and Growth*, New York: Palgrave-Macmillan.

Garbellini N. and Wirkierman A. (2013). Productivity Accounting in Vertically (Hyper)integrated Terms: Bridging the Gap between Theory and Empirics. *Metroeconomica*, 65:1, 154-190.

Harrod R. F. (1939). An Essay in Dynamic Theory. *Economic Journal*, 49, 14-33.

Hicks J. R. (1950). *A Contribution to the Trade Cycle*. Oxford: The Clarendon Press.

Kurz H. D. (1986). 'Normal' Positions and Capital Utilization. *Political Economy. Studies in the Surplus Approach*, 2:1, 37-54.

Lavoie M. (2010). Surveying Short-run and Long-run Stability Issues with the Kaleckian Model of Growth. In M. Setterfield (ed.), *Handbook of Alternative Theories of Economic Growth*. Cheltenham, UK: Edward Elgar.

Lavoie M. (2013). Convergence towards the Normal Rate of Capacity Utilization in Kaleckian Models: the Role of Non-capacity Creating Autonomous Expenditures. http://www.boeckler.de/pdf/v_2013_10_24_lavoie.pdf1-11.

Leontief W. (1970 [1986]). The Dynamic Inverse. In W. Leontief (ed.), *Input-Output Economics*. Oxford: Oxford University Press.

Löwe A. (1976). *The Path of Economic Growth*. Cambridge, UK: Cambridge University Press.

Neumann J. v. (1945-46). A Model of General Equilibrium. *Review of Economic Studies*, 13, 1-9.

Palumbo A. and Trezzini A. (2003). Growth Without Normal Capacity Utilization. *European Journal of the History of Economic Thought*, 10:1, 109-135.

Pasinetti L. L. (1988). Growing Subsystems, Vertically Hyperintegrated Sectors and the Labour Theory of Value. *Cambridge Journal of Economics*, 12, 125-134.

Schumpeter J. (1912). *Theorie der Wirtschaftlichen Entwicklung (The Theory of Economic Development)*. Leipzig: Verlag Dunker & Humbolt.

Serrano F. (1995). Long Period Effective Demand and the Sraffian Supermultiplier. *Contributions to Political Economy*, 14, 67-90.

Shaikh A. (2009). Economic Policy in a Growth Context: a Classical Synthesis of Keynes and Harrod. *Metroeconomica*, 60:3, 455-494.

Smith M. (2013). A Historical Approach to Demand-led Growth Theory. In E.S. Levrero, A. Palumbo and A. Stirati (eds), *Sraffa and the Reconstruction of Economic Theory, Volume Two: Agregate Demand, Policy Analysis and Growth*, 120-138.

Sraffa P. (1960). *Production of Commodities by Means of Commodities. Prelude to a Critique of Economic Theory*. Cambridge: Cambridge University Press.

White G. (2006). Demand-led Growth and the Classical Approach to Value and Distribution: Are They Compatible? In N. Salvadori (ed.), *Economic Growth and Distribution. On the Nature and Causes of the Wealth of Nations*, 148-178. Cheltenham, UK: Edward Elgar.

White G. (2009). Demand-led Growth in a Multi-commodity Model with Learning: Some Preliminary Results. *The University of Sydney. Economics Working Paper Series*, 1, 1-35.

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